Queen Mary University of London

SEF015: Discrete Mathematics (2022-23)

Material for the Q&A session or…*Tutorial 9 (Week 10)*

This material is for your tutorial in Week 10 and is designed to help your understanding. Please try to answer all the questions before you join your tutorial group.

Number of pages: 2

Question 1. 4. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. Compute the set $A \times B$. Write five different relations which are subsets of $A \times B$, and they are functions from A *to* B. At least one of them must be injective, surjective and bijective. Can the set $A \times B$ be a function $f : A \to B$?

Question 2*. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3\}$. Determine whether the following sets are functions from A *to* B. For those that are not functions, state everything that is preventing them from being functions.

$(a)\ f_1 = \{(1,4), (2,3), (3,2), (4,1)\}$,
$(b)\ f_2 = \{(1,2), (3,2), (4,2), (2,3)\}$,
$(c)\ f_3 = \{(1,2), (2,3), (4,3)\}$,
$(d)\ f_4 = \{(1,2), (2,1), (4,3), (2,2)\}$,
$(e)\ f_5 = \{(1,1), (2,3), (4,3), (1,2), (3,2)\}$,
$(f)\ f_6 = \{(1,3), (2,3), (4,3), (3,1)\}$.

Question 3. For each of the following functions determine, with justification, whether it is injective; surjective; bijective; or none of these.

$(a)\ f: \mathbb{N}^+ \to \mathbb{N}^+, n \mapsto n^2 + 2n + 1$,
$(b)\ g : \mathbb{Z} \to \mathbb{Z}, n \mapsto n^2 - 2n + 1$,
$(c)\ h : \mathbb{R} \to \mathbb{R}, x \mapsto x^2 - 2x + 1$,
$(d)\ k : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0} > 0, x^2 - 2x + 1$.

Question 4*. Let $A = \{1, 2, 3, 4\}$, and let $f : A \to A, g : A \to A$ and $h : A \to A$ be the following functions.

$$f = \{(1,3), (2,1), (3,4), (4,2)\},$$
$$g = \{(1,2), (2,4), (3,4), (4,3)\},$$
$$h = \{(1,2), (2,2), (3,2), (4,2)\}.$$

(a) Determine the range (or image) of each of the functions f, g, h.
(b) Determine which (if any) of the functions f, g, h is invertible. (Note that invertible means having an inverse.) For any that are, calculate the inverse.
(c) Calculate the compositions $f \circ f, f \circ g$ and $g \circ f$.
(e) Calculate $(f \circ h) \circ g$ and $f \circ (h \circ g)$ and verify that they are equal.

Question 5. Let the functions $f(x) = x^2$, $g(x) = x + 1$, where $x \in \mathbb{R}^+$. What is the range of f and g. Are f and g invertible? If yes, calculate their inverse. Also compute $f \circ g$ and $g \circ f, f \circ g^{-1}, f \circ f^{-1}, f^{-1} \circ f, g \circ g^{-1}, g^{-1} \circ g$.