(7)

	DOG(x)	DOMESTIC(x)	DOG(x) ↔ DOMESTIC(x)
line 1	T	T	T
line 2	T	F	F
line 3	F	T	F
line 4	F	F	T

(3d) is true if and only if for most things x, x is a domestic dog (line 1) or x is a wild non-dog (line 4). This formula is also true in the universe containing only three wild dogs and a lot of seagulls, where *Most dogs are domestic* is false.

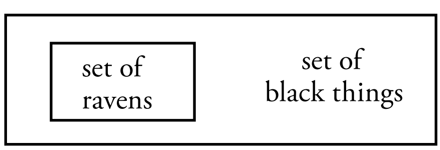
In short, none of the possible formulae gives the right truth condition for 'Most dogs are domestic', and *most* cannot be analysed in the same way as the universal and existential quantifiers.

An alternative way of analysing quantifiers, including the existential and universal, is **Generalized Quantifier (GQ) Theory**. As we shall see, GQ Theory requires the use of variables over predicates or sets, and so is second-order. (Predicates and sets were discussed in Chapter 4, but the necessary background will be reviewed again here.)

6.2 Generalized Quantifier Theory

The central idea in Generalized Quantifier theory is that **a quantifier expresses a relation between sets**. For example, *All ravens are black* expresses the relation illustrated in (8): The set of ravens is completely included in the set of black things, or, the set of ravens is a subset of the set of black things.

(8)



All ravens are black

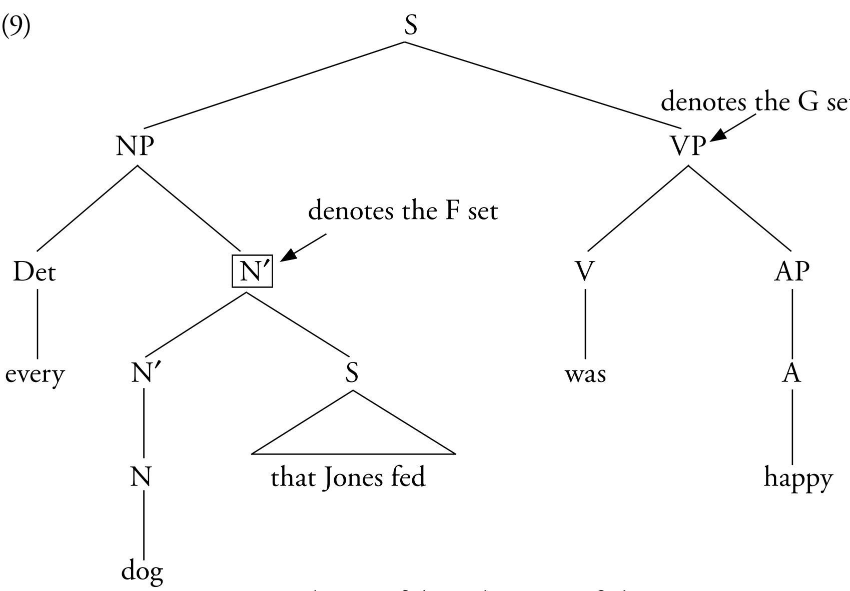
To define the quantifier determiners as generalized quantifiers we need the following ideas and symbols from set theory (A and B stand for sets).

Set Theory Terms

A = **B**:	A and B are identical: they have exactly the same members.
A ⊂ **B**:	A is a proper subset of B: all the members of A are also members of B, and B has at least one member which is not a member of A.
A ⊆ **B**:	A is a subset of B: A is a proper subset of B or A is identical to B.
|A|:	The cardinality of A, which is the number of members in A.
|A| = **9**:	The cardinality of A is 9: A has 9 members.
|A| > **|B|**:	The cardinality of A is greater than the cardinality of B: A has more members than B.
|A| ≥ **|B|**:	The cardinality of A is greater than or equal to the cardinality of B: A has at least as many members as B.
|A| ≥ **6**:	The cardinality of A is greater than or equal to 6: A has at least 6 members.
A ∩ **B**:	The intersection of A and B, which is the set of entities which are members of A and also members of B.
A – **B**:	The set of members of A which are not also members of B ('A minus B')

The definitions of quantifier determiners take the general form **Det Fs are G** or **Det F is G**. The variables F and G stand for the sets defined by 1-place predicates. The set defined by a 1-place predicate is the set of all entities of which that predicate is true – for example, the predicate DOG defines the set of dogs. Variable F stands for the set denoted by the highest N′ (pronounced 'N-bar': this is the node which combines with the determiner) and variable G stands for the set denoted by VP, as illustrated in (9):

(9)



F = the set of dogs that Jones fed
G = the set of entities which are happy

Definitions for Quantifiers: Group 1

All Fs are G $F \subseteq G$
'The set of Fs is a subset of the set of Gs'

Most Fs are G $|F \cap G| > |F - G|$
'The cardinality of the set of things which are both F and G is greater than the cardinality of the set of things which are F but not G'
'Things which are both F and G outnumber things which are F but not G'

Few Fs are G $|F - G| > |F \cap G|$
'The cardinality of the set of things which are F but not G is greater than the cardinality of the set of things which are both F and G.'
'Things which are F but not G outnumber things which are both F and G.'

Definitions for Quantifiers: Group 2

No F is G $|F \cap G| = 0$
'The cardinality of the set of things which are both F and G is zero.'
'There are no things which are both F and G.'

An F is G $|F \cap G| \geq 1$
'The cardinality of the set of things which are both F and G is greater than or equal to 1.'
'There is at least one thing which is both F and G.'

Some Fs are G $|F \cap G| \geq 2$
'The cardinality of the set of things which are both F and G is greater than or equal to 2.'
'There are at least two things which are both F and G.'

Four Fs are G $|F \cap G| = 4$
'The cardinality of the set of things which are both F and G is 4.'
'There are four things which are both F and G.'

Many Fs are G $|F \cap G| = \text{many}$
'The cardinality of the set of things which are both F and G is many (or large).'
'There are many things which are both F and G.'

Several Fs are G $|F \cap G| = \text{several}$
'The cardinality of the set of things which are both F and G is several.'
'There are several things which are both F and G.'

Few Fs are G $|F \cap G| = \text{few}$
'The cardinality of the set of things which are both F and G is few (or small).'
'There are several things which are both F and G.'

A few Fs are G $|F \cap G| = \text{a few}$
'The cardinality of the set of things which are both F and G is a few (or small).'
'There are a few things which are both F and G.'

With these definitions, quantifiers are analysed as relations between sets, or in other words, as two-place predicates taking sets as arguments. *Few* and *many* appear in both Group 1 and Group 2. The differences between the groups are discussed in the next section, and *few* and *many* are discussed in Section 6.3.2.

6.3 Different types of quantifier determiner

6.3.1 Group 1 and Group 2 determiners

Determiners in the first group express asymmetric relations, in that the order of the arguments is significant, and the sets have different roles in the relation, for example:

(10) 'All Fs are G' is not equivalent to 'All Gs are F'.
'$F \subseteq G$' is not equivalent to '$G \subseteq F$'.

'All(F, G)' is not equivalent to 'All(G, F)'.
'All dogs bark' is not equivalent to 'All barkers are dogs'.

(11) 'Most Fs are G' is not equivalent to 'Most Gs are F'.
'$|F \cap G| > |F - G|$' is not equivalent to '$|G \cap F| > |G - F|$'.

'Most(F, G)' is not equivalent to 'Most(G, F)'.
'Most leaves are green' is not equivalent to 'Most green things are leaves'.

Determiners in the second group give the cardinality of a set which is defined as the intersection of F and G, and because intersection is symmetric, the roles of the F set and the G set in the relation are not different in principle, for example:

(12) 'No F is G' is equivalent to 'No G is F'.
$|F \cap G| = |G \cap F| = 0$.
'No rose is black' is equivalent to 'No black thing is a rose'.

(13) 'An F is G' is equivalent to 'A G is F'.
$|F \cap G| = |G \cap F| \geq 1$.
'A spy is present' is equivalent to 'Someone present is a spy'.

(14) 'Some Fs are G' is equivalent to 'Some Gs are F'.
$|F \cap G| = |G \cap F| \geq 2$.
'Some plants are meat eaters' is equivalent to 'Some meat eaters are plants'.

(15) 'Four Fs are G' is equivalent to 'Four Gs are F'.
$|F \cap G| = |G \cap F| = 4$.
'Four clocks are in the hall' is equivalent to 'Four things in the hall are clocks'.

The differences reviewed above mainly arise out of the special status of the F predicate with Group 1 quantifiers: Group 1 quantifiers express a **proportion of the F set**, and are sometimes called **proportional quantifiers**.

You need to know (roughly) the size of the whole F set to know how many Fs count as all Fs, most Fs, or few Fs. For example, suppose that eight dogs were vaccinated for rabies. If there are thirty dogs altogether, it's true that few dogs were vaccinated; if there are eleven dogs altogether, it's true that most dogs were vaccinated; and if there are eight dogs altogether, it's true that all dogs were vaccinated.

Determiners which form proportional quantifiers are called **strong determiners**. Noun phrases formed with strong determiners are commonly called **strong noun phrases** or **strong NPs**.

The quantifiers in the second group express a quantity which is not a proportion. For example, for the truth of 'Several dogs were vaccinated' or 'Eight dogs were vaccinated', it matters only how many vaccinated dogs there are, and the number of dogs in total is irrelevant. These quantifiers give the cardinality of the F and G intersection, and are called **cardinal quantifiers**. Determiners which form cardinal quantifiers are **weak determiners**. Noun phrases formed with weak determiners are **weak noun phrases** or **weak NPs**.

6.3.2 The ambiguity of *few* and *many*

Few and *many* are often considered to be ambiguous between strong and weak readings. On their weak readings *few* and *many* denote a small number and a large number, respectively.

The strong reading of *few* is rather like the reading of the partitive construction, so *few fleas* on the strong reading means much the same as *few of the fleas*. This reading expresses a proportion of the group of fleas, and to know how many *few* indicates, we need to know roughly how many fleas there are altogether. Suppose a new insecticide is being tested on flies, fleas and cockroaches. After the first trial exposure the survivors are tallied.

(16) No flies and few fleas survived.

Here *few fleas* has the strong reading, expressing a small proportion, substantially less than half of the set of fleas used in the test. Say the trial used 1000 fleas and 89 survived – then *Few fleas survived* is true. But if the trial used 160 fleas and 89 survived then *Few fleas survived* is false.

The weak reading of *few* does not express a proportion, as in (17).

(17) The house seemed clean and Lee found (very) few fleas.

This sentence just means that the number of fleas Lee found was small, and the fleas Lee found are not expressed as some proportion of a given set of fleas.

The strong/weak contrasts are less clear with *many*, and speakers differ more on whether or not *many* has a strong or proportional interpretation at all. For those speakers who consider *many* to have a proportional reading, *many* is like a weaker version of *most*: *Many* denotes a proportion greater than half, and *most* a proportion which is substantially greater than half of the background set (see also Exercise 15 for the meaning of *most*). For example, consider a class of 300 students voting on assessment methods.

(18) Many students preferred assignments to tests.

For proportional-*many* speakers, this is true only if more than 150 students preferred assignments, while for some speakers (including the writer) 100 students is a sufficiently large number to count as many, even though it is only a third of the total number of students, and the sentence is true if 100 students preferred assignments.

These judgments are quite sensitive to the size of the background set. Suppose the class has 24 students and eight of them preferred assignments. In this instance I am far less confident that many students preferred assignments to tests, because eight is not a large number, even though it represents the same proportion of the total as 100 out of 300. With the class of 24 students as background, it is more likely that numbers which count as large will be the same as numbers which are greater than half, in which case cardinality and proportionality cannot be distinguished.

Suppose there are six students in the class and five prefer assignments. The assignment-preferrers are substantially more than half the class, and it is true that most students prefer assignments, but because five is an absolutely small number it seems that *Many students prefer assignments* is an inappropriate and somewhat misleading way to describe the situation. Because no large numbers at all are involved, *many* does not apply. In short, it may be that *many* is really cardinal in all uses and simply denotes a large number.

Large is a predicate which must be interpreted in relation to a comparison standard: that is, whether or not a thing counts as large depends on what kind of thing it is. A common example of this is that a small elephant is very much larger than a large butterfly. *Large* and *small* do not have absolute values. When we talk about small elephants and large butterflies we can set the scale for largeness or smallness in comparison with the typical sizes of elephants and butterflies, which will fall somewhere near the middle of a fixed range. There is a maximum size and a minimum size for elephants (although the cut-off points are fuzzy). An elephant counts as large if it is considerably larger than an average-sized or typical elephant, and small if it is considerably smaller than an average-sized or typical elephant.

The largeness of numbers cannot be judged so easily without a relevant context because there is no upper limit on numbers in general, and so no fixed range to determine the typical or average number. (Negative numbers are not used in everyday talk about quantities, so generally zero is the lower limit on the relevant range of numbers.) Which numbers count as large varies with the context. In the examples above the total class size provides a number (300, 24, 8) as a standard for comparison – the comparison standard sets the overall scale for judging numbers as large or not. With a comparison range like 1–300 the numbers which count as large may begin at around 80 or 90, which is less than half the comparison upper limit. Against a range of 1–24 the numbers which count as large will generally be the numbers which are larger than the average, or midpoint, of the range: then many and most will be the same quantities. But if the whole scale is confined to small numbers then perhaps no number on the scale can count as many, even if numbers near the top of the scale can count as most. The uses of *many* which seem to be proportional are the uses where *many* picks out a large number of members of a known 'medium-sized' background set. The background set provides a scale for judging what is a large number, and numbers which count as large coincide with numbers greater than half the background set.

6.3.3 *Few* and *a few*

Cardinal *few* and *a few* both denote a small number, but they are not interchangeable. The difference between them reflects what kind of quantity is expected in the context. This is illustrated in (19):

(19) a. Spring was late in coming, and few flowers were blooming.
b. ?Spring was late in coming, and a few flowers were blooming.
c. ?Winter was ending at last, and few flowers were blooming.
d. Winter was ending at last, and a few flowers were blooming.

In (19a, b) the first clause *Spring was late in coming* suggests, or introduces an expectation that there may be no flowers in bloom, or that the number of flowers in bloom will be smaller than one might otherwise expect for the time of year. In other words, there are only a small number or **at most a small number** of flowers in bloom. With this 'at most, only (possibly none)' expectation, *few* is appropriate as in (19a) and *a few* is anomalous as in (19b).

In (19c, d), on the other hand, the clause *Winter was ending at last* introduces the expectation that flowers will be beginning to bloom. The small number of flowers in bloom is at least as many as one might have expected, or **at least a small number**. Here *a few* is appropriate as in (19d) and *few* is anomalous as in (19c).

The 'only n flowers' or 'at most n flowers' expectation with 'Spring was late' in contrast with 'Winter was ending' is also illustrated in (20):

(20) a. Spring was late in coming, and only five tulips were blooming.
b. ?Winter was ending at last, and only five tulips were blooming.

The appropriate combinations in (19) are reversed, however, if the clauses are joined with *but*, which signals a clash in expectation between the two parts of the statement.

(21) a. ?Spring was late in coming, but few flowers were blooming.
b. Spring was late in coming, but a few flowers were blooming.

c. Winter was ending at last, but few flowers were blooming.
d. ?Winter was ending at last, but a few flowers were blooming.

The meanings of *few* and *a few* are explored further in Exercise 9.

6.3.4 *Some* and *several*

Plural *some* and *several* are both cardinal determiners of vague plurality, not specified as either large or small. *Some* is defined here as the existential quantifier with singularity or plurality marked on the N′, as in (22).

(22)	Some dog is barking.	$\lvert D \cap B\rvert \geq 1$	'at least one'
	Some dogs are barking.	$\lvert D \cap B\rvert \geq 2$	'at least two'

Several seems to differ from *some* in requiring a slightly larger number than two as the lower limit. In particular, if two dogs are barking then *Some dogs are barking* is true but *Several dogs are barking* is false.

6.4 Restricted quantifier notation

We saw in Section 6.1 that none of the first order analyses for *most*, repeated in (23), was adequate.

(23) Most dogs are domestic.
a. Most x(DOG(x) & DOMESTIC(x))
b. Most x(DOG(x) $\vee$ DOMESTIC(x))
c. Most x(DOG(x) $\rightarrow$ DOMESTIC(x))
d. Most x(DOG(x) $\leftrightarrow$ DOMESTIC(x)

Generalized Quantifier Theory offers a satisfactory semantic analysis, but there is still a notational problem – how do we form a simple representation for statements like *Most dogs are domestic*?

Ideally, a notation should also reflect the fact that the determiner usually combines with an N′ predicate to form a noun phrase. That is, natural language quantification is generally in the format *all conjurors* or *several motoring enthusiasts*, where the rest of the noun phrase specifies what kind of thing can be a value for the variable, rather than the fully general forms *everything*, *something*, and *nothing*. Another way to look at this is to compare the variables in natural language quantification with **restricted variables**. Recall that different kinds of conventional restrictions can be shown in the form of variables*: x, y,* and *z* stand for entities; *p, q,* and *r* stand for propositions, more specifically *w* stands for worlds and as we shall see *t* stands for times and *e* stands for events. In short, **the N′ part of a quantifier NP restricts the variable**. Correspondingly, the kind of quantifier expressed by a noun phrase is a **restricted quantifier**, illustrated in (24). The restricted quantifier corresponds to the whole noun phrase.

true, as the examples below indicate:

(69) Rex has been buying vintage cars in a remote country district, and was delighted with his purchases. *Several cars had not left the garage in 30 years.*
[Several x: CAR(x)] [The y: GARAGE(y)] ~ LEAVE(x, y)

(70) When the car-hire firm was wound up, *several cars had not left the garage in 30 years.*
skip this
[The x: GARAGE(x)] [Several y: CAR(y)] ~ LEAVE(y, x)

In (69), each car introduces a sub-domain in which there is a unique garage, and the quantifier is interpreted as picking out the unique garage for each car. In (70), *the garage* is interpreted from the context or the preceding discourse, and the sentence is interpreted as being about the car-hire firm's large commercial garage. As the formulae show, the difference can be represented as a difference in scope of the two quantifier determiners, *the* and *several*, as well as the different extra information added by implicature.

The most important types of scopal ambiguity with *the* do not involve another quantifier, like the examples here. They involve interactions between definite descriptions and modal expressions or certain kinds of verbs, which fall under the rubric of referential opacity, addressed in Chapter 7.

start here

6.9 Quantifiers and negative polarity items

Negative Polarity Items (**NPIs** or **negpols** for short) are expressions which can only occur in special contexts, including contexts which are in some sense in the scope of negation. Idiomatic NPIs include *budge an inch* and *lift a finger*, as illustrated in (71). The negative expression is underlined.

(71) a. Nobody lifted a finger to stop him.
b. #Several people lifted a finger to stop him.
c. I don't suppose they'll lift a finger to help.
d. #I suppose they'll lift a finger to help.
e. He won't budge an inch on this issue.
f. #He might budge an inch on this issue.
g. For all their efforts the trailer never budged an inch.
h. #After all their efforts at last the trailer budged an inch.

The commonest NPIs are *any* (*anyone*, *anything*) and *ever*.

(72) a. Sue won't ever go there again.
b. #Sue will ever go there again.
c. The office hasn't notified anyone.
d. #The office has notified anyone.

Despite their name, NPIs are not actually confined to negative contexts, and occur with some quantifier determiners (in addition to *no*). As the examples in

(73) with NPI *ever* show, the NPI may appear in N′ (the (a) examples) or in VP (the (b) examples) or in both:

(73) *every*
  a. [Everyone who has ever been to Belltree Island] will want to go back.
  b. #[Everyone who has been to Belltree Island] will ever want to go back.

(74) *no*
  a. [No one who has ever been to Belltree Island] will want to go back.
  b. [No one who has been to Belltree Island] will ever want to go back.

(75) *few* (weak *few*)
  a. [Few people who have ever been to Belltree Island] will want to go back.
  b. [Few people who have been to Belltree Island] will ever want to go back.

(76) *some*
  a. #[Someone who has ever been to Belltree Island] will want to go back.
  b. #[Someone who has been to Belltree Island] will ever want to go back.

(77) *four*
  a. #[Four people who have ever been to Belltree Island] will want to go back.
  b. #[Four people who have been to Belltree Island] will ever want to go back.

These examples show that the NPI *ever* is licensed in N′ with *every*, *no* and *few*, but not with *some* or *four*, and is licensed in VP with *no* and *few*, but not with *every*, *some* or *four*. The results are summarized in (78):

(78)

	***ever* in N′**	***ever* in VP**
every	yes	no
no	yes	yes
few	yes	yes
some	no	no
four	no	no

Ladusaw (1980) identified the contexts which license NPIs as **downward-entailing environments**. (When A entails B, if A is true then B must also be true – B is an entailment of A.) The entailing environments of interest in Ladusaw's analysis are N′ and VP. In a sentence *Det Fs are G*, N′ denotes the F set and VP denotes the G set. Whether an N′ or VP is downward-entailing or upward-entailing depends on the determiner.

The N′ environment can be tested with the frames in (79):

(79) a. If 'Det Fs are G' entails 'Det Es are G' and $E \subseteq F$, then F is **downward-entailing**. The entailment is **towards the subset**.
b. If 'Det Fs are G' entails 'Det Es are G' and $F \subseteq E$, then F is **upward-entailing**. The entailment is **towards the superset**.

Given that the set of large dogs is a subset of the set of dogs, we can use the test sentences *Det dogs are white* and *Det large dogs are white*. Entailment from the *dogs* sentence to the *large dogs* sentence is entailment towards the subset, and is a downward entailment. Entailment from the l*arge dogs* sentence to the *dogs* sentence is an entailment towards the superset, and is an upward entailment.

The VP environment can be tested with the frames in (80):

(80) a. If 'Det Fs are G' entails 'Det Fs are H' and $H \subseteq G$, then G is downward-entailing. The entailment is towards the subset.
b. If 'Det Fs are G' entails 'Det Fs are H' and $G \subseteq H$, then G is upward-entailing. The entailment is towards the superset.

Given that the set of people whistling loudly is a subset of the set of people whistling, the VP test sentences can be *Det N is/are whistling* and *Det N is/are whistling loudly*. An entailment from the *whistling* sentence to the *whistling loudly* sentence is a downward entailment, and an entailment from the *whistling loudly* sentence to the *whistling* sentence is an upward entailment.

The tests for the different determiners are shown below. Note that *few* in (83) is weak *few*.

(81) *Every* in N′: DOWNWARD
a. *Every dog is white* entails *Every large dog is white*.
b. *Every large dog is white* does not entail *Every dog is white*.

Every in VP: UPWARD
c. *Everyone is whistling* does not entail *Everyone is whistling loudly*.
d. *Everyone is whistling loudly* entails *Everyone is whistling*.

(82) *No* in N′: DOWNWARD
a. *No dogs are white* entails *No large dogs are white*.
b. *No large dogs are white* does not entail *No dogs are white*.

No in VP: DOWNWARD
c. *No one is whistling* entails *No one is whistling loudly*.
d. *No one is whistling loudly* does not entail *No one is whistling*.

(83) *Few* in N′: DOWNWARD
a. *Few dogs are white* entails *Few large dogs are white*.
b. *Few large dogs are white* does not entail *Few dogs are white*.

Few in VP: DOWNWARD
c. *Few people are whistling* entails *Few people are whistling loudly.*
d. *Few people are whistling loudly* does not entail *Few people are whistling.*

(84) *Some* in N′: UPWARD
a. *Some dogs are white* does not entail *Some large dogs are white.*
b. *Some large dogs are white* entails *Some dogs are white.*

Some in VP: UPWARD
a. *Someone is whistling* does not entail *Someone is whistling loudly.*
b. *Someone is whistling loudly* entails *Someone is whistling.*

(85) *Four* in N′: UPWARD
a. *Four dogs are white* does not entail *Four large dogs are white.*
b. *Four large dogs are white* entails *Four dogs are white.*

Four in VP: UPWARD
a. *Four people are whistling* does not entail *Four people are whistling loudly.*
b. *Four people are whistling loudly* entails *Four people are whistling.*

The downward and upward entailments are summarized in (86):

(86)

	N′	VP
every	down	up
no	down	down
few	down	down
some	up	up
four	up	up

The environments which allow negative polarity items were listed in (78), repeated here in (87). As we see, the NPI licensing environments are exactly the downward-entailing environments, as predicted.

(87)

	***ever* in N′**	***ever* in VP**
every	yes	no
no	yes	yes
few	yes	yes
some	no	no
four	no	no

6.10 Generalized quantifiers as lambda functions

Given that a quantifier determiner expresses a relation between sets, it is a function that takes two predicates to form a proposition. The two arguments to the determiner are the predicate expressed by N′, which is of type <e, t>,