Essential Foundation Mathematical Skills

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Overview

ILOs

→ Today's lecture is on **Monomials and Polynomials**;

After today's lecture, you are expected to understand the concepts of <u>monomials</u> and <u>how to construct polynomials & their arithmetic</u>.

Introduction

Definition (Monomial)

A **monomial** is an expression in which variables and constants may stand alone or be multiplied.

$\rightarrow$ A monomial cannot have a variable in the denominator.
$\rightarrow$ You can think of a monomial as being one term.

Example:
Here are some monomials:

$$\begin{aligned} 5, \\ x^3, \\ -2x^5, \\ x^2y. \end{aligned}$$

Introduction

Definition (Polynomial)

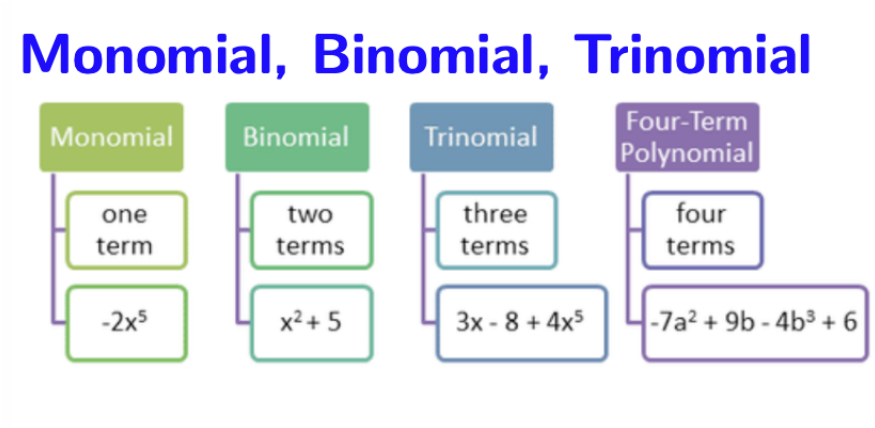
A **polynomial** is defined as an expression which is composed of variables, constants and exponents, combined using mathematical operations.

$\rightarrow$ The prefix "poly" means many.

Example:
Here are some polynomials:

$$x^2 + 5,$$
$$3x - 8 + 4x^5,$$
$$-7a^2 + 9b - 4b^3 + 6.$$

Introduction



Basic Polynomial Identities

Basic Polynomial Identities

$$x(y+z) = xy + xz$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$x^2 - y^2 = (x-y)(x+y)$$

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

Polynomial Remainder

Given two polynomials $f(x)$ and $g(x)$, we can write:

$$f(x) = g(x)q(x) + r(x),$$

where $q(x)$ and $r(x)$ are polynomials and the degree of $r(x)$ is less than that of $g(x)$.

$\rightarrow$ The polynomials q(x) and r(x) are called the **quotient** and the **remainder**, respectively, of the division $f(x) \div g(x)$.

$\rightarrow$ They are computed with the long division algorithm.

Examples and exam-style questions

Exercises

Multiply.

$c^3\,c^5$; $(x^3)^2$; $(x\,y)^3$

$(-x^2)^3$; $(-x^3)^2$; $-(x^3)^2$

$(-a^2b)(-3ab^2)(-7)$; $4xy(-xy)(-xy^2)$

$(a\,b^2\,c^3)^4$; $(-aba^2b)^2(3bcbc)^3$

$$\left(\frac{3}{4}xz\right)\left(-\frac{2}{9}x^2yz\right)\left(-\frac{12}{5}y^2z\right)$$

$$\frac{\alpha}{2}\left(\frac{-\,\alpha\beta}{2}\right)^5 64\,(-\beta^2)$$

Collect like terms.

$-1-3x^2-8x^2+4x+3-x+4x^2$

$2a^5-3a^3+a^5-3a^3-a^5-2a^5+5a^3$

$4ab+ab^2-2a-b^2+3ab-ab^2-7ab$

$-3x^2y-8x^2y+4xy^2+2-xy^2+4x^2y$

$c^4d^2-c^3d+3c^2d-2c^4d^2+3c^3d-cd$

Examples and exam-style questions

Exercise

Expand, collecting like terms.

$(-a-b)^2; \qquad (2x+y)^2$

$(-a+b)^2; \qquad -(x-3)^2$

$(-\alpha\beta+1)^2; \qquad (-6\theta+3\delta^2-\theta)^2$

$3^2(2x-1)^2; \qquad (ab^5+5)^2$

$(a^3+3b^2)(a^3-3b^2)$

$(d^3+5)(d^3-5); \qquad (7a^2+2)(2-7a^2)$

Examples and exam-style questions

Exercise

Compute (q, r), the quotient and remainder of polynomial division

$(x-1) \div (1-x); \qquad (x+1) \div (x-1)$

$(z-1) \div (z+1); \qquad (-3z+2) \div (z+2)$

$(2b-1) \div (3b+1); \qquad (-7c+3) \div (3c+4)$

$(a^2-1) \div (a+1); \qquad (a^2+1) \div (a+1)$

$(x^2-7x+3) \div (x+2)$

$(x^3+28) \div (x+3)$

$(-x^{10}+1) \div x^3$

$(y^4-16y^2+3y) \div (-4+y)$

Examples and exam-style questions

Exercise

Compute the quotient of polynomial division

$(x^4 - 4x + 1) \div (x - 2)$

$(x^4 - x + 1) \div (-x + 3)$

$(-y^4 - y^3 + 1) \div (y + 2)$

$(3a^6 + 5a^4) \div (a^3 - 3)$

$(-2x^4 + 9x^2 + 2) \div (x - 2)$

$(4y^5 - y^4 + y^2 + 1) \div (y^3 + y^2 - 3)$

$(Z^4 - 2Z^3 - 6Z^2 + 1) \div (-Z + 3)$

$(z^6 - z^3 - 1) \div (z^3 - 3z)$

$(a^4 - a^3 + a + 1) \div (-a - 2)$

$(c^4 - c^3 + c + 1) \div (-c + 3)$

$(2X^5 - X^4 + 2) \div (X^3 - 3X + 1)$

Examples and exam-style questions

Question: Compute the quotient of polynomial division.

$$(c^4 - c^3 + c + 1) \div (-c + 3)$$

Question: Compute (q, r), the quotient and remainder of polynomial division.

$$(5x^3 - x) \div (3x + 1)$$