

Picard-Lindelöf Theorem.

Let \mathcal{D} be the rectangular domain in the xy plane defined as $\mathcal{D} = (|x - a| \leq A, |y - b| \leq B)$ and suppose $f(x, y)$ is a function defined on \mathcal{D} which satisfies the following conditions:

- (i) $f(x, y)$ is continuous and therefore bounded in \mathcal{D}
- (ii) the parameters A and B satisfy $A \leq B/M$ where $M = \max_{\mathcal{D}} |f(x, y)|$
- (iii) $|\frac{\partial f}{\partial y}|$ is bounded in \mathcal{D} .

Then there exists a unique solution on \mathcal{D} to the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(a) = b.$$

Exact first-order ODEs:

If the equation

$$P(x, y) + Q(x, y) \frac{dy}{dx} = 0$$

is exact, its solution can be found in the form $F(x, y) = \text{Const.}$ where

$$P = \frac{\partial F}{\partial x} \quad \text{and} \quad Q = \frac{\partial F}{\partial y}$$

Some derivatives

In the table below, a , b and c are constants.

$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$1/\cos^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$1/\cosh^2 x$
$\log x$	$\frac{1}{x}$

End of Appendix.