Coursework 2 2023-2024

1. ## Dynamical systems

   CLOZE 0.10 penalty

   a) Which of the following systems of ODEs is autonomous and dynamical?

   MULTI 1 point Multiple Shuffle

   - I
   - II ✓
   - III,

   where
   I: $y_1' = 3y_1 e^t, \quad y_2' = y_1 - y_2$
   II: $\dot{y}_1 = y_2 e^{y_1} - \ln(1 + |y_2|), \quad \dot{y}_2 = (y_1 y_2)^{5/2}$
   III: $\frac{dy_1}{dx} = \ln(3 + |y_1|), \quad \frac{dy_2}{dx} = -y_2$

   b) Find out which of the following options are equilibria of the dynamical system,

   $$\dot{y}_1 = e^{y_1 y_2 - 1} - 1, \quad \dot{y}_2 = (y_1 - 4)y_2$$

   MULTI 2 points Multiple Shuffle

   - I
   - II
   - III ✓
   - IV

   where I:$(y_1^*, y_2^*) = (1, 0)$; II:$(y_1^*, y_2^*) = (4, 0)$;
   III:$(y_1^*, y_2^*) = (4, 1/4)$; IV:$(y_1^*, y_2^*) = (1, 1)$.

2. ## Phase portrait 2

   CLOZE 0.10 penalty

   Consider a system of two linear first-order ordinary differential equations: $\dot{y}_1 = y_1 - y_2, \quad \dot{y}_2 = 2y_1 - y_2$.

   a) The corresponding eigenvalues are

   MULTI 1 point Multiple Shuffle

- $\lambda_1 = 1$, $\lambda_2 = -1$ • $\lambda_1 = i$, $\lambda_2 = -i$ ✓   • $\lambda_1 = 1+i$, $\lambda_2 = 1-i$

b) The corresponding eigenvectors of this linear ODE system are:

- I and II   • I and IV ✓   • II and III   • III and IV ✓

where

I:$u_1 = \begin{pmatrix} 1+i \\ 2 \end{pmatrix}$

II:$u_2 = \begin{pmatrix} 1-i \\ 3 \end{pmatrix}$

III:$u_1 = \begin{pmatrix} 2i \\ 2(1+i) \end{pmatrix}$

IV:$u_2 = \begin{pmatrix} 2 \\ 2(1+i) \end{pmatrix}$

c) The phase portrait for this system of ODEs is

- Stable node   • Unstable focus with spiral out
- Centre ✓   • Stable focus with spiral in

3. **Stability**

For which value of a the system of ODEs
$\dot{y}_1 = \sinh(y_1) + a\tanh(y_2)$, $\dot{y}_2 = -2\cos(y_1) + 2e^{y_1+y_2} + \tanh(y_2)$, linearised around $(y_1, y_2) = (0,0)$ displays an unstable focus?

(a) $0 < a < 2$
(b) $a = 2$
(c) $a < -1/2$ (100%)
(d) $-1/2 < a < 0$

4. **Lyapunov function 2**

Which of the following functions $V(y_1, y_2)$ is a Lyapunov function for the dynamical system with equilibrium point at $(0,0)$
$\dot{y}_1 = -2y_1 y_2^2 e^{(y_1 y_2)^2} - 6y_1$, $\dot{y}_2 = -2y_1^2 y_2 e^{(y_1 y_2)^2} - 2y_2$

(a) $V(y_1, y_2) = y_1^4 + (y_2 - 1)^2$

2

(b) $V(y_1, y_2) = e^{(y_1 y_2)^2}$

(c) $V(y_1, y_2) = e^{(y_1 y_2)^2} + y_2^2 - 1 + 3y_1^2$ (100%)

(d) $V(y_1, y_2) = y_1^2 e^{(y_1 y_2)^2} + y_2^2 - 3y_1$

Total of marks: 10