SEF015: Discrete Mathematics (2022-23)

Material for the Q&A session *or…Tutorial 5 (Week 6)*

This material is for your tutorial in Week 6 and is designed to help your understanding. Please try to answer all the questions before you join your tutorial group.

Number of pages: 2

Question 1*. Write the following sets using propositional functions (that is write them in the form {x ∈ X | p(x)} for some propositional function p(x))

(a) {0, 2, 4, 6, 8, . . .}.
(b) The set of positive integers divisible by both 5 and 13.
(c) The set of (real) roots of the polynomial $1 + 2x + 3x^2 + 4x^3 + 5x^4$. (Hint: You do not need to factor the polynomial or find its roots. Just write down the set.

Question 2*. Let A , B be subsets of some universal set U. Use logical arguments to prove that

$$(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B).$$

Question 3*. Let A , B be subsets of some universal set U. Using logical arguments and Venn diagrams, prove that the following:

$$A - B = A \cap B^c, \quad (A \cap B)^c = A^c \cup B^c.$$

Question 4. Prove that $\neg p \vee q$ and $p \rightarrow q$ are equivalent statements.

Question 5. Let A, B, C be sets. What is the logical representation of $A \subseteq B$. Use a logical argument to prove that

$$(A \subseteq C) \wedge (B \subseteq C) \Leftrightarrow (A \cup B) \subseteq C.$$

Illustrate using Venn diagram.

Question 6. State whether the following are true and explain why.

If this is true, this can be done by arguing the proposition is true or demonstrating the negation is false.

If it is false, then it can be done by demonstrating the proposition is false or that the negation is true.

Assume that $n \in Z$ and $x, y, \varepsilon, \delta \in R$. If a condition is written after $\forall$ or $\exists$, this implies that a smaller set is taken, for example $\forall\ \varepsilon > 0$ means for all positive (and real) ε.

(a) $\forall\ x, \exists\ y, 3x^2 - 2y = 5$,

(b) $\exists\ x, \forall\ y, 3x^2 - 2y = 5$,

(c) $\forall\ x, \exists\ y, 3x^2 - 2y^2 = 5$,

(d) $\exists\ x, \forall\ y, 3x^2 - 2y^2 = 5$,

(e) $\forall\ \varepsilon > 0, \exists\ n, 2 - n < \varepsilon$.

Question 7. Write down the negation of the following propositional functions and state whether the statement or its negation is true. You can assume n is an integer and x and y are real numbers.

(a) $\forall\ n, (n \leq 10) \wedge (n > 0)$,

 (b) $\exists\ n, (n \neq 0) \rightarrow (n^2 > 0)$,

 (c) $\exists\ x, \exists\ y, (y = 3x - 10) \wedge (y = -x + 2)$,

 (d) $\forall\ x, \exists\ y, x^2 + y^2 = 1$.

Question 8. Let A, B be subsets of some universal set U. Using logical arguments, prove that

$A \subseteq B \Leftrightarrow B^c \subseteq A^c$.

Question 9. Write down the power set $\wp(X)$ of $X = \{a, b, c\}$.