MTH5112 Linear Algebra I
MTH5212 Applied Linear Algebra
(2023/2024)

COURSEWORK 7

WebWork submission of **exercise marked** $(*)$ due:
11.59am on Wednesday 06 December 2023

You should also attempt all of the other exercises in order develop your mathematical reasoning and skill in constructing arguments and proofs; model solutions will be posted on QMPlus after the submission date.

Exercise $(*)$ 1. Solve WeBWork **Set 7** at:

`https://webwork.qmul.ac.uk/webwork2/MTH5112-2023`.

Log in with your '`ah***`' QMUL ID as username, and your student number as password, see Coursework 0 for further instructions.

Exercise 2. Let $L : V \to W$ be a linear transformation, and suppose that $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ spans V. Prove that the set $\{L(\mathbf{v}_1), \dots, L(\mathbf{v}_n)\}$ spans $\mathrm{im}(L)$.

Exercise 3. Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator defined by

$$L \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 16x_1 - 20x_2 - 16x_3 \\ -10x_2 \\ 48x_1 - 48x_3 \end{pmatrix}.$$

(1) Find bases for the image and the kernel of L.
(2) Hence, determine the rank and the nullity of L, and verify that the Rank–Nullity Theorem holds for this linear transformation.
(3) Find the matrix $A \in \mathbb{R}^{3\times 3}$ associated with L, i.e., such that

$$L(\mathbf{x}) = A\mathbf{x}$$

for all $\mathbf{x} \in \mathbb{R}^3$.