SEF015: Discrete Mathematics

Tutorial 5 - Solutions -

Question 1

a) $\{ n \in \mathbb{Z} : n \geq 0 \wedge 2 | n \}$

b) $\{ n \in \mathbb{Z} : n > 0 \wedge 65 | n \}$

c) $\{ x \in \mathbb{R} : 1 + 2x + 3x^2 + 4x^3 + 5x^4 = 0 \}$

Question 2

$x \in (A \setminus B) \cup (B \setminus A) \iff (x \in A \land x \notin B) \lor (x \in B \land x \notin A)$

$\iff [(x \in A \land x \notin B) \lor x \in B] \land [(x \in A \land x \notin B) \lor x \notin A]$

$\because$ Distributive Law

$\iff [(x \in A \lor x \in B) \land (x \notin B \lor x \in B)] \land$

$[(x \in A \lor x \notin A) \land (x \notin B \lor x \notin A)]$

$\because$ Distributive Law

$\iff [x \in A \cup B \land x \in U] \land [x \in U \land (\neg x \in B \lor \neg x \in A)]$

$\iff x \in [(A \cup B) \cap U] \land [x \in U \land \neg (x \in B \land x \in A)]$

$\because$ De Morgan Law

$\iff x \in (A \cup B) \land [x \in U \land \neg (x \in A \land x \in B)]$

$\because$ Commutative Law

$\iff x \in (A \cup B) \land [x \in U \land x \notin A \cap B]$

$\iff x \in (A \cup B) \land x \notin (A \cap B)$

$\iff x \in (A \cup B) \setminus (A \cap B)$

So, $\boxed{(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)}$

Question 3

$$x \in A \setminus B \iff x \in A \wedge x \notin B$$

$$\iff x \in A \wedge (x \in U \wedge x \notin B)$$

$$\iff x \in A \wedge x \in B^c$$

$$\iff x \in A \cap B^c$$

So, $\boxed{A \setminus B = A \cap B^c}$

Question 3 (continued)

$$x \in (A \cap B)^c \iff x \in \mathcal{U} \wedge x \notin A \cap B$$

$$\iff x \in \mathcal{U} \wedge \neg(x \in A \cap B)$$

$$\iff x \in \mathcal{U} \wedge \neg(x \in A \wedge x \in B)$$

$$\iff x \in \mathcal{U} \wedge (\neg x \in A \vee \neg x \in B)$$

$\because$ De Morgan Law

$$\iff (x \in \mathcal{U} \wedge \neg x \in A) \vee (x \in \mathcal{U} \wedge \neg x \in B)$$

$\because$ Distributive Law

$$\iff (x \in \mathcal{U} \wedge x \notin A) \vee (x \in \mathcal{U} \wedge x \notin B)$$

$$\iff x \in A^c \vee x \in B^c$$

$$\iff x \in A^c \cup B^c$$

So, $\boxed{(A \cap B)^c = A^c \cup B^c}$

Homework: Try the other similar law:

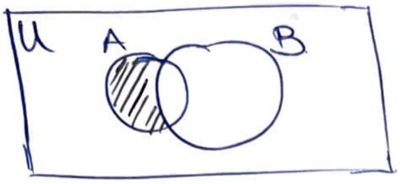
$$(A \cup B)^c = A^c \cap B^c$$

Can you show this?

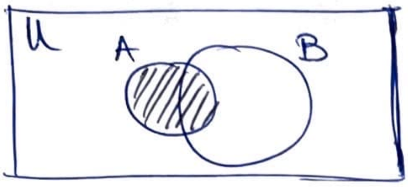
Question 3 (continued)

Venn diagrams.

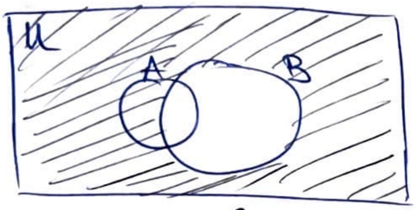
$$\boxed{A \setminus B = A \cap B^c}$$



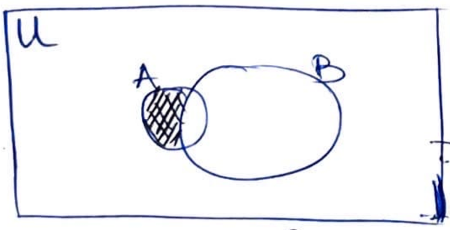
$A \setminus B$



A



B^c



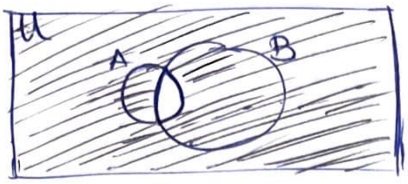
$A \cap B^c$.

Both Venn diagrams have same shaded region.
(for the case when $A \cap B \neq \emptyset$)

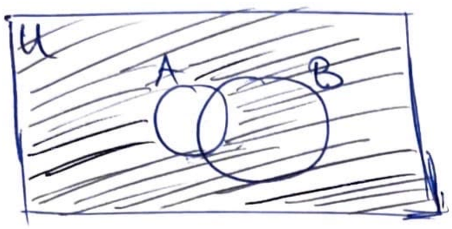
Question 3 (continued)

Venn diagrams

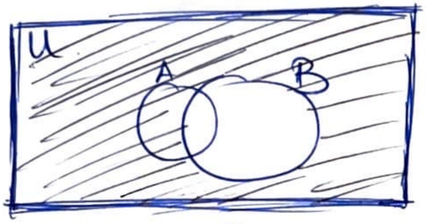
$$\boxed{(A \cap B)^c = A^c \cup B^c}$$



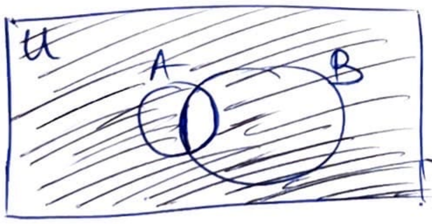
$(A \cap B)^c$



A^c



B^c



$A^c \cup B^c$

Both diagrams are the same.. Therefore,

$$\boxed{(A \cap B)^c = A^c \cup B^c}$$

Question 1

p	q	$\neg p$	$\neg p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

The highlighted columns in the truth table are same which shows that these two statements are equivalent.

Question 5

$$(A \subseteq C) \wedge (B \subseteq C) \iff (\forall x, x \in A \rightarrow x \in C) \wedge (\forall x, x \in B \rightarrow x \in C)$$

$\because A \subseteq B \iff (\forall x, x \in A \rightarrow x \in B)$

$$\iff \forall x, [(x \in A \rightarrow x \in C) \wedge (x \in B \rightarrow x \in C)]$$

Note that:

$[\forall x, p(x)] \wedge [\forall x, q(x)]$

is equivalent to

$\forall x, (p(x) \wedge q(x))$.

This is not TRUE for $\vee$ ("or")!

$$\iff \forall x, [(x \notin A \vee x \in C) \wedge (x \notin B \vee x \in C)]$$

$$\iff \forall x, [(x \notin A \wedge x \notin B) \vee x \in C]$$

$\because$ Distributive Law

$$\iff \forall x, [(\neg x \in A \wedge \neg x \in B) \vee x \in C]$$

$$\iff \forall x, [\neg (x \in A \vee x \in B) \vee x \in C]$$

$\because$ De Morgan Law

$$\iff \forall x, [\neg (x \in A \cup B) \vee x \in C]$$

Question 5 (continued)

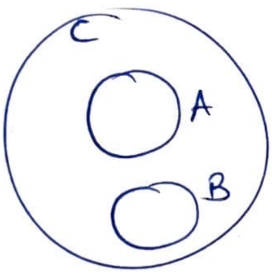
$$(A \subseteq C) \land (B \subseteq C) \Leftrightarrow \forall x, (x \notin A \cup B \lor x \in C)$$

$$\Leftrightarrow \forall x, (x \in A \cup B \to x \in C)$$

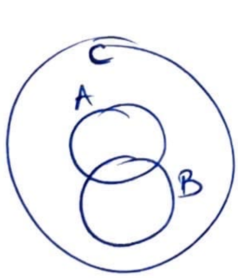
$$\Leftrightarrow (A \cup B) \subseteq C$$

So, $\boxed{(A \subseteq C) \land (B \subseteq C) \Leftrightarrow (A \cup B) \subseteq C}$

Venn diagram :



, if $A \cap B = \emptyset$



, if $A \cap B \neq \emptyset$

Reminder:

$\forall$: for all / for any / for every

$\exists$: there is | there exists
there are | there exist

$\therefore$: Therefore.

$\because$: Because.

Please check the MathHubs on our QMPlus page.