Solution 9

Q.1 $1+4+7+\cdots+(3n-5)+(3n-2)=\dfrac{n(3n-1)}{2}$ — ①

(i) $P(1)$ is true ?

L.H.S : $n=1 \Rightarrow 3(1)-2 = 1$

R.H.S : $n=1 \Rightarrow \dfrac{1(3(1)-1)}{2} = \dfrac{2}{2} = 1$

So L.H.S = R.H.S , So $P(1)$ is true.

(ii) we suppose that ① is true for $n=k$, i.e

$$1+4+7+\cdots+(3k-2) = \frac{k(3k-1)}{2} \quad — ②$$

add $3k+1$ on both sides of ②,

$$1+4+7+\cdots+(3k-2)+(3k+1) = \frac{k(3k-1)}{2}+3k+1$$

$$1+4+7+\cdots+(3k+1) = \frac{3k^2-k+6k+2}{2}$$

$$1+4+7+\cdots+(3k+1) = \frac{3k^2+5k+2}{2}$$

$$1+4+7+\cdots+(3(k+1)-2) = \frac{3k^2+3k+2k+2}{2}$$

$$= \frac{3k(k+1)+2(k+1)}{2}$$

which is true for $n=k+1$, $= \dfrac{(k+1)(3k+2)}{2}$

So ① is true for all $n \in \mathbb{N}$. $= [(k+1)(3(k+1)-1)]/2$

Q.2 $$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1} \quad -(1)$$

(i) L.H.S : $n=1 \Rightarrow \frac{1}{(2-1)(2+1)} = \frac{1}{3}$

R.H.S : $n=1 \Rightarrow \frac{1}{2(1)+1} = \frac{1}{3}$

So P(1) is true as L.H.S = R.H.S .

(ii) Suppose (1) is true for $n=k$, i.e

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{[2(k)-1][2(k)+1]} = \frac{k}{2k+1} \quad -(2)$$

adding $\frac{1}{(2k+1)(2k+3)}$ on both sides of (2),

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2k-1)(2k+1)}+\frac{1}{(2k+1)(2k+3)} = \frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$$

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2k+1)(2k+3)} = \frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2+2k+k+1}{(2k+1)(2k+3)}$$

$$= \frac{2k(k+1)+1(k+1)}{(2k+1)(2k+3)}$$

$$= \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)} = \frac{k+1}{2(k+1)+1}$$

So (1) is true for $n=k+1$. Hence (1) is true for all $n \in \mathbb{N}$.

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Q.3 $8^n - 3^n$ is divisible by 5 $\forall n \geq 1$.

(i) when $n=1$, the statent is

$8-3=5$ is divisible by 5, which is true. So $P(1)$ is true. (For $n=0$, it is also true)

(ii) Suppose the statent is true for $n=k$, ie

$8^k - 3^k$ is divisible by 5.

Now we need to ckeck whether the statent is true for $n=k+1$. So

$8^{k+1} - 3^{k+1}$ is divisible by 5 ??

So $8^{k+1} - 3^{k+1} = 8 \cdot 8^k - 3 \cdot 3^k$

$= (5+3)8^k - 3 \cdot 3^k$

$= 5 \cdot 8^k + 3 \cdot 8^k - 3 \cdot 3^k$

$= 5 \cdot 8^k + 3(8^k - 3^k)$ — (3)

$5 \cdot 8^k$ is divisible by 5.

$3(8^k - 3^k)$ is also divisible by 5 as $8^k - 3^k$ is divisible by 5.

So $5 \cdot 8^k + 3(8^k - 3^k)$ (the sum of two nos divisible by 5 is also divisible by 5) is divisible by 5. Hence proved.

Q4: Suppose the statement,

$$P(n) : 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{(n(n+1))^2}{4} \quad \text{— (1)}$$

(i) $n = 1 \implies$

$$1^3 = \frac{(1(1+1))^2}{4}$$

$$1 = \frac{4}{4}$$

$$1 = 1$$

which is true for $n=1$.

(ii) Suppose the statement is true for $n=k$, i.e

$$P(k) : 1^3 + 2^3 + 3^3 + \cdots k^3 = \frac{(k(k+1))^2}{2^2} \quad \text{— (2)}$$

adding $(k+1)^3$ on both sides,

$$1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$1^3 + 2^3 + 3^3 + \cdots + (k+1)^3 = \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$$

$$= \frac{(k+1)^2[k^2 + 4(k+1)]}{4}$$

$$= \frac{(k+1)^2(k^2 + 4k + 4)}{4}$$

$$1^3 + 2^3 + 3^3 + \cdots + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4} \quad \text{— (3)}$$

(3) shows that the statement is also true for $n = k+1$. Hence, the statement (1), is true for all $n \in \mathbb{N}$.

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Q.5 Suppose the statement

$$P(n): \quad 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}} = 2\left(1 - \frac{1}{2^n}\right) \quad \text{—— (1)}$$

(i) $P(1)$:

$$\frac{1}{2^{1-1}} = 2\left(1 - \frac{1}{2^1}\right)$$

$$\frac{1}{2^0} = 2\left(1 - \frac{1}{2}\right)$$

$$\frac{1}{1} = 2\left(\frac{1}{2}\right)$$

$1 = 1$, So $P(1)$ is true.

(ii) Suppose the statement (1) is true for $n = k$, i.e

$$P(k): \quad 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{k-1}} = 2\left(1 - \frac{1}{2^k}\right) \quad \text{—— (2)}$$

adding $\frac{1}{2^k}$ on both sides,

$$1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{k-1}} + \frac{1}{2^k} = 2\left(1 - \frac{1}{2^k}\right) + \frac{1}{2^k}$$

$$1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^k} = 2 - \frac{2}{2^k} + \frac{1}{2^k}$$

$$= 2 - \frac{1}{2^k}$$

$$= 2 - \frac{2}{2^{k+1}}$$

$$1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^k} = 2\left(1 - \frac{1}{2^{k+1}}\right) \quad \text{—— (3)}$$

(3) shows that the statement is true for $n = k+1$

Hence, the statement (1) is true for all positive integers.