Coursework 2 2022-2023

1. **Dynamical systems**

CLOZE 0.10 penalty

a) Which of the following systems of ODEs is autonomous and dynamical?

MULTI 1 point Multiple Shuffle

• I ✓ • II • III,

where
I: $\dot{y_1} = e^{y_1} - \sin(y_2), \quad \dot{y_2} = y_1 + y_2$
II: $\frac{dy_1}{dx} = 5y_1, \quad \frac{dy_2}{dx} = -y_2$
III: ${y_1}' = 3y_2, \quad {y_2}' = y_1 y_2 - y_2$

b) Find out which of the following options are equilibria of the dynamical system,

$$\dot{y_1} = e^{y_1 y_2} - 1, \quad \dot{y_2} = (y_1 - 3)y_2$$

MULTI 2 points Multiple Shuffle

• I (50%) • II (50%) • III • IV

where I:$(y_1^*, y_2^*) = (5, 0)$; II:$(y_1^*, y_2^*) = (3, 0)$;
III:$(y_1^*, y_2^*) = (0, 1)$; IV:$(y_1^*, y_2^*) = (3, 2)$.

2. **Phase portrait**

CLOZE 0.10 penalty

Consider a system of two linear first-order ordinary differential equations: $\dot{y_1} = y_2, \quad \dot{y_2} = -9y_1$.

a) The corresponding eigenvalues are

MULTI 1 point Multiple Shuffle

• $\lambda_1 = 3$, $\lambda_2 = -3$ • $\lambda_1 = 3i$, $\lambda_2 = -3i$ ✓ • $\lambda_1 = 9$, $\lambda_2 = -9$

b) The corresponding eigenvectors of this linear ODE system are

MULTI | 1 point | Multiple | Shuffle

• I and III ✓ • II and III ✓ • I and IV • I and III ✓

where

I:$u_2 = \begin{pmatrix} i \\ 3 \end{pmatrix}$

II:$u_2 = \begin{pmatrix} 1 \\ -3i \end{pmatrix}$

III:$u_1 = \begin{pmatrix} 1 \\ 3i \end{pmatrix}$

IV:$u_1 = \begin{pmatrix} 3i \\ -1 \end{pmatrix}$

c) The phase portrait for this system of ODEs is

MULTI | 1 point | Multiple | Shuffle

• Stable node • Unstable focus with spiral out

• Centre ✓ • Stable focus with spiral in

3. **Stability**

MULTI | 2 points | 0.10 penalty | Single | Shuffle

For which value of a the system of ODEs
$\dot{y_1} = \tanh(y_1) + a\sin(y_2)$, $\dot{y_2} = -2\cos(y_1) + 2e^{y_1} + 3y_2$ has an unstable focus at $(y_1, y_2) = (0, 0)$

(a) $0 < a < 2$
(b) $a = 2$
(c) $a < -1/2$ (100%)
(d) $-1/2 < a < 0$

4. **Lyapunov function**

MULTI | 2 points | 0.10 penalty | Single | Shuffle

Which of the following functions $V(y_1, y_2)$ is a Lyapunov function for the dynamical system
$\dot{y_1} = (4 - y_1)e^{y_1 y_2}$, $\dot{y_2} = y_1^2 - y_1^2 y_2$

(a) $V(y_1, y_2) = (y_1 - 4)^4 + (y_2 - 1)^2$ (100%)

(b) $V(y_1, y_2) = y_1^4 + y_2^2$
(c) $V(y_1, y_2) = -y_1^3 + e^{y_2}$
(d) $V(y_1, y_2) = -y_1^4 + (y_2 - 1)^3$

Total of marks: 10