Queen Mary
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SEF015: Discrete Mathematics (2022-23)

Tutorial 3 (Week 4) - Solutions

Question 1. Since $f(2) = 2^3 + 2.2^2 - 5.2 - 6 = 8 + 8 - 10 - 6 = 0$, so 2 is a root and $x - 2$ is a factor of $f(x)$ or $x - 2$ divides $f(x)$. The division of $f(x)$ by $x - 2$ gives $x^2 + 4x + 3$ (with remainder 0). Now, by inspection (or the quadratic formula), we can find $x^2 + 4x + 3 = (x + 3)(x + 1)$. Therefore

$$x^3 + 2x^2 - 5x - 6 = (x + 3)(x + 1)(x - 2).$$

Question 2. The truth table for $\neg(\neg p \wedge q)$ is

p	q	$\neg p$	$\neg p \wedge q$	$\neg(\neg p \wedge q)$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Question 3. The truth table for $(p \wedge r) \vee (q \wedge r)$ is

p	q	r	$p \wedge r$	$q \wedge r$	$(p \wedge r) \vee (q \wedge r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	F	T
T	F	F	F	F	F
F	T	T	F	T	T
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

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Question 7. The negation, converse, inverse, and contrapositive of the implication
p → q are given below:

Negation (N): $\neg(p \to q) \equiv p \wedge \neg q$;
Converse (Cv): $q \to p$;
Inverse (In): $\neg p \to \neg q$;
Contrapositive (Co): $\neg q \to \neg p$.

p	q	$\neg p$	$\neg q$	$p \to q$	N	Cv	In	Co
F	F	T	T	T	F	T	T	T
F	T	T	F	T	F	F	F	T
T	F	F	T	F	T	T	T	F
T	T	F	F	T	F	T	T	T

It can be seen that the inverse and converse are logically equivalent, and the implication and contrapositive are logically equivalent.

Question 8:

(a) "If the grass is green, then it has rained in the last month".

– Converse: "If it has rained in the last month, then the grass is green".
– Inverse: "If the grass is not green, then it has not rained in the last month".
– Contrapositive: "If it has not rained in the last month, then the grass is not green".
– Negation: "The grass is green and it has not rained in the last month".

(b) "(Let n be an integer.) If n is odd then n^2 is odd".

– Converse: "If n^2 is odd then n is odd".
– Inverse: "If n is even then n^2 is even".
– Contrapositive: "If n^2 is even then n is even".
– Negation: "n is odd and n^2 is even".

(c) "(Let $f(x)$ be a polynomial.) If $f(a) = 0$ then $(x - a)$ divides f".

– Converse: "If $(x - a)$ divides f then $f(a) = 0$".
– Inverse: "If $f(a) \neq 0$ then $(x - a)$ does not divide f".
– Contrapositive: "If $(x - a)$ does not divide f then $f(a) \neq 0$".
– Negation: "$f(a) = 0$ and $(x - a)$ does not divide f".

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