MTH5112 Linear Algebra I
MTH5212 Applied Linear Algebra
(2023/2024)

COURSEWORK 6

WebWork submission of **exercise marked** $(*)$ due:
11.59am on Wednesday 22 November 2023

You should also attempt all of the other exercises in order develop your mathematical reasoning and skill in constructing arguments and proofs; model solutions will be posted on QMPlus after the submission date.

Exercise $(*)$ 1. Solve WeBWork **Set 6** at:

`https://webwork.qmul.ac.uk/webwork2/MTH5112-2023`.

Log in with your 'ah***' QMUL ID as username, and your student number as password, see Coursework 0 for further instructions.

Exercise 2. Let V be a finite dimensional vector space, and let U and W be subspaces of V. Recall (from CW2 and lectures notes week 3, page 31) that $L \cap M$ and

$$L + M = \mathrm{Span}(L \cup M) = \{\mathbf{u} + \mathbf{v} : \mathbf{u} \in L, \mathbf{v} \in M\}$$

are both subspaces of V. Prove the *modular law*

$$\dim(L + M) + \dim(L \cap M) = \dim(L) + \dim(M).$$

Exercise 3. Let

$$A = \begin{pmatrix} 1 & -1 & 3 & 1 & 2 \\ 2 & -2 & 6 & 3 & 0 \\ 3 & -3 & 9 & 4 & 2 \end{pmatrix}.$$

Find bases for the row space, column space, and null space of A. Hence, determine the rank and the nullity of A, and verify that the Rank–Nullity Theorem holds for this particular matrix.

Exercise 4. Use the Rank–Nullity Theorem to prove that an $n \times n$ matrix A is invertible if and only if rank$(A) = n$.

Exercise 5. Is it possible to construct matrices with the following properties?

(a) A 4×3 matrix with rank 1 and nullity 2?
(b) A 3×4 matrix with rank 2 and nullity 1?

If you think the answer is "yes", give an example; if you think the answer is "no", explain why such a matrix cannot exist.