MTH6140 Linear Algebra II

Coursework 2

1. Let V be a finitely generated vector space, and let $B \subseteq V$ be a finite list of vectors. Show that B is a basis of V if, and only if, B is a maximal linearly independent list.

2. Let U and W be subspaces of the vector space V. Show that their sum

$$U + W = \{u + w : u \in U \text{ and } w \in W\}$$

is a subspace of V containing U and W.

3. Let $V = \mathbb{R}^2$, and let $\mathbf{v}_1 = (\alpha, 0), \mathbf{v}_2 = (0, \beta)$. When is $(\mathbf{v}_1, \mathbf{v}_2)$ a basis of V? Please justify your answer.

4. Let X, Y, D be subspaces of the real vector space $V = \mathbb{R}^2$ given by

$$X := \{(x, 0) : x \in \mathbb{R}\},$$
$$Y := \{(0, y) : y \in \mathbb{R}\},$$
$$D := \{(x, x) : x \in \mathbb{R}\}.$$

Show that $V = X \oplus Y = X \oplus D = Y \oplus D$.