MTH5123 **Differential Equations**

Formative Assessment Week 6 part **G.Bianconi**

- This Formative Assessment consists of two parts:
  - **I.** Practice problems. You will get help on this Formative Assessment in the tutorial of week 3. You should work on this before you go to this session.
  - **II.** Mock Quiz Week 6.
- A selection of solutions to the listed problems will be posted on QMPlus by the end of Week 6. You are expected to seek solutions to the remaining problems using the Reading List and making use of the tutorial sessions.
- I encourage all students to learn and check their computational answers using math softwares such as Mathematica, MATLAB, etc. Using numerical software is a fun practice and will help you to visualise your solutions (– sketching solutions will be tested in the final exam).

I. Practice Problems

A. Find the solution to the following IVP for the given ODE

$$x^2\frac{d^2y}{dx^2} - 2y = 0, \quad y(1) = 0,\ y'(1) = 1.$$

B. Consider the following boundary value problem (BVP)

$$\frac{1}{\cos x}\frac{d^2y}{dx^2} + \left(\frac{\sin x}{\cos^2 x}\right)\frac{dy}{dx} = 0\ ,\ y(0) = 0\ ,\ y\left(\frac{\pi}{4}\right) = 2$$

Show that the left-hand side of the ODE can be written down in the form $\frac{d}{dx}\left(r(x)\frac{dy}{dx}\right)$ for some function $r(x)$. Use this fact to determine the solution to the above BVP.

C. Find the solution to the following Boundary Value Problem for the second order inhomogeneous differential equation

$$\frac{d^2y}{dx^2} = x\ ,\ y(-1) = 0\ ,\ y(1) = 0.$$

D. Find the solution of the following Boundary Value Problem for the second order linear inhomogeneous differential equation,

$$(x+1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = f(x)\ ,\ f(x) = -1,\ y(0) = 0\ ,\ y'(1) = 0.$$

Hint: the left-hand side of the ODE can be written down in the form $\frac{d}{dx}\left(r(x)\frac{dy}{dx}\right)$ for some function $r(x)$ and use this fact to determine the general solution of the associated homogeneous ODE $y_h(x)$. Based on $y_h(x)$, using the variation of parameter method to find the general solution to the inhomogeneous ODE $y_g(x)$. Useful formula: $\int \ln z dz = z(lnz - 1) + c$.

II. Homework

Train for Coursework 1 with Mock Quiz Week 6.