CWK 6 Solutions

7.1.3 $\dot{r} = r(1-r^2)(4-r^2) \quad \dot{\theta} = 2 - r^2.$

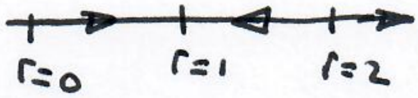
So $\dot{r} > 0 \; - \; 0 < r < 1$

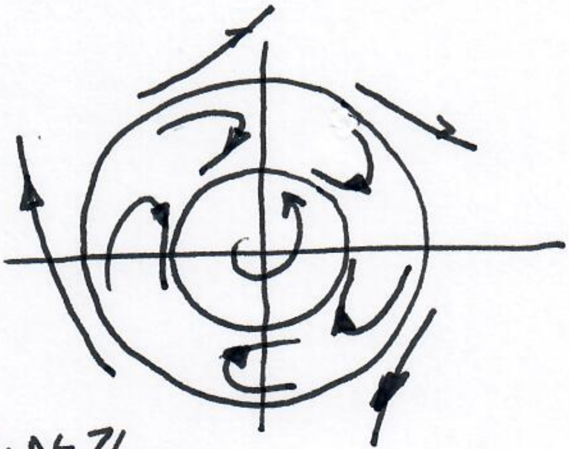
$\dot{r} = 0 \; - \; r = 1$

$\dot{r} < 0 \; - \; 1 < r < 2$

$\dot{r} = 0 \; - \; r = 2$

$\dot{r} < 0 \; - \; r > 2$





7.1.4. $\dot{r} = 0$, $r = 0$, $r = n\pi$, $n \in \mathbb{Z}$

$\dot{r}$:

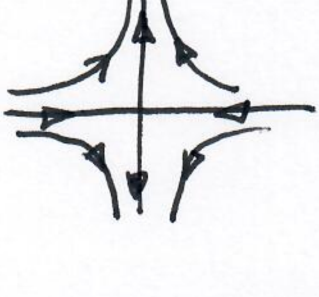
, $\dot{\theta} = 1$ (anti-clockwise)

$\therefore$ Phase portrait is a set of limit cycles $r = n\pi$ with an unstable spiral at $r=0$ and stable/unstable spirals consecutively

7.2.1 $V = x^2 + y^2$, $\dot{x} = -\dfrac{\partial V}{\partial x} = -2x$, $\dot{y} = -\dfrac{\partial V}{\partial y} = -2y$

$\dot{x} = -2x$, $\dot{y} = -2y$ is a stable star phase portrait

7.2.2. $V = x^2 - y^2$ $\dot{x} = -2x$, $\dot{y} = 2y$, this is a set of hyperbolic trajectories at a saddle with stable manifold x-axis, and unstable manifold, y-axis



7.2.10 Let $\dot{x} = y - x^3$, $\dot{y} = -x - y^3$ & $L = ax^2 + by^2$

$$\dot{L} = \frac{dL}{dt} = \frac{\partial L}{\partial x}\cdot\dot{x} + \frac{\partial L}{\partial y}\cdot\dot{y} = 2ax(y - x^3) + 2by(-x - y^3)$$

$$= 2(a-b)xy - 2ax^4 - 2by^4 \quad \text{Let } a = b = 1$$

then $\dot{L} = -2x^4 - 2y^4$, N Definite & L P. definite

$\therefore$ The trajectories globally attract to $L = 0$ i.e. $x = y = 0$

& there are no ~~...~~ closed orbits.

7.2.12 Let $\dot{x} = -x + 2y^3 - 2y^4$ and $\dot{y} = -x - y + xy$

Let $L = x^m + ay^n$, for $a > 0$ & m, n integers to be chosen.

$$\dot{L} = mx^{m-1}(-x + 2y^3 - 2y^4) + nay^{n-1}(-x - y + xy)$$

$$= -mx^m + 2mx^{m-1}y^3 - mx^{m-1}y^4$$

$$- naxy^{n-1} - nay^n + naxy^n$$

So can we choose a, b both positive, retain terms ($\checkmark$)

and lose terms ($\times$). Observation gives for

$n = 4$ & $m = 2$

$$\dot{L} = -2x^2 - 4ay^4 + (na - 2m)xy^4 + (2m - na)xy^3$$

Choose $a = 2m/n = 1$ to get

$$\dot{L} = -2x^2 - 4y^4 \text{ (ND) } \& \quad L = x^2 + y^4 \text{ (PD}$$

No periodic trajectories.