Queen Mary
University of London

SEF015: Discrete Mathematics (2022-23)

Tutorial 1 (Week 2) – Solutions

Question 1.

$$(1 + x^3 + x^5)(1 - x^3 - x^5) = 1(1 - x^3 - x^5) + x^3(1 - x^3 - x^5) + x^5(1 - x^3 - x^5)$$

$$= 1 - x^3 - x^5 + x^3 - x^6 - x^8 + x^5 - x^8 - x^{10}$$

$$= 1 - x^6 - 2x^8 - x^{10}$$

$$= - x^{10} - 2x^8 - x^6 + 1$$

Question 2. *i)* $x + 2x^2 + 3x^3 + 4x^4 + \cdots + 100x^{100} = \sum_{i=1}^{100} ix^i$,

$$ii)\ 5 + 10x + 15x^2 + 20x^3 + 25x^4 = \sum_{i=1}^{5} 5ix^{i-1} \ \ or$$

$$5 + 10x + 15x^2 + 20x^3 + 25x^4 = \sum_{i=0}^{4} 5(i+1)x^i$$

Question 3.

$$\left(\sum_{i=0}^{2} a_i x^i\right)\left(\sum_{j=0}^{3} b_j x^j\right) = (a_0 + a_1x + a_2x^2)(b_0 + b_1x + b_2x^2 + b_3x^3)$$
$$= a_0b_0 + (a_0b_1 + a_1b_0)x + (a_0b_2 + a_1b_1 + a_2b_0)x^2$$
$$+ (a_0b_3 + a_1b_2 + a_2b_1)x^3 + (a_1b_3 + a_2b_2)x^4 + a_2b_3x^5$$

Question 4.

$$(x - 1)(x^2 - 2)(x^3 - 3)(x^4 - 4) = \prod_{i=1}^{4}(x^i - i) = \prod_{j=1}^{4}(x^j - j)$$

Question 5. Divide $8x^6 - 6x^4 + 8x^3 - 7x + 2$ by $2x^3 + x + 1$, finding the quotient $q(x)$ and remainder $r(x)$. Verify that the quotient times $2x^3 + x + 1$ plus the remainder is the original polynomial.

$$
\begin{array}{l}
2x^3 + x + 1 \;\; \overline{)\, 8x^6 - 6x^4 + 8x^3 - 7x + 2} \;(\, 4x^3 - 5x + 2 \\
\qquad\qquad\qquad\;\; \pm 8x^6 \pm 4x^4 \pm 4x^3 \\
\qquad\qquad\qquad\;\; \overline{\qquad - 10x^4 + 4x^3 - 7x + 2} \\
\qquad\qquad\qquad\qquad\;\; \mp 10x^4 \mp 5x^2 \mp 5x \\
\qquad\qquad\qquad\qquad\;\; \overline{\qquad 4x^3 + 5x^2 - 2x + 2} \\
\qquad\qquad\qquad\qquad\qquad \pm 4x^3 \qquad \pm 2x \pm 2 \\
\qquad\qquad\qquad\qquad\qquad \overline{\qquad 5x^2 - 4x \qquad}
\end{array}
$$

Verification:

$$(4x^3 - 5x + 2)(2x^3 + x + 1) + 5x^2 - 4x = 8x^6 - 6x^4 + 8x^3 - 7x + 2$$

Question 6. Divide $x - 4$ by $x^3 - 7x^2 + 8x + 16$.

$$
\begin{array}{l}
x - 4 \;\; \overline{)\, x^3 - 7x^2 + 8x + 16} \;(\, x^2 - 3x - 4 \\
\qquad\quad\; \pm x^3 \mp 4x^2 \\
\qquad\quad\; \overline{\qquad - 3x^2 + 8x + 16} \\
\qquad\qquad\quad \mp 3x^2 \pm 12x \\
\qquad\qquad\quad \overline{\qquad - 4x + 16} \\
\qquad\qquad\qquad\quad \mp 4x \pm 16 \\
\qquad\qquad\qquad\quad \overline{\qquad 0 \qquad}
\end{array}
$$

Question 7.

$$i)\ Commutative\ property$$

$$ii) Associative\ property$$

Question 8. As we discussed in detail during our tutorials,

$i)$

$$a0 = a(0+0) \Rightarrow$$

$$a0 = a0 + a0 \xRightarrow{-(a0)}$$

$$0 = a0$$

$ii)$ Based on $i)$,

$$a(b-b) = 0 \Rightarrow$$

$$ab + a(-b) = 0 \xRightarrow{-(ab)}$$

$$a(-b) = -(ab)$$

$iii)$ Based on $ii)$, setting $b = 1$ yields

$$a(-1) = -a$$

$iv)$ From $ii)$ we have

$$a(-b) = -(ab) \xRightarrow{*(-1))}$$

$$(-1)\, a(-b) = (-1)(-ab) \xRightarrow{(iii)}$$

$$(-a)(-b) = ab$$

$iv)$ Let us assume $ab = 0$ and $a \neq 0$ and $b \neq 0$. Since $a \neq 0$ and $b \neq 0$, $a^{(-1)}$ and $b^{(-1)}$ exist. Thus, assuming $ab = 0$,

$$ab = 0 \xRightarrow{*(a^{(-1)}b^{(-1)})}$$

$$a^{(-1)}b^{(-1)}ab = 0 \Rightarrow$$

$$(a^{(-1)}a)b^{(-1)}b = 0 \Rightarrow$$

$$1 \cdot 1 = 0 \Rightarrow$$

$$1 = 0$$

,which is false. Therefore, if $ab = 0$, then $a = 0$ or $b = 0$.