MTH5112 Linear Algebra I
MTH5212 Applied Linear Algebra
(2023/2024)

COURSEWORK 8

WebWork submission of **exercise marked** $(*)$ due:
11.59am on Monday 06 December 2023

You should also attempt all of the other exercises in order develop your mathematical reasoning and skill in constructing arguments and proofs; model solutions will be posted on QMPlus after the submission date.

Exercise (*) 1. Solve WeBWork **Set 8** at:

`https://webwork.qmul.ac.uk/webwork2/MTH5112-2023`.

Log in with your 'ah***' QMUL ID as username, and your student number as password, see Coursework 0 for further instructions.

Exercise 2. Let A be a square matrix. Prove the following:

(a) 0 is an eigenvalue of A if and only if A is not invertible.
(b) If A is invertible, then λ is an eigenvalue of A if and only if λ^{-1} is an eigenvalue of A^{-1}.
(c) If λ is an eigenvalue of A and n is a positive integer, then λ^n is an eigenvalue of A^n.

Exercise 3. Let

$$A = \begin{pmatrix} 4 & 3 & 3 \\ -4 & -3 & -4 \\ 2 & 2 & 3 \end{pmatrix}.$$

(a) Verify that $(3, -4, 2)^T$ is an eigenvector of A, and find the corresponding eigenvalue.
(b) Determine all of the eigenvalues of A, and find bases for the corresponding eigenspaces. (Hint: you can use the eigenvalue from (a) to help factorise the characteristic polynomial.)
(c) Using (b), explain why A is diagonalisable and find a matrix P that diagonalises A. (Remember what this means: P should be invertible and $P^{-1}AP$ should be diagonal.)
(d) Using (c), compute A^5.

Exercise 4. For each of the following matrices, find all of the eigenvalues and find bases for the corresponding eigenspaces. Decide whether the matrix is diagonalisable. If it is, find a matrix that diagonalises it; if it is not, explain why.

(a) $\begin{pmatrix} 6 & 4 & 2 \\ -7 & -6 & -5 \\ 4 & 4 & 4 \end{pmatrix}$, (b) $\begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$, (c) $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$.