

PROBLEM SET 2 FOR MTH 6151

1. Find the general solutions (by any method) to:

(i) $U_x - 2U_t = 0$.

(ii) $\pi U_x + U_y = 0$.

Here, and in the next question $\pi = 3.1415\dots$

2. Find the solutions (by any method) to:

(i) $U_x - 2U_t = 0$, $U(0, t) = \cos t$. Explain what type of problem is this (initial/boundary value)?

(ii) $\pi U_x + U_y = 0$, $U(x, 0) = x^2$. Explain what type of problem is this (initial/boundary value)?

3. Consider the PDE $U_x + U_t = 1 + e^t$.

(i) Verify that $x + e^t$ is a solution to this PDE.

(ii) Find the general solutions to this inhomogeneous PDE. (Hint: Use the principle of superposition)

4. Use the change of coordinates

$$\tilde{x}(x, y) = ax + by,$$

$$\tilde{y}(x, y) = bx - ay.$$

seen in the lectures to solve the constant coefficients pde

$$aU_x + bU_y + cU = 0.$$

5. Use the method of change of coordinates to solve the equation

$$U_x + 2U_y + (2x - y)U = 2x^2 + 3xy - 2y^2.$$

Hint: look at Problem 4 to figure out the right change of coordinates.

6. Use the method of change of coordinates to solve the equation

$$U_x + U_y + U = e^{x+2y}.$$

Hint: look at Problem 4 to figure out the right change of coordinates.

7. Solve the initial value problem

$$\begin{cases} U_x - U_t - U = 0 \\ U(x, 0) = 2. \end{cases}$$

8. Use the method of characteristics to find the general solutions for the pde

$$U_x + U_y = 1.$$

9. Use the method of characteristics to solve the boundary value problem

$$xU_t - tU_x = 1,$$

$$U(0, t) = 0.$$