

- This coursework consists of three parts:
 - I. Practice problems
 - II. Mock Quiz
 - III. More Practice to connect week 9 and 10 learning
 - A selection of solutions to the listed problems will be posted on QMPlus by the end of Week 9 and discussed during the tutorials.
 - I encourage all students to learn and check their computational answers using math software such as Mathematica, MATLAB, etc. Using numerical software is a fun practice and will help you to visualise your solutions.
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I. Practice Problems

A. Determine the eigenvalues and eigenvectors of the following matrices:

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$$

B. Find and sketch the solution of the following initial value problems

1) $y_1 = -\frac{1}{2}y_1 + \frac{5}{2}y_2$, $y_2 = \frac{5}{2}y_1 - \frac{1}{2}y_2$, $y_1(0) = a$, $y_2(0) = b$.

2) $y_1 = -y_1 + 5y_2$, $y_2 = -y_1 + y_2$, $y_1(0) = 0$, $y_2(0) = 4$.

C.

(1) Linearize $\dot{y}_1 = y_1 + e_2^y - \cos y_2$, $\dot{y}_2 = 3y_1 - y_2 - \sin y_2$ around the fixed point at $y_1 = y_2 = 0$ and find the eigenvalues.

(2) Linearize the following equation $\dot{y}_1 = -2y_1 - 3y_2 + y_1^5$, $\dot{y}_2 = y_1 + y_2 - y_2^2$ around the fixed point at $y_1 = y_2 = 0$ and find the eigenvalues.

D. (1) Compute all equilibria of the non-linear ODE system

$$\dot{y}_1 = -y_1 + 3y_2 - y_1^2 + 3y_1y_2, \quad \dot{y}_2 = -3y_1 - y_2,$$

and linearise this ODE systems around its equilibria separately and write down their matrix forms.

(2) Determine the general solution of the linearised system at $y = 0$. Find the function $x(t)$ that solves the initial value problem for the system above specified by the initial conditions

$$y_1(0) = a, \quad y_2(0) = b$$

and express it in terms of real-valued functions. Sketch the trajectories of this autonomous system in phase space when $a = b = 1$.

II. Mock Quiz

Train yourself for Coursework 2 by answering Mock Quiz Week 9.

III. More Practice: graphing trajectories and analysis of dynamical systems

Consider the dynamical system given by

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -2y_1 + 2y_2 \end{cases}$$

- 1) Rewrite the system in matrix form and find the eigenvalues and eigenvectors of the associated coefficient matrix.
- 2) Find the general solution of the system of ODEs, justifying your answer.
- 3) Sketch the trajectory corresponding to the initial condition $y_1(0) = 0$, $y_2(0) = 1$.