1. (1 point) `Library/WHFreeman/Holt_linear_algebra/Chaps_1-4/2.2.61.pg`

Let $\mathbf{u}_4$ be a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
Select the best statement.

- A. $\text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} = \text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
- B. There is no obvious relationship between $\text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $\text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
- C. We only know that $\text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} \subseteq \text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$.
- D. $\text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} = \text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ when $\mathbf{u}_4$ is a scalar multiple of one of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
- E. none of the above

2. (1 point) `Library/WHFreeman/Holt_linear_algebra/Chaps_1-4/2.3.50.pg`

Let $\mathbf{u}_4$ be a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly dependent set of vectors unless one of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is the zero vector.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly dependent set of vectors.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly dependent or linearly dependent set of vectors depending on the vectors chosen.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly dependent or linearly dependent set of vectors depending on the vector space chosen.
- F. none of the above

3. (1 point) `Library/TCNJ/TCNJ_LinearIndependence/problem1.pg`
Suppose $S = \{r, u, d\}$ is a set of linearly independent vectors.

If $x = 2r + 2u + 5d$, determine whether $T = \{r, u, x\}$ is a linearly independent set.

[?] 1. Is T linearly independent or dependent?

If T is dependent, enter a non-trivial linear relation below. Otherwise, enter 0's for the coefficients.

$\_\_\_\_\, r + \_\_\_\_\, u + \_\_\_\_\, x = 0.$

4. (1 point) `Library/WHFreeman/Holt_linear_algebra/Chaps_1-4/2.3.47.pg`

Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ be a linearly independent set of vectors.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is never a linearly independent set of vectors.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is always a linearly independent set of vectors.
- D. none of the above

5. (1 point) `Library/Rochester/setLinearAlgebra8VectorSpaces/ur_la_8_5.pg`

Find the dimensions of the following linear spaces.
(a) $\mathbb{R}^{6\times 2}$ ___
(b) P_4 ___
(c) The space of all diagonal 3×3 matrices ___

6. (1 point) `Library/Rochester/setLinearAlgebra10Bases/ur_la_10_34.pg`
The set

$$B = \left\{ \begin{bmatrix} -1 & -2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \right\}$$

is a basis of the space of upper-triangular 2×2 matrices.

Find the coordinates of $M = \begin{bmatrix} -7 & -3 \\ 0 & -8 \end{bmatrix}$ with respect to this basis.

$$[M]_B = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

7. (1 point) `Library/Rochester/setLinearAlgebra15TransfOfLinSpaces/ur_la_15_1.pg`
Which of the following transformations are linear? Select all of the linear transformations. There may be more than one correct answer. Be sure you can justify your answers.

- A. $T(A) = \begin{bmatrix} 4 & 3 \\ -2 & 6 \end{bmatrix} A$ from $\mathbb{R}^{2\times 4}$ to $\mathbb{R}^{2\times 4}$
- B. $T(A) = A^T$ from $\mathbb{R}^{4\times 6}$ to $\mathbb{R}^{6\times 4}$

- C. $T(A) = A\begin{bmatrix} 2 & -5 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 6 & 8 \end{bmatrix} A$ from $\mathbb{R}^{2\times 2}$ to $\mathbb{R}^{2\times 2}$
- D. $T(A) = \det(A)$ from $\mathbb{R}^{6\times 6}$ to $\mathbb{R}$
- E. $T(A) = A\begin{bmatrix} -7 & 1 \\ 4 & 9 \end{bmatrix}$ from $\mathbb{R}$ to $\mathbb{R}^{2\times 2}$
- F. $T(A) = A + I_4$ from $\mathbb{R}^{4\times 4}$ to $\mathbb{R}^{4\times 4}$

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8. (1 point) `Library/Hope/Multi1/04-01-Linear-transformations/Lin_trans_05.pg`

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(\langle x,y\rangle) = 4x - 6y + 2$. Is f a linear transformation?

(1) $f(\langle x_1,y_1\rangle + \langle x_2,y_2\rangle) =$ ________. (Enter x_1 as x1, etc.)
$f(\langle x_1,y_1\rangle) + f(\langle x_2,y_2\rangle) =$ ____ + ____.
Does $f(\langle x_1,y_1\rangle + \langle x_2,y_2\rangle) = f(\langle x_1,y_1\rangle) + f(\langle x_2,y_2\rangle)$ for all $\langle x_1,y_1\rangle, \langle x_2,y_2\rangle \in \mathbb{R}^2$?

- choose
- Yes, they are equal
- No, they are not equal

(2) $f(c\langle x,y\rangle) =$ ________.
$c(f(\langle x,y\rangle)) =$ __ (____).
Does $f(c\langle x,y\rangle) = c(f(\langle x,y\rangle))$ for all $c \in \mathbb{R}$ and all $\langle x,y\rangle \in \mathbb{R}^2$?

- choose
- Yes, they are equal
- No, they are not equal

(3) Is f a linear transformation?

- choose
- f is a linear transformation
- f is not a linear transformation

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9. (1 point) `Library/Rochester/setLinearAlgebra14TransfOfRn/ur_la_14_10.pg`

Consider a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^2$ for which

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 8 \\ 7 \end{bmatrix},\quad T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\quad T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \end{bmatrix}.$$

Find the matrix A of T.

$$A = \begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$$

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10. (1 point) `Library/Rochester/setLinearAlgebra15TransfOfLinSpaces/ur_la_15_7.pg`

Find the matrix A of the linear transformation $T(f(t)) = 9f'(t) + 6f(t)$ from P_2 to P_2 with respect to the standard basis for P_2, $\{1, t, t^2\}$.

$$A = \begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$$

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11. (1 point) `Library/Rochester/setLinearAlgebra11Eigenvalues/ur_la_11_9.pg`

Supppose A is an invertible $n \times n$ matrix and $\vec{v}$ is an eigenvector of A with associated eigenvalue -4. Convince yourself that $\vec{v}$ is an eigenvector of the following matrices, and find the associated eigenvalues.

(1) The matrix A^2 has an eigenvalue ____.

(2) The matrix A^{-1} has an eigenvalue ____.

(3) The matrix $A - 9I_n$ has an eigenvalue ____.

(4) The matrix $-7A$ has an eigenvalue ____.

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12. (1 point) `Library/Rochester/setLinearAlgebra11Eigenvalues/ur_la_11_8.pg`

Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 6 & -4 & -15 \\ -1 & 3 & 9 \\ 4 & -4 & -13 \end{bmatrix}.$$

From smallest to largest, the eigenvalues are $\lambda_1 < \lambda_2 < \lambda_3$ where

$\lambda_1 =$ __ has an eigenvector $\begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$,

$\lambda_2 =$ __ has an eigenvector $\begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$,

$\lambda_3 =$ __ has an eigenvector $\begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$.

Note: you may want to use a graphing calculator to estimate the roots of the polynomial which defines the eigenvalues.

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13. (1 point) `Library/Hope/Multi1/05-04-Diagonalization/DiagR_04.pg`

Let

$$A = \begin{bmatrix} -13 & -4 & 28 \\ 4 & 3 & -8 \\ -6 & -2 & 13 \end{bmatrix}.$$

If possible, find an invertible matrix P so that $D = P^{-1}AP$ is a diagonal matrix. If it is not possible, enter the identity matrix for P and the matrix A for D. You must enter a number in every answer blank for the answer evaluator to work properly.

$$P = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}.$$

$$D = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}.$$

Is A diagonalizable over $\mathbb{R}$?

- choose
- diagonalizable
- not diagonalizable

Be sure you can explain why or why not.

14. (1 point) `Library/TCNJ/TCNJ_LengthOrthogonality/problem3.pg`

Let W be the set of all vectors $\begin{bmatrix} x \\ y \\ x+y \end{bmatrix}$ with x and y real. Find a basis of $W^{\perp}$.

$$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}.$$

15. (1 point) `Library/Rochester/setLinearAlgebra17DotProductRn/ur_la_17_17.pg`

Find the orthogonal projection of

$$\vec{v} = \begin{bmatrix} 0 \\ 0 \\ -9 \\ 0 \end{bmatrix}$$

onto the subspace W of $\mathbb{R}^4$ spanned by

$$\begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}.$$

$$\text{proj}_W(\vec{v}) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$$

16. (1 point) `Library/Rochester/setLinearAlgebra20LeastSquares/ur_la_20_2.pg`

Find the least-squares solution $\vec{x}^*$ of the system

$$\begin{bmatrix} 2 & -1 \\ -2 & 1 \\ 4 & 5 \end{bmatrix} \vec{x} = \begin{bmatrix} 13 \\ -17 \\ -5 \end{bmatrix}.$$

$$\vec{x}^* = \begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$$