Probability

Answers

"A coin is tossed 15 times.
Head is recorded 10 times and tail is recorded 5 times"

$$P(\text{Head}) = \frac{10}{15}$$

$$= \frac{2}{3}$$

$$P(\text{Tail}) = \frac{5}{15}$$

$$= \frac{1}{3}$$

"What is the theoretical probability of the Head when "A coin is tossed"?

$\hookrightarrow P(\text{Head}) = \frac{1}{2}$

Remember the analysis on the theoretical and empirical probability.
Key words: unbiased, fair coin .

Example:

What is the probability of getting five sixes when five dice are rolled?

Five dice produce $6^5 = 7,776$ outcomes.

Only one outcome is all sixes.

Therefore,

$$P(\text{five sixes}) = \frac{1}{7,776}$$

Note: It is recommended you define the relevant event of interest at the start, e.g. A denoting the set $A = \{(6,6,6,6,6)\}$ before you calculate $P(A)$.

☺.

Example:

What is the probability that there will be at least one head in five tosses of a fair coin?

Five coins produce $2^5 = 32$ outcomes

However, only TTTTT does not contain at least one head, so

$$P(\text{at least one } H) = \frac{31}{32}$$

with H denoting head.

Example (on recording the number of squirrels on my way to work).

What is the probability that I will see at least two squirrels on my next journey?

Answer:

If x is the number or squirrels I see, then

$$P(x \geq 2) = \frac{17}{20}$$

$$= 0.85,$$

given that $5+2+7+1+0+1+1=17$ days out of the 20 days I saw two or more (at least two) squirrels.

• Consider a sample space Ω and three events A, B and C.

Find the set expression for the following:

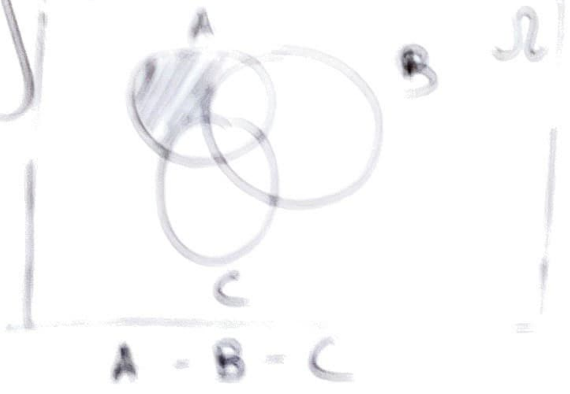
1) Among A, B and C, only A occurs.

2) At least one of the events A, B or C occurs

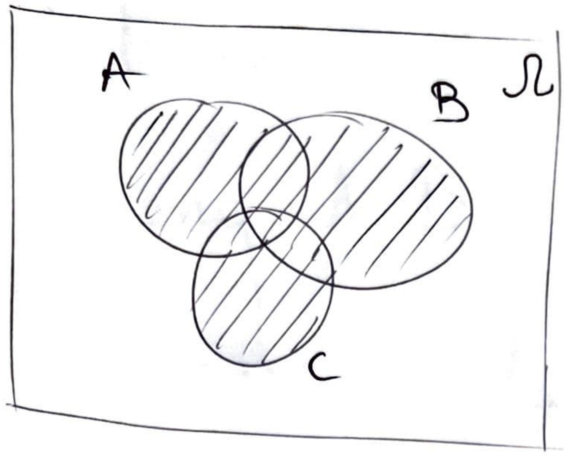
3) A or C occurs, but not B.

4) At most two of the events A, B or C occur.

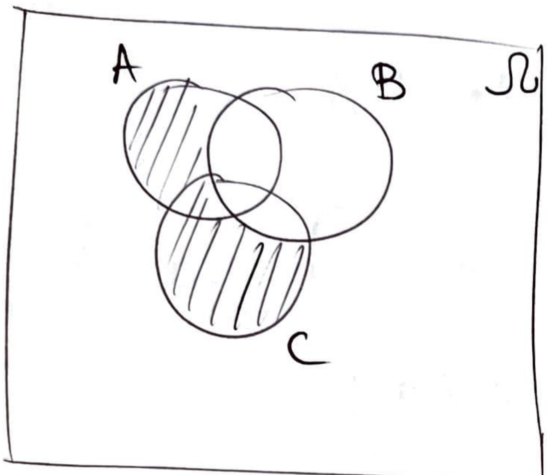
By drawing the following Venn diagrams



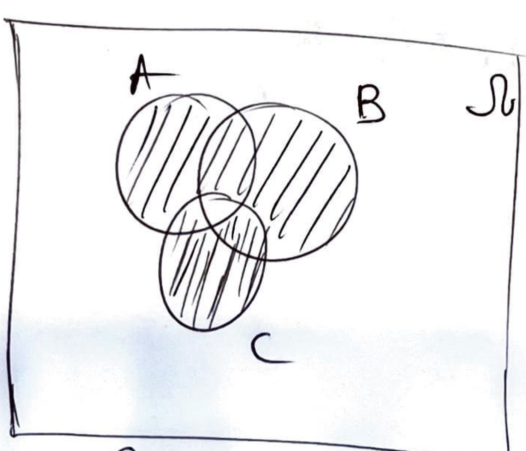
A - B - C



$A \cup B \cup C$



$(A \cup C) - B$



$(A \cap B \cap C)^c$

, we have :

1) Among A, B and C, only A occurs:

$$A - B - C = A - (B \cup C)$$

2) At least of the events A, B or C occurs:

$$A \cup B \cup C$$

3) A or C occurs, but not B :

$$(A \cup C) - B$$

4) At most two of the events A, B or C occur :

$$(A \cap B \cap C)^c = A^c \cup B^c \cup C^c$$

Example:

We know that:

There is 60 percent chance that it will rain today.

There is 50 percent chance that it will rain tomorrow.

There is 30 percent chance that it does not rain either day.

Find the probabilities below:

1) The probability that it will rain today or tomorrow.

2) The probability that it will rain today and tomorrow.

3) The probability that it will rain today but not tomorrow.

4) The probability that it either will rain today or tomorrow, but not both.

1) Let A denote the event that it will rain today and B the event that it will rain tomorrow.

Then, $P(A) = 0.6 \left(= \frac{60}{100}\right)$

$P(B) = 0.5 \left(= \frac{50}{100}\right)$

$P(A^c \cap B^c) = 0.3 \left(= \frac{30}{100}\right)$

The probability that it will rain today or tomorrow can be written as:

$P(A \cup B)$.

By using Venn diagrams, $A \cup B = \Omega \setminus A^c \cap B^c$ with Ω the universal set; in this contect the sample space.

Therefore

$$P(A \cup B) = 1 - P(A^c \cap B^c)$$

$$= 1 - 0.3$$

$$= 0.7$$

2) The probability that it will rain today and ~~to~~tomorrow is $P(A \cap B)$.
Therefore,

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 0.6 + 0.5 - 0.7$$

$$= 0.4$$

(based on the properties of the probability as explained in the lectures),

3) The probability that it will rain today but not tomorrow is $P(A \cap B^c)$. Then,

$$P(A \cap B^c) = P(A - B)$$

$$= P(A) - P(A \cap B)$$

$$= 0.6 - 0.4$$

$$= 0.2$$

(based on the Venn diagrams which could be of help (Please show))

4) The probability that it will either rain today or tomorrow but not both. This can be expressed as

$$P(A-B)+P(B-A).$$

From question (3) we have already found that $P(A-B)=0.2$.

Following the same reasoning we have,

$$P(B-A)=P(B)-P(B\cap A)$$

$$=0.5-0.4$$

$$=0.1.$$

Therefore

$$P(A-B)+P(B-A)=0.2+0.1$$

$$=0.3$$

Example:

We pick a random number from $\{1, 2, \dots, 10\}$ and this random number is denoted by η.

Assume that all outcomes are equally likely.

Let A be the event that η is less than 7.

Let B be the event that η is an even number.

Are A and B independent?

We have $A = \{1, 2, 3, 4, 5, 6\}$

$B = \{2, 4, 6, 8, 10\}$

$A \cap B = \{2, 4, 6\}$.

Then,

$$P(A) = 0.6$$

$$P(B) = 0.5$$

$$P(A \cap B) = 0.3$$

$$P(A) \cdot P(B) = 0.6 \cdot 0.5$$
$$= 0.3$$
$$= P(A \cap B)$$

Therefore, A and B are independent.

This means that knowing that B has happened does not change our belief/expectation about the probability of A.