Queen Mary
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Macro for Policy

Consumption and policy impacts

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Agenda

1. Euler equation
2. Permanent income hypothesis
3. Liquidity constraints/precautionary saving
4. Empirical framework (consumption function)

The basic model of consumption

$$\max_{\{c_{t+i}, i=0,1,\cdots\}} E_t\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+\rho}\right)^i u(c_{t+i})\right]$$

subject to

$$A_{t+i+1} = (1+r)A_{t+i} + y_{t+i} - c_{t+i}$$

$$\lim_{j\to\infty}\left(\frac{1}{1+r}\right)^j A_{t+j} \geq 0$$

A_t is given.

- No-Ponzi-game condition / transversality condition:
- Intertemporal budget constraint:

$$A_t = \frac{A_{t+1}}{(1+r)} - \frac{y_t}{(1+r)} + \frac{c_t}{(1+r)} = \frac{\frac{A_{t+2}}{(1+r)} - \frac{y_{t+1}}{(1+r)} + \frac{c_{t+1}}{(1+r)}}{(1+r)} - \frac{y_t}{(1+r)} + \frac{c_t}{(1+r)} \Rightarrow \text{keep substituting up to period } t+j$$

The basic model of consumption

- Intertemporal budget constraint:

$$A_t = \frac{A_{t+1}}{(1+r)} - \frac{y_t}{(1+r)} + \frac{c_t}{(1+r)} = \frac{\frac{A_{t+2}}{(1+r)} - \frac{y_{t+1}}{(1+r)} + \frac{c_{t+1}}{(1+r)}}{(1+r)} - \frac{y_t}{(1+r)} + \frac{c_t}{(1+r)} = \gg \left(\frac{1}{1+r}\right)^j A_{t+i} - \frac{1}{1+r}\sum_{i=0}^{j-1}\left(\frac{1}{1+r}\right)^i y_{t+i} + \frac{1}{1+r}\sum_{i=0}^{j-1}\left(\frac{1}{1+r}\right)^i c_{t+i}$$

$$A_t + \frac{1}{1+r}\sum_{i=0}^{j-1}\left(\frac{1}{1+r}\right)^i y_{t+i} = \left(\frac{1}{1+r}\right)^j A_{t+i} + \frac{1}{1+r}\sum_{i=0}^{j-1}\left(\frac{1}{1+r}\right)^i c_{t+i}$$

Taking limit:

$$A_t + \frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i y_{t+i} = \frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i c_{t+i}$$

Euler equation (optimal consumption dynamics)

$$L(C_t, C_{t+1}, \cdots) = E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+\rho}\right)^i u(c_{t+i}) + \lambda \left[A_t + \frac{1}{(1+r)} \sum_{i=0}^{\infty} \frac{y_{t+i}}{(1+r)^i} - \frac{1}{(1+r)} \sum_{i=0}^{\infty} \frac{c_{t+i}}{(1+r)^i}\right]$$

- $\left(\frac{1}{1+\rho}\right)^i E_t u'(c_{t+i}) = \lambda \frac{1}{(1+r)} \frac{1}{(1+r)^i}$

- $i = 0 \quad \rightarrow \quad u'(c_t) = \lambda \frac{1}{(1+r)}$
- $i = 1 \quad \rightarrow \left(\frac{1}{1+\rho}\right) E_t u'(c_{t+1}) = \lambda \frac{1}{(1+r)^2}$

- [Euler equation] $u'(c_t) = \left(\frac{1+r}{1+\rho}\right) E_t u'(c_{t+1})$

Permanent Income Hypothesis

$$u'(c_t) = \left(\frac{1+r}{1+\rho}\right) E_t u'(c_{t+1})$$

Suppose $r = p$, so $u'(c_t) = \ E_t u'(c_{t+1})$

Denote the Euler equation error: $u'(c_{t+1}) = u'(c_t) + \eta_{t+1}$

- Bab Hall (1978): Stochastic Implications of the Life-Cycle Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*

- Quadratic utility function: $u(c) = c - \frac{b}{2}c^2$, so $u'(c) = 1 - bc$
- This yields: $c_{t+1} = c_t + \varepsilon_{t+1}$
- This also implies: $E[c_{t+1}] = c_t$

Consumption function

Intertemporal budget constraint:

$$A_t + \frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t[y_{t+i}] = \frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t[c_{t+i}]$$

$$A_t + \frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t[y_{t+i}] = \frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i c_t$$

$$\frac{1}{r}c_t = A_t + \frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t[y_{t+i}]$$

$$c_t = r[A_t + H_t] = y_t^p$$

- *How does uncertainty on future incomes affect consumption?*

Uncertainty and income shocks

$$c_t = rA_t + \frac{r}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t[y_{t+i}] = r[A_t + H_t] = y_t^p$$

- *Consumption is the annuity value of total wealth (return on financial and human wealth).*
- *How does uncertainty on future incomes affect consumption?*
- *Consumption responds to unanticipated changes in the consumer's expected permanent income.*

$$\begin{aligned} c_{t+1} &= c_t + \eta_{t+1} \\ &= c_t + r\left[\frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i (E_{t+1} - E_t) y_{t+1+i}\right] \end{aligned}$$

Consumption response to unanticipated income change

Consider an auto-regressive process of income:

- $y_{t+1} = \rho y_{t+1} + (1-\rho)\bar{y} + \varepsilon_{t+1}$
- $E_{t+1}[y_{t+1+i}] - E_t[y_{t+1+i}] = \rho^i \varepsilon_{t+1}$

Response of consumption to shocks (unpredictable shocks)

- $\Delta c_t = r\frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i (E_{t+1}-E_t)y_{t+1+i} = r\frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i \rho^i \varepsilon_{t+1}$

$$= \varepsilon_{t+1} r \frac{1}{1+r}\sum_{i=0}^{\infty}\left(\frac{\rho}{1+r}\right)^i$$

$$= \varepsilon_{t+1}\frac{r}{1+r}\frac{(1+r)}{(1+r-\rho)}$$

$$= \varepsilon_{t+1}\frac{r}{(1+r-\rho)}$$

- *What if* $\rho = 1$
- *What if* $\rho = 0$

Liquidity constraints

PIH assumes frictionless credit markets.

If liquidity constraints bind consumer responds strongly to income changes (hand-to-mouth).

Kaplan and Violante (2010) How Much Consumption Insurance beyond Self-Insurance? *AMERICAN ECONOMIC JOURNAL: MACROECONOMICS*

1. Poor hand-to-mouth (little illiquid wealth and liquid wealth)
2. Wealthy hand-to-mouth (large illiquid wealth, but no liquid wealth)

The marginal propensity to consume varies depending on the ability to borrow.

- 0.05 (transitory income shock) and 0.77 (permanent income shock)
- 0.18 (transitory income shock) and 0.93 (permanent income shock)