Queen Mary University of London

Main Examination period 2022 – May/June – Semester B

MTH6112: Actuarial Financial Engineering

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment:

- **You may use books and notes.**
- **You may use calculators and computers, but you must show your working for any calculations you do.**
- **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
- **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. To be eligible for IFoA exemption, **your must submit your exam within the first 3 hours of the assessment period**.

Examiners: C. Sutton, L. Fang

Question 1 [18 marks]. Sam is considering an investment in the shares of Derby Industries Inc but does not know much about the company. In a personal finance blog Sam reads that the author believes the current Derby Industries share price of $23 per share accurately reflects the company's balance sheet and cashflow. Therefore Sam uses a stochastic process, X_t to model the share price where t is measured in days. The model used is

$$X_t = 23 + W_t$$

where W_t is a standard Wiener Process.

(a) Find the value of $\mathbb{E}[X_t]$ for $t \geq 0$. **[2]**

(b) Determine whether X_t is a martingale with respect to its natural filtration. **[4]**

(c) What is the probability that the share price will be between $22 and $24 after 5 days under this model? **[4]**

(d) What are the weaknesses of this model when evaluating the risk of investing in Derby Industries shares? **[8]**

Question 2 [22 marks]. Consider the following stochastic differential equation

$$dX_t = Y_t dt + zW_t$$

where Y_t is a stochastic process, z is a constant and W_t represents standard Brownian Motion.

(a) Write down Ito's lemma for $f(X_t, t)$ where f is a suitable function. **[3]**

(b) Determine $df(X_t, t)$ where $f(X_t, t) = e^{2tX_t}$, simplifying your answer where possible. **[3]**

(c) If Y_t is replaced with a constant y, what function $g(X_t, t)$ is required such that the application of Ito's lemma leads to $g(X_t, t)$ representing Geometric Brownian Motion? **[1]**

(d) State the probability distribution that X_t follows under the function in (c) above. **[3]**

(e) If $y = 0.052, z = 0.149$ and $X_0 = 150$, find a 95% confidence interval for X_{15}. **[5]**

(f) If X_t is to be used to model the value of an equity portfolio, how realistic is it to use a constant y rather than a stochastic process Y_t? **[7]**

Question 3 [15 marks]. A European call option, with value c_t at time t, is written on a non-dividend paying stock, with price S_t at time t. The call option matures at time T and the strike price is K. The continuously compounded risk-free rate is r.

A portfolio contains one call option and $Ke^{-(T-t)r}$ cash.

(a) Prove that, at time T, the value of the portfolio will always be greater than or equal to the value of the share, S_T. **[3]**

(b) State the upper and lower bound for the value of the call option, c_t. **[4]**

The prices of a stock follow a geometric Brownian motion with parameters $\mu = 0.3$ and $\sigma = 0.2$. Presently, the stock's price is £50. Consider a call option having nine months until its expiration time and having a strike price of £45.

(c) What is the probability that the call option will be exercised? **[4]**

(d) If the interest rate is 3%, find the price of the call option using the Black-Scholes formula. **[4]**

Question 4 [25 marks]. A short rate of interest is governed by the Vasicek model, i.e.

$$dr_t = -a(r_t - \mu)dt + \sigma dB_t$$

where B_t is a standard Brownian motion and $a, \mu > 0$ are constants.

A stochastic process $\{X_t : t \geq 0\}$ is defined by $X_t = e^{at+b} \cdot r_t$, where b is a constant.

(a) Derive an equation for dX_t. **[5]**

(b) Solve the equation to find X_t. **[5]**

(c) Prove that:

$$r_t = \mu + (r_0 - \mu)e^{-at} + \sigma \int_0^t e^{a(s-t)} dB_s.$$

[5]

(d) Determine the probability distribution of r_t and the limiting distribution for large t. **[5]**

(e) Derive, in the case where $s < t$, the conditional expectation $E[r_t | F_s]$, where $\{F_s : s \geq 0\}$ is the filtration generated by the Brownian motion B_s. **[5]**

Question 5 [20 marks]. A credit analyst wishes to model the probability that a bond issued by a shipping company defaults, using a time varying default transition intensity $\lambda(t)$.

(a) Draw diagram representing a two-state model that could be used in this scenario. [**2**]

The credit analyst uses a quadratic form for the default transition intensity

$$\lambda(t) = \frac{2 + 10t - t^2}{300}$$

where $1 \leq t \leq 10$.

(b) Calculate the probability that the bond does not default between times 2 and 6. [**6**]

(c) Explain how the model could be modified to allow for the default transition intensity to depend on economic growth as well as time. [**6**]

A credit rating agency has given the bond a B rating.

(d) Explain how the two-state model could be extended to use information from the credit rating agency including how default transition intensities are estimated. [**6**]

End of Paper.