Coursework 3

1. Suppose X and Y have joint density function

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-x-y} & \text{if } x > y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

   Calculate the conditional density function $f_{Y|X=x}(y)$.
   In lectures we calculated the $f_{X|Y=y}$.

2. Suppose that X, Y, Z are random variables. If $a \in \mathbb{R}$, prove that

   a) $\mathrm{Cov}(aX, Y) = \mathrm{Cov}(X, aY) = a\,\mathrm{Cov}(X, Y)$

   b) $\mathrm{Cov}(a + X, Y) = \mathrm{Cov}(X, a + Y) = \mathrm{Cov}(X, Y)$

   c) $\mathrm{Cov}(X, Y + Z) = \mathrm{Cov}(X, Y) + \mathrm{Cov}(X, Z)$

3. Suppose we throw a dice. Define the events A to be "the outcome is an odd number", B to be "the outcome is 2", C to be "the outcome is either a 5 or a 6".

   a) What is the sample space Ω?

   b) Are the events A and B independent?

   c) Are the events A and C independent?

   d) Are the events A,B and C independent?

4. Suppose that X, Y are discrete, independent random variables. You are reminded that this means that for any x, y from the range of X and Y

$$P(X = x,\ Y = y) = P(X = x) \times P(Y = y).$$

   Find $E(XY \mid Y = y)$.

5. Suppose X and Y have joint density function

$$f_{X,Y}(x,y) = \begin{cases} e^{-x-y} & \text{if } x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

   Find whether X and Y are independent.