MTH5129 Probability & Statistics II

Coursework 4

1. Suppose X and Y are jointly continuous random variables with joint density function

$$f_{X,Y}(x, y) = ce^{-|x|-|y|}, \quad \forall\ (x, y) \in \mathbb{R}^2$$

   Find whether X and Y are independent.

   (c is a constant chosen to make $f_{X,Y}$ a density function).

2. Suppose that X and Y are two random variables.

   a) Prove that if X and Y are independent then

$$E(X^k Y^m) = E(X^k)\, E(Y^m),$$

   b) Deduce that if X and Y are independent then $\mathrm{Corr}(X, Y) = 0$.

3. Suppose we choose independently X and Y to be two $Uniform(0, 1)$ random variables. Use their **convolution** to find the probability density function of their sum $Z = X + Y$.

4. Let X have the probability density function

$$f_X(x) = \begin{cases} 4x^3, & \text{for } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

   Find the probability density function of $Y = 2X - 1$ using the cumulative distribution function method.

5. Suppose X and Y are two independent $Uniform(0, 1)$ random variables. Use the **cumulative distribution function method** to find the probability density function of their sum $U = X + Y$.