Coursework 1 Year 2022/2023 final

1. **Linearity of ODEs**

   MATCHING | 4 points | 0.10 penalty | Shuffle

   Find the right match for the following ODEs in the dropdown menu

$y' + y'' = 2x^2y$ •	······	• 2nd-order linear homogeneous ODE
$5y' = x - e^x y$ •	······	• 1st-order linear ODE
$2x^2yy' + 2xy^2 + 3 = 0$ •	······	• 1st-order Exact ODE
$xy' = (-5y + x)\sin(y/x)$ •	······	• Scale-invariant ODE
$y'' = \frac{x}{y}$ •	······	• None of the above forms
$y'' = 5 + y$ •	······	• 2nd-order linear inhomogeneous ODE
$x^2y'' = 5y$ •	······	• 2nd-order Euler-type ODE
$ye^{xy} - 2\frac{y^2}{x^3} + y'xe^{xy} + 2\frac{y}{x^2}y' = 0$ •	······	• 1st-order Exact ODE
$y''' = \frac{x}{y}$ •	······	• None of the above forms
$y'e^x = y\sin(x) + \tanh(y)$ •	······	• 1st-order nonlinear ODE

2. **IVP**

   CLOZE | 0.10 penalty

   a) Solving the initial value problem (IVP) $y' = y^{3/4}/x, \quad y(1) = 0$ implies finding a solution $y(x)$ of the differential equation that passes through the point (x_0, y_0) with

   MULTI | 1 point | Single | Shuffle

   • $x_0 = 1, y_0 = e$ • $x_0 = 1, y_0 = 0$ ✓ • $x_0 = 0, y_0 = 1$

   b) If this IVP has a unique solution, it means that

   MULTI | 1 point | Single | Shuffle

   - there exists a rectangular region of the two dimensional xy plane whose center is the point (x_0, y_0) where the solution to the IVP is unique. ✓
   - in the whole xy plane, there exist one and only one solution passing through this point.
   - there is only one unique solution to the ODE in the xy plane.

c) Does the IVP satisfy the hypotheses of the Picard-Lindelöf theorem?

MULTI | 1 point | Multiple | Shuffle

- no ✓
- yes

d) How many solutions has the IVP in a)?

MULTI | 1 point | Multiple | Shuffle

- none
- more than one ✓
- one

3. **Scale Invariant ODE**

MULTI | 2 points | 0.10 penalty | Single | Shuffle

The general solution of the 1st-order ODE, $y'x^2 = -y^2$ is

(a) $y(x) = -\frac{2x}{2+Cx}$ (100%)
(b) $y(x) = Cx + \frac{1}{x}$
(c) $y(x) = \frac{2x}{C+x}$
(d) $y(x) = \frac{1}{1+x}$

Total of marks: 10