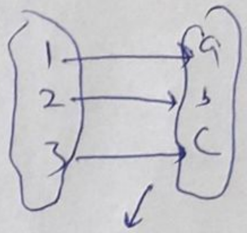
Help for Tutorial 9 "Functions"

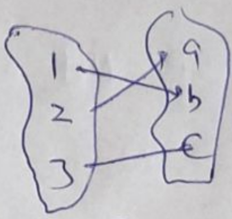
Q.1 $A \times B = \{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c),$
$(3,a),(3,b),(3,c)\}$

$f_1 = \{(1,a),(2,b),(3,c)\} \subset A \times B$

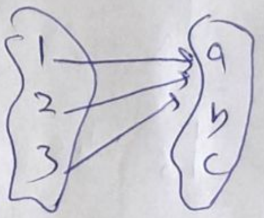


Injective,
surjective
bijective

$f_2 = \{(1,b),(2,a),(3,c)\} \subset A \times B$

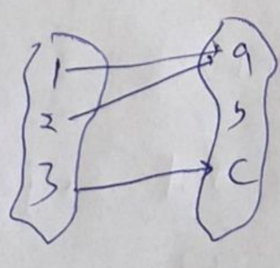


$f_3 = \{(1,a),(2,a),(3,a)\} \subset A \times B$

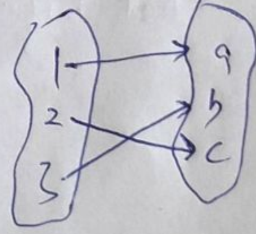


Not injective, ~~No Bi~~
Not bijective
Not surjective.

$f_4 = \{(1,a),(2,a),(3,c)\} \subset A \times B$



$f_5 = \{(1,a),(2,c),(3,b)\} \subset A \times B$



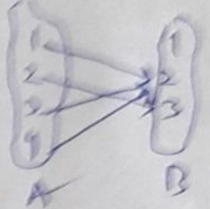
Injective, surjective,
bijective.

Q.2

a) f_1 is not a function as $f(1) = 4$ and $4 \notin B$.

b) f_2 is a function.

(Not injective, Not surjective, Not bijective).



c) f_3 is Not a function $\because$ $f(3)$ is not listed.

d) f_4 is Not a function $\because$ 2 has not a unique image.

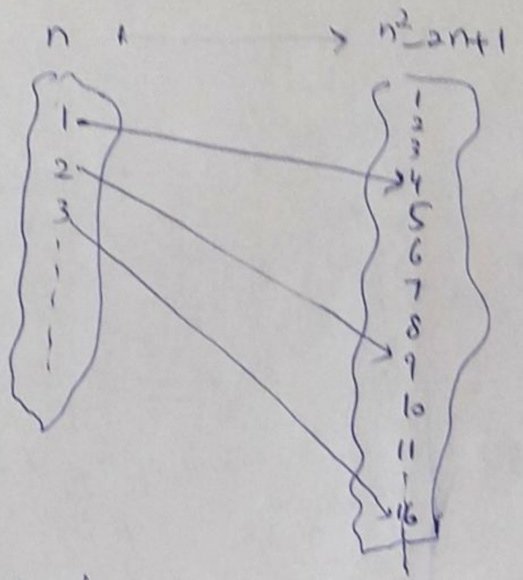
e) f_5 is not a function $\because$ 1 has no a unique image.

f) f_6 is a function.

Q. 3 (a) $f : \mathbb{N}^+ \longrightarrow \mathbb{N}^+$, $n \longmapsto n^2 + 2n + 1$

Actually $n^2 - 2n + 1 = (n+1)^2$

f is injective. f is not surjective, so not bijective.



Injective, Not surjective, Not bijective.

b) $g : \mathbb{Z} \to \mathbb{Z}, \quad n \mapsto n^2 - 2n + 1$

$n^2 - 2n + 1 = (n-1)^2$

g is not injective $\because$ $g(2) = 1 = g(0)$

g is not surjective $\because$ 3 is not the image of any element of domain set.

So not bijective.

c) h is not injective, ~~however~~ h is also not surjective. $h(0) = h(2) = 1$

d) K is not injective $k(0) = k(2) = 1$.
k is surjective.

4) (a) $\text{range}(f) = \{1,2,3,4\}$,

$\text{range}(g) = \{2,3,4\}$,

$\text{range}(h) = \{2\}$.

b) Only f is invertible.

$f^{-1} = \{(1,2),(2,4),(3,1),(4,3)\}$.

c) $f \circ f = \{(1,4),(2,3),(3,2),(4,1)\}$

$f \circ g = \{(1,1),(2,2),(3,2),(4,4)\}$

$g \circ f = \{(1,4),(2,2),(3,3),(4,4)\}$

d) $f \circ h = \{(1,1),(2,1),(3,1),(4,1)\}$,

$(f \circ h) \circ g = \{(1,1),(2,1),(3,1),(4,1)\}$,

$h \circ g = \{(1,2),(2,2),(3,2),(4,2)\}$,

$f \circ (h \circ g) = \{(1,1),(2,1),(3,1)(4,1)\}$.

So $(f \circ h) \circ g$ and $f \circ (h \circ g)$ coincide in this case.

(5)

Q.5. Range $(f) = \mathbb{R}^+$

Range (g) ; $y > 1$, $y \in \mathbb{R}$.

f and g are bijective functions, so invertible

To find the inverse,

suppose $y = f(x)$ — (i)

$\Rightarrow x = f^{-1}(y)$ — (ii)

using (i)

$y = x^2 \Rightarrow x = \sqrt{y}$

$\Rightarrow f^{-1}(y) = \sqrt{y}$ using (ii)

$\Rightarrow \boxed{f^{-1}(x) = \sqrt{x}}$

Similarly,

$y = g(x) \Rightarrow x = g^{-1}(y)$

So $y = x + 1$

$\Rightarrow x = y - 1$

$\Rightarrow g^{-1}(y) = y - 1$

$\boxed{g^{-1}(x) = x - 1}$

$$f \circ g = f(g(x)) = f(x+1) = (x+1)^2$$

$$g \circ f = g(f(x)) = g(x^2) = x^2 + 1$$

$$f \circ g^{-1} = f(g^{-1}(x)) = f(x-1) = (x-1)^2$$

$$f \circ f^{-1} = f(f^{-1}(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$$

$$f^{-1} \circ f = f^{-1}(f(x)) = f^{-1}(x^2) = \sqrt{x^2} = x$$

$$g \circ g^{-1} = g(g^{-1}(x)) = g(x-1) = (x-1)+1 = x$$

$$g^{-1} \circ g = g^{-1}(g(x)) = g^{-1}(x+1) = (x+1)-1 = x$$