Coursework 1 Year 2023/2024 final

1. **Type of ODEs 1**

   MATCHING | 2 points | 0.10 penalty | Shuffle

   Find the right match for the following ODEs in the dropdown menu

$y' + y'' = 2x^2y$ •	······	• 2nd-order linear ODE
$10y' - y = e^x$ •	······	• 1st-order linear ODE
$2\tanh(y)y' + 2xy + 3 = 0$ •	······	• 1st-order nonlinear ODE
$y'' = \tanh(y/x)$ •	······	• 2nd-order nonlinear ODE

2. **Type of ODEs 2**

   MATCHING | 2 points | 0.10 penalty | Shuffle

   Find the right match for the following ODEs in the dropdown menu

$y'' = \frac{x}{y}$ •	······	• None of the above forms
$y' = 5 + \sin(y/x)$ •	······	• Scale-invariant ODE
$9x^2y'' = y + 3xy'$ •	······	• 2nd-order Euler-type ODE
$2xy^2e^{(xy)^2} + 2x^2ye^{(xy)^2}y' = 0$ •	······	• 1st-order exact ODE

3. **IVP**

   CLOZE | 0.10 penalty

   a) Solving the initial value problem (IVP) $y' = (y+2)^{3/5}/(x+2)$, $y(2) = -2$ implies finding a solution $y(x)$ of the differential equation that passes through the point (x_0, y_0) with

   MULTI | 1 point | Single | Shuffle

   • $x_0 = 1, y_0 = e$ • $x_0 = -2, y_0 = 2$ • $x_0 = 2, y_0 = -2$ ✓

   b) If this IVP has a unique solution, it means that

   MULTI | 1 point | Single | Shuffle

   - there exists a rectangular region of the two dimensional xy plane whose center is the point (x_0, y_0) where the solution to the IVP is unique. ✓
   - in the whole xy plane, there exist one and only one solution passing through this point.
   - there is only one unique solution to the ODE in the xy plane.

c) Does the IVP satisfy the hypotheses of the Picard-Lindelöf theorem?

MULTI | 1 point | Multiple | Shuffle

- no ✓
- yes

d) How many solutions has the IVP in a)?

MULTI | 1 point | Multiple | Shuffle

- none
- more than one ✓
- one

4. **Reducible ODE**

MULTI | 2 points | 0.10 penalty | Single | Shuffle

The general solution of the 1st-order ODE, $(x+3y-2)y' = e^{-3(x+3y-2)^2} - y - 1/3(x-2)$ is

(a) $y(x) = \ln(3x + C) - 2$
(b) $y(x) = \left[\frac{1}{\sqrt{3}} \ln(6x + C) + (2 - x)\right] /3$
(c) $y(x) = \frac{1}{3\sqrt{3}} \left[\pm\sqrt{\ln(18(x + C))} + (2 - x)\sqrt{3}\right]$ (100%)
(d) $y(x) = \left[\frac{1}{\sqrt{3}} \ln(6x + C) \pm (2 - x)\right] /3$

Total of marks: 10