Help for Solving Questions of Week 9 Tutorial

Q.1:

$S = \{a, b, c\}$, $|S| = 3$

$T = \{b, d, e, x, z\}$, $|T| = 5$

$S \cup T = \{a, b, c, d, e, x, z\}$, $|S \cup T| = 7$

$S \cap T = \{b\}$, $|S \cap T| = 1$

So

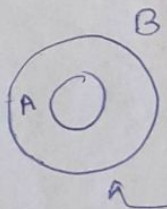
$$|S \cup T| = |S| + |T| - |S \cap T|$$

$$7 = 3 + 5 - 1$$

$$7 = 7 \quad \text{(verified)}$$

Q.2:

$|A| = 4$, $|B| = 10$



$0 \le |A \cap B| \le 4 \qquad \because A \cap B \subseteq A$

So if $|A \cap B| = n$ such that $n \in \{0, 1, 2, 3, 4\}$

then

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B| = 4 + 10 - n = 14 - n$$

Thus $|A \cup B| \in \{10, 11, 12, 13, 14\}$

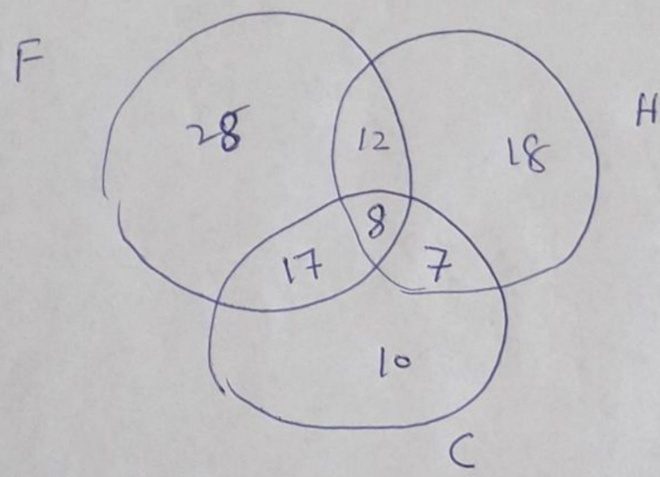
So $10 \le |A \cup B| \le 14$

Q.3:

$|F| = 65$, $|H| = 45$, $|C| = 42$

$|(F \cap H)| = 20$, $|F \cap C| = 25$, $|H \cap C| = 15$

$|F \cap H \cap C| = 8$, ~~AUBUC~~ $|F \cup H \cup C| = ?$



$65 - (12+8+17) = 28$

$45 - (12+8+7) = 18$

$42 - (17+8+7) = 10$

So total students $= 28 + 18 + 10 + 12 + 8 + 17 + 7 = 100$

Verification using Inclusion-Exclusion formula:

$|F \cup H \cup C| = |F| + |H| + |C| - |F \cap H| - |H \cap C| - |F \cap C| + |F \cap H \cap C|$

$= 65 + 45 + 42 - 20 - 25 - 15 + 8 = 100$

Q-4: $A = \{1, 2, 3\}$, $B = \{a, b\}$

$A \times B = \{(1,a),(1,b),(2,a),(2,b),(3,a),(3,b)\}$

$B \times A = \{(a,1),(b,1),(a,2),(b,2),(a,3),(b,3)\}$

$A \times A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

$B \times B = \{(a,a),(a,b),(b,a),(b,b)\}$

Q.5:

$A = \{1, 2, 3\}$

(i) $R = \{(1,1),(1,2),(1,3),(3,3)\}$

Not reflexive $\because (2,2) \notin R$

Not symmetric $\because (1,2) \in R$ but $(2,1) \notin R$.

Transitive , Antisymmetric

(ii) $S = \{(1,1),(1,2),(2,2),(2,3)\}$

Not reflexive $\because (3,3) \notin S$

Not symmetric $\because (1,2) \in S$ but $(2,1) \notin S$.

Not transitive , $(1,2) \wedge (2,3) \in S$ but $(1,3) \notin S$

Antisymmetric

Q.6:

$S = \{1, 2, 3, 4, 5\}$

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (3,5), (4,3), (4,4), (4,5), (5,3), (5,4), (5,5)\}$

→ R is reflexive ∵ $(1,1), (2,2), (3,3), (4,4), (5,5) \in R$

→ R is symmetric ∵ $(1,2), (2,1) \in R$, $(3,4), (4,3) \in R$ etc

→ R is transitive ∵ $(3,4), (4,5), (3,5) \in R$ etc.

So R is an equivalance relation.

Q.7: $S = \{1, 2, 3, 4\}$

a) $R_1 = \{(1,1), (2,2), (3,3), (4,4), (1,2), (1,3), (3,4), (1,4)\}$

→ R_1 is reflexive.

→ R_1 is antisymmetric

→ R_1 is transitive.

So R_1 is a partial order.

However, $2 \not R_1 3$. So R_1 is not a total order.

b) Do yourself

c)

Scanned By Camera Scanner