# Coursework 1 Year 2023/2024 final

## 1. Type of ODEs 1

MATCHING 2 points 0.10 penalty Shuffle

Find the right match for the following ODEs in the dropdown menu

$$y'+y''=2x^2y$$
 • · · · · · · 2nd-order linear ODE  $10y'-y=e^x$  • · · · · · · • 1st-order linear ODE  $2\tanh(y)y'+2xy+3=0$  • · · · · · • 1st-order nonlinear ODE

$$y'' = \tanh(y/x) \bullet \cdots \bullet \text{ 2nd-order nonlinear ODE}$$

### 2. Type of ODEs 2

MATCHING 2 points 0.10 penalty Shuffle

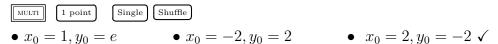
Find the right match for the following ODEs in the dropdown menu

$$y'' = \frac{x}{y} \bullet \cdots \cdots \bullet$$
 None of the above forms  $y' = 5 + \sin(y/x) \bullet \cdots \cdots \bullet$  Scale-invariant ODE  $9x^2y'' = y + 3xy' \bullet \cdots \cdots \bullet 2$ nd-order Euler-type ODE  $2xy^2e^{(xy)^2} + 2x^2ye^{(xy)^2}y' = 0 \bullet \cdots \cdots \bullet 1$ st-order exact ODE

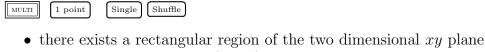
#### 3. **IVP**

CLOZE 0.10 penalty

a) Solving the initial value problem (IVP)  $y' = (y+2)^{3/5}/(x+2)$ , y(2) = -2 implies finding a solution y(x) of the differential equation that passes through the point  $(x_0, y_0)$  with



b) If this IVP has a unique solution, it means that



- whose center is the point  $(x_0, y_0)$  where the solution to the IVP is unique. ✓ • in the whole xy plane, there exist one and only one solution passing
- through this point. • there is only one unique solution to the ODE in the xy plane.

c) Does the IVP satisfy the hypotheses of the Picard-Lindelöf theorem?

MULTI 1 point Multiple Shuffle

• no √

• yes

d) How many solutions has the IVP in a)?

Multiple Shuffle

• none

 $\bullet \;$  more than one  $\checkmark$ 

 $\bullet$  one

#### 4. Reducible ODE

MULTI 2 points 0.10 penalty Single Shuffle

The general solution of the 1st-order ODE,  $(x+3y-2)y' = e^{-3(x+3y-2)^2} - y - 1/3(x-2)$  is

(a)  $y(x) = \ln(3x + C) - 2$ 

(b) 
$$y(x) = \left[\frac{1}{\sqrt{3}}\ln(6x+C) + (2-x)\right]/3$$

(c) 
$$y(x) = \frac{1}{3\sqrt{3}} \left[ \pm \sqrt{\ln(18(x+C))} + (2-x)\sqrt{3} \right]$$
 (100%)

(d) 
$$y(x) = \left[ \frac{1}{\sqrt{3}} \ln(6x + C) \pm (2 - x) \right] / 3$$

Total of marks: 10