Naive Set Theory

PART 1

Naive Set Theory

A set is a collection of distinct objects.

↳ The collection must be well-defined.

↳ Capital letters are usually used to denote sets.

↳ There are two ways to specify a particular set:

1) The set A of the first 6 odd integers is denoted as:

$$A = \{1, 3, 5, 7, 9, 11\}$$

or alternatively,

2) $A = \{x \mid x \text{ is an odd integer, } 1 \leq x \leq 11\}$

Members belonging to a set are denoted as:

$a \in S$, with a belonging to the set S

If $A = \{1, 3, 5, 7, 9, 11\}$,

$1 \in A$, $5 \in A$ but $4 \notin A$

(4 does <u>not</u> belong to A)

Examples of well-known sets

- $\mathbb{N} = \{0, 1, 2, 3, \dots\}$, set of Natural numbers
- $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$, set of Integers
- $\mathbb{Q} = \left\{ \frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0 \right\}$
- $\mathbb{R}$, the set of Real numbers

- $\{(x,y): x \in \mathbb{R}, y \in \mathbb{R}\}$

  The set of points in the (Euclidean) plane $\mathbb{R}^2$

- The empty set: $\{\ \}$ or $\emptyset$

Universal Set

All sets of interest (under investigation) are assumed to belong to some larger set, called the universal set, which is usually denoted by U.

Subsets

A set A is said to be a subset of a set B if: every element in set A is also an element of set B.

We can also say that <u>A is contained in B</u> or <u>B contains A</u>.

We denote it as:

$$\boxed{A \subseteq B}$$

The sets A and B are said to be <u>equal</u> if: both have the same elements <u>or</u> <u>equivalently if</u>: each is contained in the other, i.e.

$$\boxed{A = B \text{ if and only if } A \subseteq B \text{ and } B \subseteq A.}$$

If A is <u>not</u> a subset of B,
then there exists at least one element
of A that does not belong to B.
We write : $A \not\subseteq B$.

<u>Examples</u>:

- $\{1,2\} \subseteq \{1,2,3\}$
- $\{1,2,3\} \subseteq \{1,2,3\}$
- $\emptyset \subseteq \{1,2,3\}$
- $\{1\} \subseteq \{1,2,3\}$
- $\{1,4\} \not\subseteq \{1,2,3\}$
- $\{1,\{2,3\}\} \not\subseteq \{1,2,3\}$
- $\{2,3\} \subseteq \{1,\{2,3\}\}$
- $1 \in \{1,2,3\}$ <u>but</u> $1 \not\subseteq \{1,2,3\}$

(*) $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$, $\emptyset \subseteq A \subseteq U$.

Operations on Sets

Let A and B be sets.

- **Union** [or]

$$A \cup B = \{ x : x \in A \vee x \in B \}$$

- **Intersection** [and]

$$A \cap B = \{ x : x \in A \wedge x \in B \}$$

- **Difference**

$$A \setminus B = \{ x : x \in A \wedge x \notin B \}$$

- **Complement**

$$A^c = A' = U \setminus A = \{ x : x \notin A \}$$

Examples

$U = \{1, 2, 3, \dots, 30\}$

$A = \{2, 4, 6, \dots, 30\}$

$B = \{4, 8, 12, \dots, 28\}$

$C = \{1, 3, 5, \dots, 29\}$

$D = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$.

Homework :

$A \cup B = \dots$

$A \cup C = \dots$

$A \cap B = \dots$

$A \cap C = \dots$

$C \cap D = \dots$

$A \cap D = \dots$

$A \setminus B = \dots$

$C \setminus D = \dots$

$A^c = \dots$

$B^c = \dots$

Some properties of set operations:

For any sets $A, B, C, \ldots$

- **Commutative law**

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

- **Associative law**

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

- **Distributive law**

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Also,

$$A \cup A = A \cap A = A \cup \emptyset = \emptyset \cup A = A \setminus \emptyset = A$$

Disjoint Sets

Two sets A and B are said to be disjoint if:
they have no elements in common.
For example,

$$A = \{1, 2\}, \quad B = \{4, 5, 6\}.$$

Mathematically formulated,
A and B are disjoint, if $A \cap B = \emptyset$.

Proper Subset

A is said to be a proper subset of B, if A is a subset of B and $A \neq B$.

It is denoted by $\boxed{A \subset B}$

e.g. $A = \{1, 2, 3\}$, $B = \{1, 2, 3, \dots, 10\}$,

$$A \subset B.$$