

MTH5112 Linear Algebra I

COURSEWORK 5

WebWork submission of **exercise marked (*)** due:
11.59am on Wednesday 22 November 2023

You should also attempt all of the other exercises in order to develop your mathematical reasoning and skill in constructing arguments and proofs; model solutions will be posted on QMPlus after the submission date.

Exercise (*) 1. Solve WeBWork **Set 5** at:

<https://webwork.qmul.ac.uk/webwork2/MTH5112-2023/>.

Log in with your 'ah***' QMUL ID as username, and your student number as password, see Coursework 0 for further instructions.

Exercise 2. Consider the following subsets of \mathbb{R}^3 :

$$\mathcal{B}_1 = \{(1, -2, 0)^T, (0, 1, 1)^T, (-3, 6, 1)^T\} \quad \text{and} \quad \mathcal{B}_2 = \{(1, 3, -1)^T, (2, 0, 1)^T, (1, 0, 1)^T\}.$$

- Prove that \mathcal{B}_1 and \mathcal{B}_2 are bases for \mathbb{R}^3 .
- What is the transition matrix from \mathcal{B}_2 to the standard basis?
- Determine the transition matrix from the standard basis to \mathcal{B}_1 , and hence determine the transition matrix from \mathcal{B}_2 to \mathcal{B}_1 .
- If $\mathbf{x} \in \mathbb{R}^3$ has coordinate vector $(1, -3, 2)^T$ with respect to the basis \mathcal{B}_2 , then what is the coordinate vector of \mathbf{x} with respect to \mathcal{B}_1 ?

Exercise 3. Let $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3 \in P_2$ be given by

$$\mathbf{p}_1(t) = t^2 - 4t + 2, \quad \mathbf{p}_2(t) = t + 3, \quad \mathbf{p}_3(t) = 1.$$

- Prove that $\mathcal{B} = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ is a basis for P_2 .
- If $\mathbf{p} \in P_2$ has coordinate vector $[\mathbf{p}]_{\mathcal{B}} = (-1, 3, 2)^T$ with respect to the basis \mathcal{B} , then what is \mathbf{p} ?
- Consider the polynomial $\mathbf{q} \in P_2$ given by $\mathbf{q}(t) = -t^2 + 6$. Find the coordinates of \mathbf{q} with respect to the basis \mathcal{B} .