SEF015: Discrete Mathematics (2022-23)

Tutorial 4 (Week 5) - Solutions

Question 1. For the sets A, B, C, D,

$$A = \{1, 2, 3, \ldots, 20\},$$
$$B = \{2, 4, 6, \ldots, 20\},$$
$$C = \{x : x \text{ is a prime number between 4 and 20}\},$$
$$D = \text{The set of first five positive odd numbers.}$$

which of the following is true and which is false?

a.	$A \subseteq B$	F
b.	$B \subseteq C$	F
c.	$C \subseteq A$	T
d.	$A \subseteq D$	F
e.	$D \subseteq A$	T
f.	$D \subseteq B$	F
g.	$1 \in A$	T
h.	$\{1\} \in A$	F
i.	$\{1, 2, 3, \ldots, 20\} \in A$	F
j.	$\{1, 2, 3, \ldots, 20\} \subseteq A$	T
k.	$19 \notin C$	F
l.	$\{9\} \notin D$	T
m.	$A \cap B = B = B \cap A$	T
n.	$A \cup B = A = B \cup A$	T
o.	$C \cup D = \varphi$	F
p.	$B \cap D = \varphi$	T

Question 2. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Write down the following sets explicitly:

a) $\{x \in U \mid x > 9\}$;

$\{10\}$

b) $\{x \in U \mid x > 2\}$;

$\{3,4,5,6,7,8,9,10\}$

c) $\{ x \in U \mid x = 3k \ for \ some \ k \in Z \}$;

   {0,3,6,9}

d) $\{ x \in U \mid x = 11k \ for \ some \ k \in Z \}$;

   {0}

e) $\{x \in U \mid (2 < x < 6) \lor (x = 6k \ for \ some \ k \in Z)\}$

   {3,4,5} ∪ {0,6} ={0,3,4,5,6}

Question 3. Let $A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$, $C = \{b, e, f, g\}$.
Verify the following:
   a) $A \cup B = B \cup A$

$$A \cup B = \{a, b, c, d, e, g\} = B \cup A$$

   b) $A \cap B = B \cap A$

$$A \cap B = \{a, c, e\} = B \cap A$$

   c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$A \cup (B \cap C) = \{a, b, c, d, e\} \cup (\{a, c, e, g\} \cap \{b, e, f, g\})$$
$$= \{a, b, c, d, e\} \cup \{ e, g\} = \{a, b, c, d, e, g\}$$

$$(A \cup B) \cap (A \cup C) = \{a, b, c, d, e, g\} \cap \{a, b, c, d, e, f, g\}$$
$$= \{a, b, c, d, e, g\}$$

Left hand side is same as right hand side.

Question 4. Let $\xi = \{0, 1, 2, 3, \ldots, 100\}$, here ξ is the universal set and with the following sets
$$A = \{1, 2, 3, \ldots, 50\},$$
$$B = \{0, 2, 4, 6, \ldots, 100\},$$
$$C = \{x: x \text{ is a prime number between } 1 \text{ and } 20\},$$
$$D = \text{The set of first } 10 \text{ positive multiples of } 10.$$

Investigate:

a) $A - B = \{1, 2, 3, \ldots, 50\} - \{0, 2, 4, 6, \ldots, 100\} = \{1, 3, 5, \ldots, 49\}$

   $B - A = \{0, 2, 4, 6, \ldots, 100\} - \{1, 2, 3, \ldots, 50\} = \{0, 52, 54, 56, \ldots, 100\}$

b) $A^c = \{0, 1, 2, 3, \ldots, 100\} - \{1, 2, 3, \ldots, 50\} = \{0, 51, 52, 53, \ldots, 100\}$

   $B^c = \{0, 1, 2, 3, \ldots, 100\} - \{0, 2, 4, 6, \ldots, 100\} = \{1, 3, 5, \ldots, 99\}$

   $D^c = \{0, 1, 2, 3, \ldots, 100\} - \{10, 20, 30, \ldots, 100\}$
   $= \{0, 1, 2, 3, \ldots, 9, 11, 12, 13, \ldots 19, 21, 22, 23, \ldots 29, 31, 32, 33, \ldots 39, 41, 42, 43, \ldots, 99\}$