MTH5129 Probability & Statistics II

Coursework 2

1. Suppose that X and Y have joint density function $f_{X,Y}$ given by

$$f_{X,Y}(x,y) = \begin{cases} ce^{-2x-3y} & \text{if } x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

   a) Use the properties of the joint density function to show that $c = 6$.

   b) Find the following probabilities

      i. $P(5 < X < 6 \text{ and } 7 < Y < 9\}$

      ii. $P(-2 < X < 3 \text{ and } 7 < Y < 9\}$

      iii. $P(X > Y > 0\}$

   c) Find the marginal densities f_X and f_Y.

2. Suppose that X and Y have joint density function $f_{X,Y}$ given by

$$f_{X,Y}(x,y) = \begin{cases} ce^{-2x-3y} & \text{if } y > x > 0 \\ 0 & \text{otherwise} \end{cases}$$

   where c is a constant.

   a) Show that $c = 15$.

   b) Find the probabilities

      i. $P(2 < X < 3 \text{ and } 7 < Y < 9\}$

      ii. $P(5 < X < 6 \text{ and } 5 < Y < 6\}$

      iii. $P(X > 2Y > 0\}$

      iv. $P(X > 2Y\}$

   c) Find the marginal densities f_X and f_Y.

3. Suppose that X and Y have joint density function $f_{X,Y}$ given by

$$f_{X,Y}(x,y) = ce^{-|x|-|y|}$$

   where c is a constant.

   a) Find $f_{X,Y}(x,y)$ for all cases of values of x and y (for example, when $x, y > 0$, when $x > 0$ but $y < 0$, etc.).

   b) Find c.

   c) Find the probability that $P(-1 < X < 2 \text{ and } 3 < Y < 4)$.

   d) Find the marginal densities f_X and f_Y.