Ivan Tomasic **MTH5112-MTH5212**

Assignment Alternative_Assessment_2020 due 08/29/2020 at 09:05am BST

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1. (5 points) `local/Library/TCNJ/TCNJ_VectorEquations/problem1.pg`

Let $\mathbf{w},\mathbf{x},\mathbf{y},\mathbf{z}$ be vectors and suppose $\mathbf{z} = -3\mathbf{x} - 2\mathbf{y}$ and $\mathbf{w} = -6\mathbf{x} - 3\mathbf{y} - 2\mathbf{z}$.

Mark the statements below that must be true.

- A. $\text{Span}(\mathbf{y}) = \text{Span}(\mathbf{w})$
- B. $\text{Span}(\mathbf{x},\mathbf{z}) = \text{Span}(\mathbf{y},\mathbf{w})$
- C. $\text{Span}(\mathbf{x},\mathbf{y}) = \text{Span}(\mathbf{x},\mathbf{w},\mathbf{z})$
- D. $\text{Span}(\mathbf{x},\mathbf{y}) = \text{Span}(\mathbf{w})$

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2. (10 points) `Library/TCNJ/TCNJ_LinearIndependence/problem2.pg`

Determine whether or not the following sets S of 2×2 matrices are linearly independent.

$\boxed{?}$ 1. $S = \left\{ \begin{pmatrix} -4 & -2 \\ -6 & -2 \end{pmatrix}, \begin{pmatrix} 16 & 8 \\ 24 & 8 \end{pmatrix} \right\}$

$\boxed{?}$ 2. $S = \left\{ \begin{pmatrix} -4 & -2 \\ -6 & -2 \end{pmatrix}, \begin{pmatrix} 16 & 32 \\ 16 & 8 \end{pmatrix}, \begin{pmatrix} 1 & -3 \\ 9 & 10 \end{pmatrix}, \begin{pmatrix} -2 & -4 \\ 8 & -4 \end{pmatrix}, \begin{pmatrix} 17 & -31 \\ \pi & e^2 \end{pmatrix} \right\}$

$\boxed{?}$ 3. $S = \left\{ \begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix}, \begin{pmatrix} -4 & 1 \\ 1 & -2 \end{pmatrix}, \begin{pmatrix} -4 & -2 \\ 1 & 0 \end{pmatrix} \right\}$

$\boxed{?}$ 4. $S = \left\{ \begin{pmatrix} -4 & -2 \\ -6 & -2 \end{pmatrix}, \begin{pmatrix} 16 & 32 \\ 16 & 8 \end{pmatrix} \right\}$

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3. (10 points) `Library/Rochester/setLinearAlgebra9Dependence/ur_la_9_1b.pg`

Are the vectors $\begin{bmatrix} -4 \\ -2 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 4 \\ -2 \end{bmatrix}$ linearly independent?

- Choose
- linearly dependent
- linearly independent

If they are linearly dependent, find scalars that are not all zero such that the equation below is true. If they are linearly independent, find the only scalars that will make the equation below true.

$$\_\_\begin{bmatrix} -4 \\ -2 \\ 5 \end{bmatrix} + \_\_\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + \_\_\begin{bmatrix} 4 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

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4. (5 points) `local/Library/WHFreeman/Holt_linear_algebra/Chaps_1-4/2.3.52.pg`

Assume $\mathbf{u}_4$ is not a linear combination of $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3\}$.
Select the most accurate statement.

- A. $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4\}$ is always a linearly dependent set of vectors.
- B. $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4\}$ is a linearly independent set of vectors unless one of $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3\}$ is the zero vector.
- C. $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4\}$ could be a linearly independent or linearly dependent set of vectors depending on the vector space chosen.
- D. $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4\}$ is never a linearly dependent set of vectors.
- E. $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4\}$ is a linearly dependent set precisely when $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3\}$ is a linearly dependent set.
- F. none of the above

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5. (10 points) `Library/Rochester/setLinearAlgebra10Bases/ur_la_10_36.pg`

Find a basis for the space of 2×2 lower triangular matrices.

$$\text{Basis} = \left\{ \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \right\}$$

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6. (10 points) `local/Library/Rochester/setLinearAlgebra14TransfOfRn/ur_la_14_1.pg`

If $L:\mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation such that

$$L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 3 \\ -4 \end{bmatrix},\ L\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix},\ L\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \left[\ - \right.$$

then $L\left(\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

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7. (5 points) `local/Library/Rochester/setLinearAlgebra4InverseMatrix/ur_Ch2_1_3.pg`

Suppose that $L:\mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that

$$L\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -3 \\ 9 \end{bmatrix},\quad L\left(\begin{bmatrix} -6 \\ -5 \end{bmatrix}\right) = \begin{bmatrix} 33 \\ -3 \end{bmatrix},$$

Find the matrix associated with L in the standard basis.

In other words, find the matrix such that for any $\mathbf{v} \in \mathbb{R}^2$, the linear transformation L is given by $L(\mathbf{v}) = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \mathbf{v}$.

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8. (10 points) `local/Library/Rochester/setLinearAlgebra15TransfOfLinSpaces/ur_la_15_4.pg`

The matrices

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

form a basis for the linear space $V = \mathbb{R}^{2\times 2}$. Write the matrix of the linear transformation $L : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$ such that $L(A) = 9A + 2A^T$ relative to this basis.

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

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9. (10 points) `Library/Rochester/setLinearAlgebra11Eigenvalues/ur_la_11_19.pg`

The matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

has two real eigenvalues, one of multiplicity 1 and one of multiplicity 2. Find the eigenvalues and a basis for each eigenspace.

The eigenvalue λ_1 is ____ and a basis for its associated eigenspace is $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

The eigenvalue λ_2 is ____ and a basis for its associated eigenspace is $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

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10. (5 points) `local/Library/TCNJ/TCNJ_Eigenvalues/problem1.pg`

Suppose A is an $n \times n$ matrix.

Determine which of the following statements are true:

- A. To find the eigenvalues of A, reduce A to echelon form.
- B. A number c is an eigenvalue of A if and only if the equation $(A - cI)\mathbf{x} = 0$ has a nontrivial solution $\mathbf{x}$.
- C. Finding an eigenvector of A might be difficult, but checking whether a given vector is in fact an eigenvector is easy.
- D. If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$, then λ is an eigenvalue of A.
- E. A matrix A is not invertible if and only if 0 is an eigenvalue of A.

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11. (10 points) `Library/NAU/setLinearAlgebra/diag3x3.pg`

Let $A = \begin{bmatrix} 2 & -4 & 12 \\ 3 & -5 & 9 \\ 0 & 0 & -2 \end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

$P = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$, $D = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

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12. (10 points) `local/Library/TCNJ/TCNJ_OrthogonalProjections/problem1.pg`

Given $\mathbf{v} = \begin{bmatrix} -5 \\ 8 \\ -5 \\ -7 \end{bmatrix} \in \mathbb{R}^4$, find the closest point to $\mathbf{v}$ in the subspace spanned by vectors $\begin{bmatrix} 3 \\ 4 \\ -6 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -6 \\ 4 \\ -42 \end{bmatrix}$.

Enter the coordinates of this closest point to five decimal places:

$$\begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$