Queen Mary University of London

SEF015: Discrete Mathematics (2022-23)

Tutorial 2 (Week 3) - Solutions

Question 1. Since $f(2) = 2^3 + 2.2^2 - 5.2 - 6 = 8 + 8 - 10 - 6 = 0$, so 2 is a root and $x - 2$ is a factor of $f(x)$ or $x - 2$ divides $f(x)$. The division of $f(x)$ by $x - 2$ gives $x^2 + 4x + 3$ (with remainder 0). Now, by inspection (or the quadratic formula), we can find $x^2 + 4x + 3 = (x + 3)(x + 1)$. Therefore

$$x^3 + 2x^2 - 5x - 6 = (x + 3)(x + 1)(x - 2).$$

Question 2. The truth table for $\neg(\neg p \wedge q)$ is

p	q	$\neg p$	$\neg p \wedge q$	$\neg(\neg p \wedge q)$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Question 3. The truth table for $(p \wedge r) \vee (q \wedge r)$ is

p	q	r	$p \wedge r$	$q \wedge r$	$(p \wedge r) \vee (q \wedge r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	F	T
T	F	F	F	F	F
F	T	T	F	T	T
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

Question 4. The truth table for $(p \vee \neg q) \leftrightarrow r$ is

p	q	$\neg q$	$p \vee \neg q$	r	$(p \vee \neg q) \leftrightarrow r$
T	T	F	T	T	T
T	T	F	T	F	F
T	F	T	T	T	T
T	F	T	T	F	F
F	T	F	F	T	F
F	T	F	F	F	T
F	F	T	T	T	T
F	F	T	T	F	F

Question 5. The truth table for $p \rightarrow (\neg r \vee q)$ is

p	q	r	$\neg r$	$q \vee \neg r$	$\neg r \vee q$	$p \rightarrow (\neg r \vee q)$
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	F	F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	F	T
F	F	F	T	T	T	T

Question 6. a) $\neg p \vee (p \vee q)$

p	$\neg p$	q	$p \vee q$	$\neg p \vee (p \vee q)$
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T

a) $(p \rightarrow r) \rightarrow (p \rightarrow (q \vee r))$

p	q	r	$p \rightarrow r$	$q \vee r$	$p \rightarrow (q \vee r)$	a) $(p \rightarrow r) \rightarrow (p \rightarrow (q \vee r))$
T	T	T	T	T	T	T
T	T	F	F	T	T	T
T	F	T	T	T	T	T
T	F	F	F	F	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

a) See the last column in both the above tables, all true. So the statements $\neg p \vee (p \vee q)$ and $(p \rightarrow r) \rightarrow (p \rightarrow (q \vee r))$ are tautologies.