Queen Mary University of London

Macro for Policy

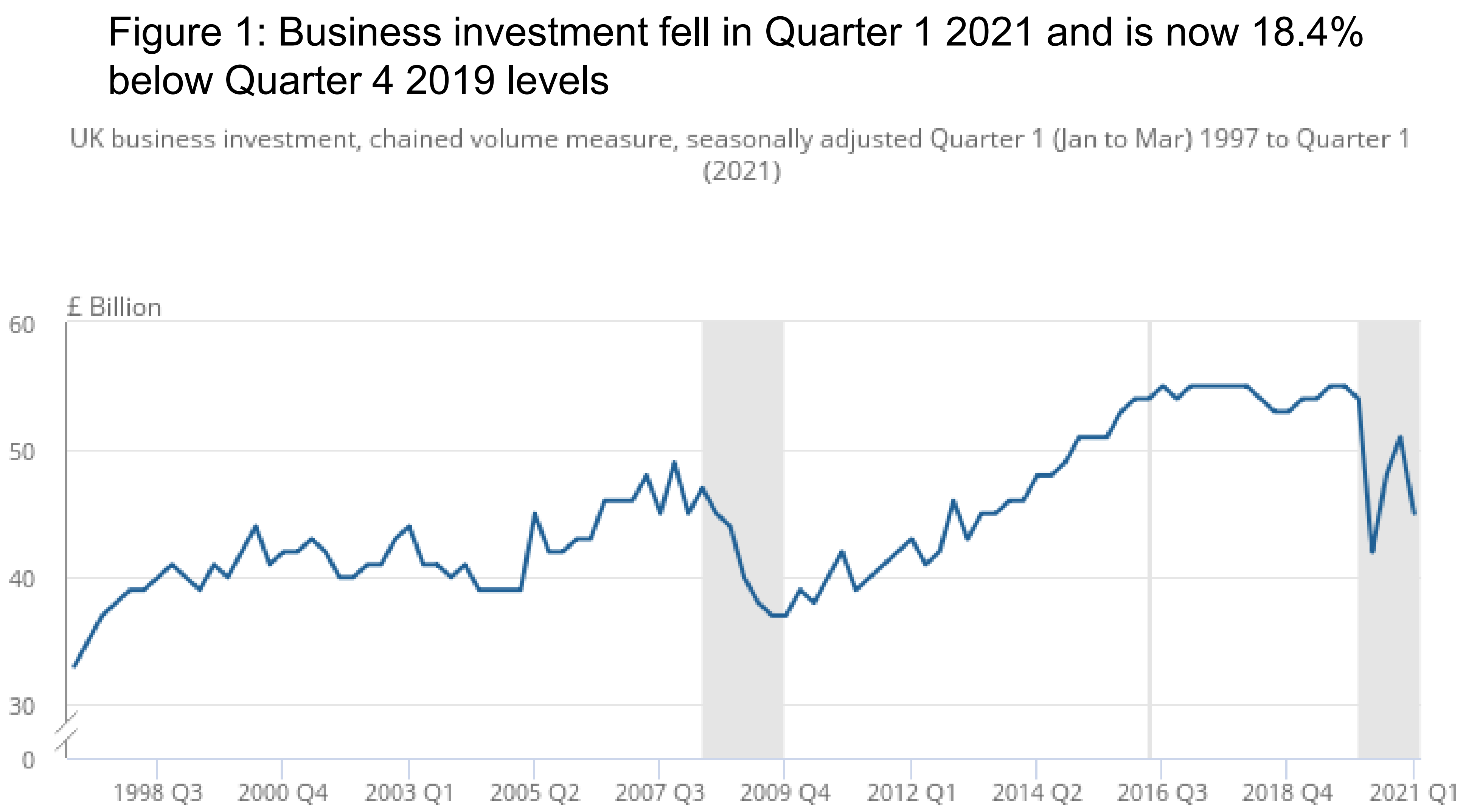
Taxes and investment

Lecturer: Dr. Tatsuro Senga
School of Economics and Finance

Agenda

1. Investment in the UK
2. Basic investment model (the User Cost of Capital and Tobin's Q model)
3. Nonconvex adjustment costs and financial frictions
4. Uncertainty

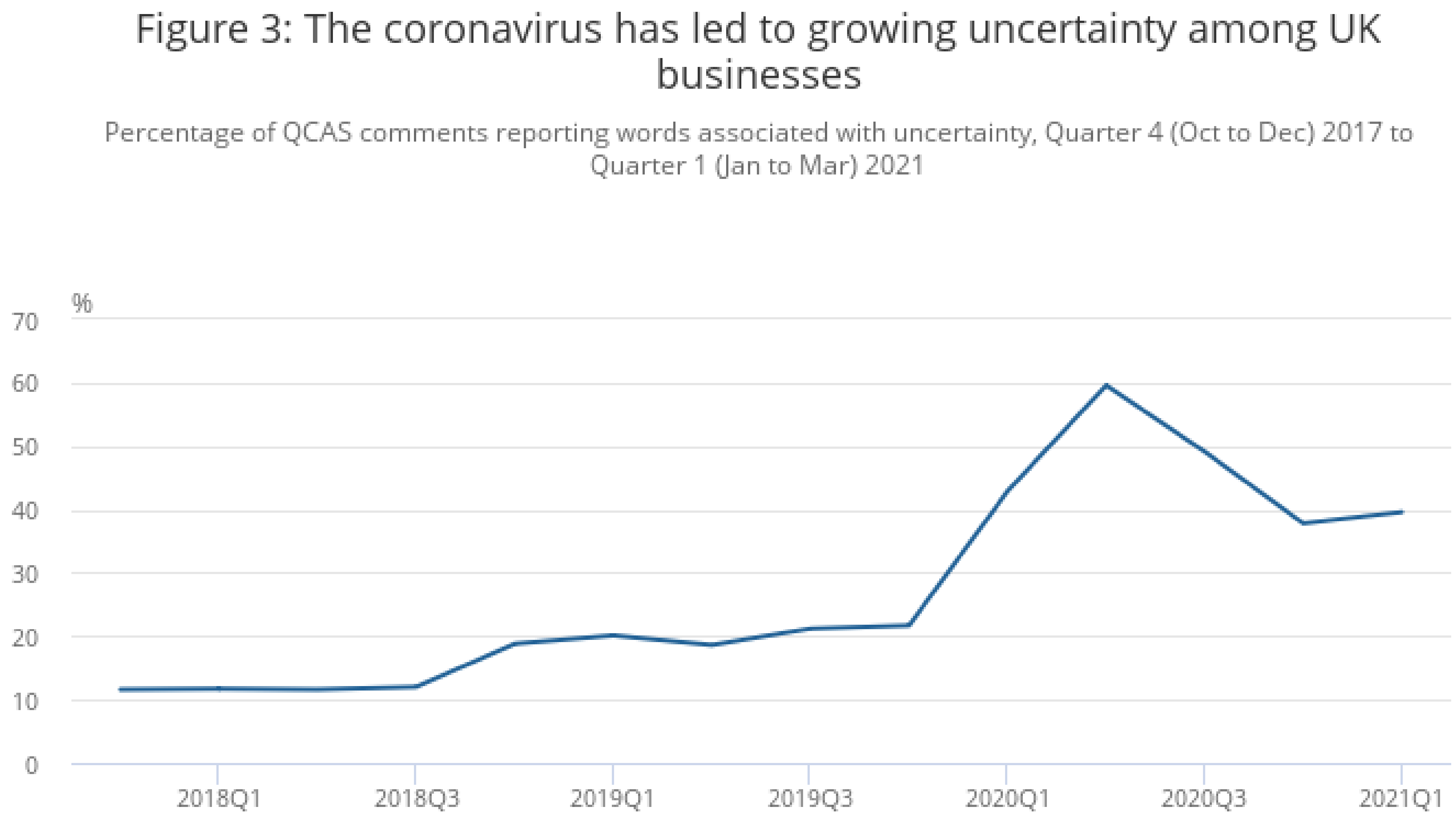
Aggregate investment in the UK



"Business investment is estimated to have fallen by 11.9% in Quarter 1 (Jan to Mar) 2021; in comparison with pre-pandemic levels in Quarter 4 (Oct to Dec) 2019, it is 18.4% lower."

Source: Office for National Statistics – Business Investment provisional results

Investment under uncertainty



"The coronavirus has led to many businesses delaying or cancelling their investment; the number of respondents mentioning investment delays rose by a further 2.3% in Quarter 1 2021 from the 25.3% seen in Quarter 4 2020."

Source: Office for National Statistics – Business Investment provisional results

Agenda

1. Investment in the UK
2. **Basic investment model (the User Cost of Capital and Tobin's Q model)**
3. Nonconvex adjustment costs and financial frictions
4. Uncertainty

The basic model of investment

- Production function:

$$y_t = k_t^{\alpha}, \qquad 0 < \alpha < 1$$

- Capital accumulation:

$$k_{t+1} = (1-\delta)k_t + i_t$$

- Quadratic adjustment costs:

$$-\frac{\phi}{2}(\frac{i_t}{k_t})^2 k_t$$

Individual firm's problem, discounting future at interest rate r

$$\max_{\{i_t, k_{t+1}\}} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[k_s^{\alpha} - i_s - \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2 k_s\right]$$

such that

$$k_{t+1} = (1-\delta)k_t + i_t$$

The basic model of investment

$$\max_{\{i_t, k_{t+1}\}} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[k_s^{\alpha} - i_s - \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2 k_s\right]$$

such that

$$k_{t+1} = k_t + i_t \quad (\delta = 0)$$

- Let q_t be the current-valued Lagrange multiplier:

$$L_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[k_s^{\alpha} - i_s - \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2 k_s - q_s[k_{s+1} - k_s - i_s]\right]$$

The basic model of investment

$$L_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[k_s^{\alpha} - i_s - \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2 k_s - q_s[k_{s+1} - k_s - i_s]\right]$$

- First order conditions:

$$\frac{\partial L_t}{\partial i_s} = -1 - \phi\left(\frac{i_s}{k_s}\right) + q_s \dashrightarrow \quad q_s - 1 = \phi\left(\frac{i_s}{k_s}\right)$$

$$\frac{\partial L_t}{\partial k_{s+1}} = -q_s + \frac{1}{1+r}\left(\alpha k_{s+1}^{\alpha-1} + \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2 + q_{s+1}\right)$$

$$\dashrightarrow \quad q_s = \frac{1}{1+r}\left(\alpha k_{s+1}^{\alpha-1} + \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2 + q_{s+1}\right)$$

Investment Euler equation

$$q_s = \frac{1}{1+r}\left(\alpha k_{s+1}^{\alpha-1} + \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2 + q_{s+1}\right)$$

The date s shadow price of an extra capital is the discounted sum of:

- the capital's marginal product next period
- the capital's marginal contribution to lower adjustment costs next period
- the shadow price of an extra capital next period

Tobin's marginal q

$$q_t = \frac{1}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[\alpha k_{s+1}^{\alpha-1} + \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2\right]$$

- The shadow price of installed capital equals its discounted stream of future marginal products and its marginal contributions to the reduction of adjustment costs.

The User Cost Model

- First order conditions:

$$q_s - 1 = \phi\left(\frac{i_s}{k_s}\right)$$

$$q_s = \frac{1}{1+r}\left(\alpha k_{s+1}^{\alpha-1} + \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2 + q_{s+1}\right)$$

- $\phi = 0$

$$q_s = 1$$

$$\alpha k_{s+1}^{\alpha-1} = r$$

Empirical Performance of the user cost model

$$\frac{i_{it}}{k_{it}} = \alpha + \boldsymbol{\beta u_{it}} + \gamma X_{it} + \varepsilon_{it}$$

- u_{it} (user cost) can include relative price of capital, taxes, depreciation.
- β turned out to be small in various past studies (Hall and Jorgensen, 1967; Cummins, Hassett, and Hubbard, 1994; Chirinko, Fazarri, and Meyer, 1999)
- Other variables like cash flow are often correlated with $\frac{i_{it}}{k_{it}}$, with a significant and large coefficient $\boldsymbol{\gamma}$.s

The Q-Theory Model

- $\phi > 0$

$$q_s - 1 = \phi\left(\frac{i_s}{k_s}\right)$$

$$q_t = \frac{1}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[\alpha k_{s+1}^{\alpha-1} + \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2\right]$$

Key implication 1:

investment is positively related to q_t*:* $\frac{i_t}{k_t} = \frac{1}{\phi}(q_t - 1)$.

The Q-Theory Model

- $\phi > 0$

$$q_s - 1 = \phi\left(\frac{i_s}{k_s}\right)$$

$$q_t = \frac{1}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[\alpha k_{s+1}^{\alpha-1} + \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2\right]$$

Key implication 1:

investment is positively related to q_t*:* $\frac{i_t}{k_t} = \frac{1}{\phi}(q_t - 1)$*.*

The Q-Theory Model

- $\phi > 0$

$$q_s - 1 = \phi\left(\frac{i_s}{k_s}\right)$$

$$q_t = \frac{1}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[\alpha k_{s+1}^{\alpha-1} + \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2\right]$$

Key implication 2:

q_t is the marginal value of capital to the firm.

The Q-Theory Model

- $\phi > 0$

$$q_s - 1 = \phi\left(\frac{i_s}{k_s}\right)$$

$$q_t = \underbrace{\frac{1}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[\alpha k_{s+1}^{\alpha-1} + \frac{\phi}{2}\left(\frac{i_s}{k_s}\right)^2\right]}_{=\frac{\partial V(k_{t+1})}{\partial k_{t+1}}}$$

Key implication 2:

q_t is the marginal value of capital to the firm.

How to measure marginal q?

Value of firms in the stock market

Assuming the firm's stock market valuation (V_{it}^{E}) equals its fundamental value (V_{it})

Marginal and Average q

Hayashi (1982) show marginal q is equal to average q, under constant returns in production and investment.

$$q_{it} = \frac{V_{it}}{k_{it}}.$$

Empirical Performance of the Q-theory Model

$$\frac{i_{it}}{k_{it}} = \alpha + \boldsymbol{\beta} \boldsymbol{q}_{it} + \boldsymbol{\gamma} \boldsymbol{X}_{it} + \boldsymbol{\varepsilon}_{it}$$

- one can use stock market information to measure q_{it}.
- β turned out to be small in various past studies (Summers, 1981; Cummins, Hassett, and Hubbard, 1994); Erickson and Whited, 2000).
- Other variables like cash flow are often correlated with $\frac{i_{it}}{k_{it}}$, with a significant and large coefficient $\boldsymbol{\gamma}$.

Agenda

1. Investment in the UK
2. Basic investment model (the User Cost of Capital and Tobin's Q model)
3. **Nonconvex adjustment costs and financial frictions**
4. Uncertainty

Attempts to improve investment models 1

Financial frictions to acquiring investment funds

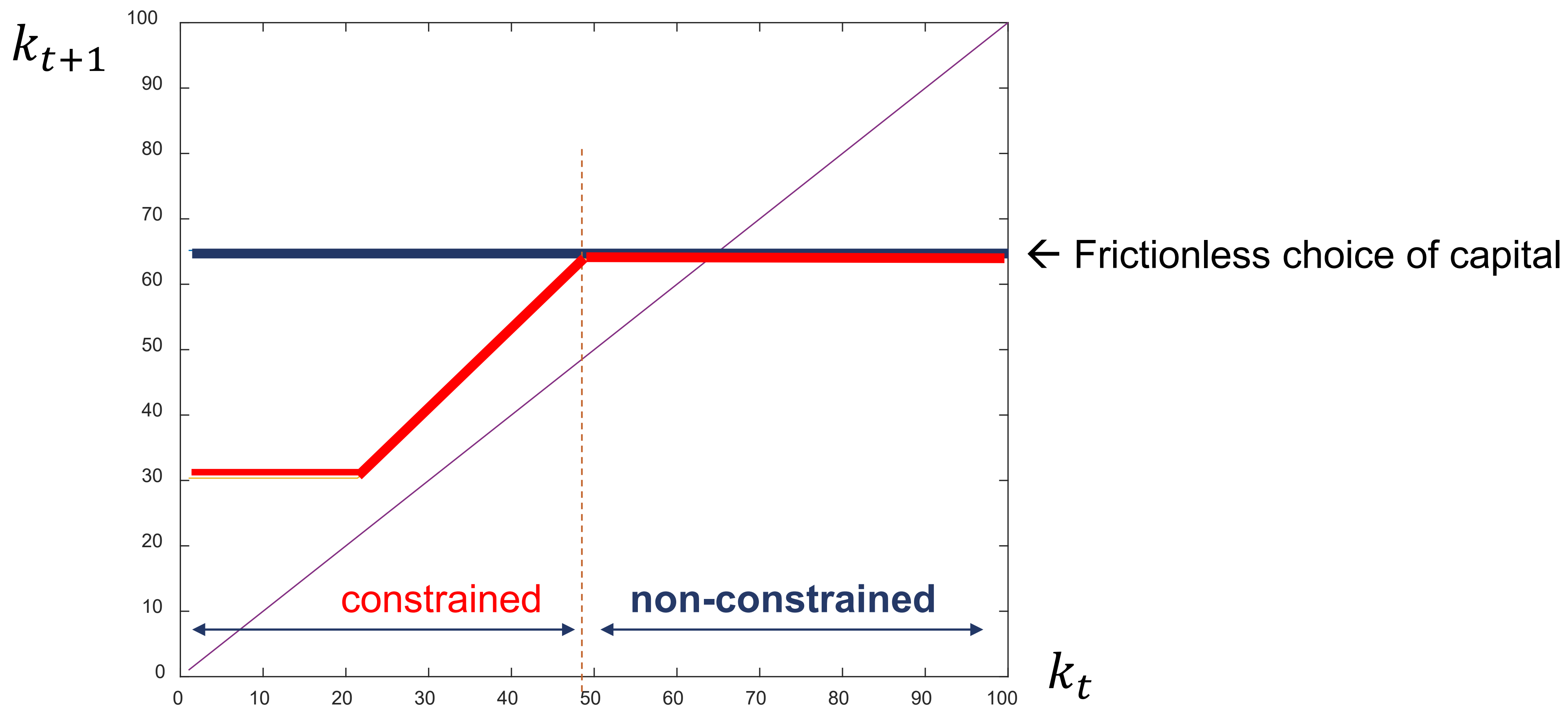
- Firms are more informed about their investment projects than financial intermediaries like banks.
- Risk associated with investment would be absorbed by intermediaries in the event of default.
- This adds costs for firms when they raise funds externally (monitoring costs, expected losses for bankrupt etc).

Financial frictions

$$\max_{\{i_t, k_{t+1}\}} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} [k_s^\alpha - i_s + \eta(i_s, k_s)]$$

such that

$$k_{t+1} = k_t + i_t \quad (\delta = 0) \quad \text{and} \quad \eta(i_s, k_s) = \begin{cases} \eta_1(k_s^\alpha - i_s) & \text{if } k_s^\alpha < i_s \\ 0 & \text{if } k_s^\alpha \geq i_s \end{cases}$$



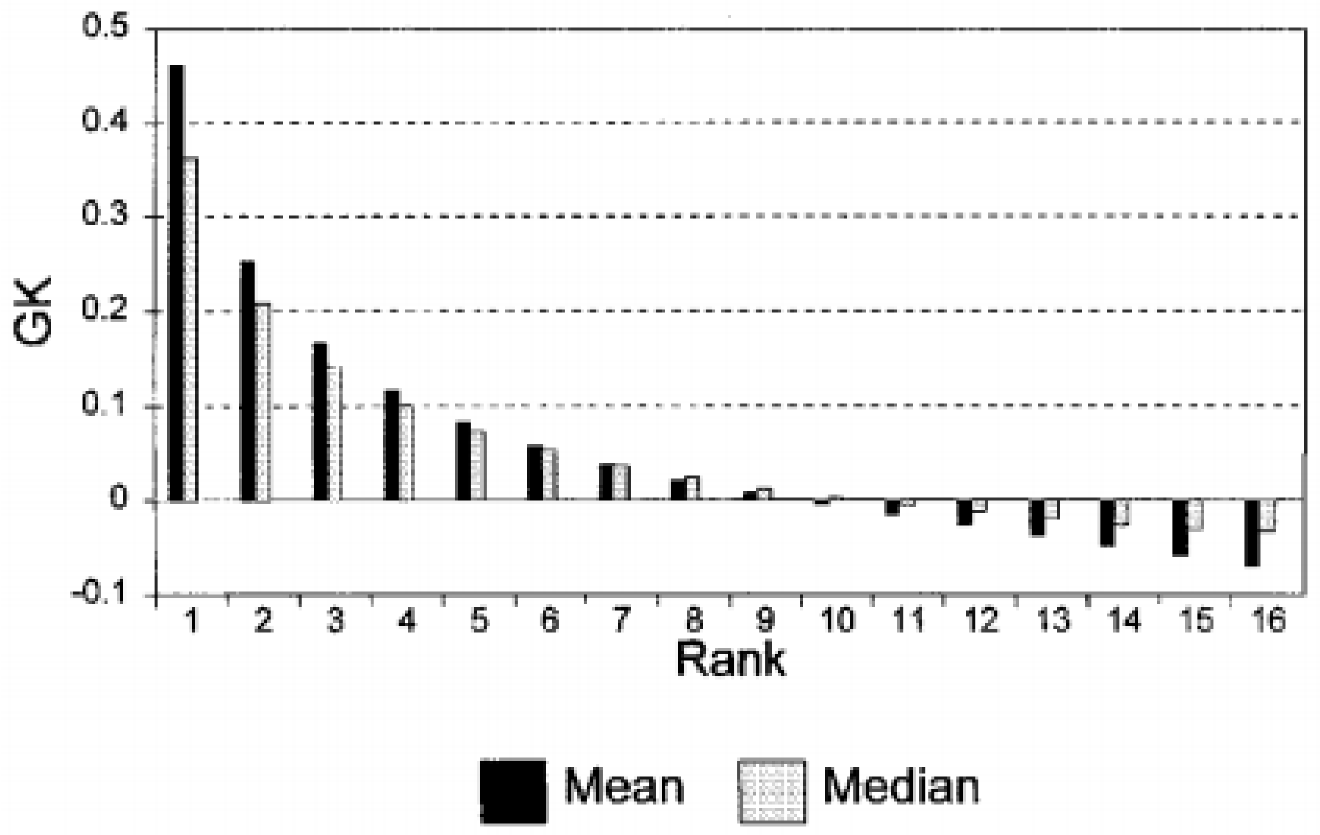
Attempts to improve investment models 2

Lumpy investment

- Doms and Dunne (1993) study the investment behavior of 12,000 U.S. manufacturing plants from 1972 to 1988

a. over 50 % of the plants with capital growth close to 50 percent in a single year.

b. over 25% of an average plant's investment is concentrated in a single year.



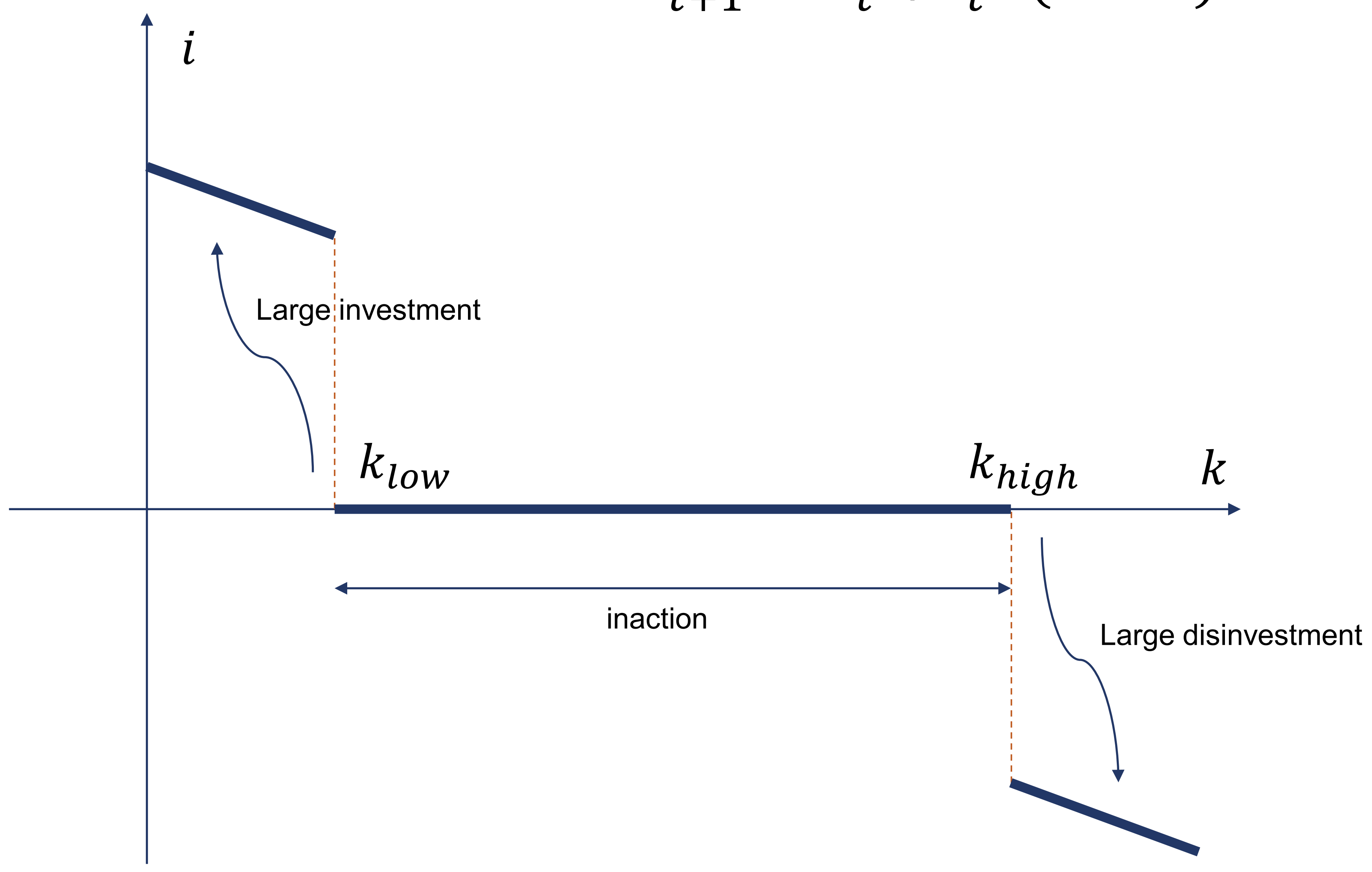
$$GK_{it} = \frac{i_{it} - \delta k_{it-1}}{0.5 \times (k_{it-1} + k_{it})}$$

Non-convex adjustment costs

$$\max_{\{i_t, k_{t+1}\}} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} [k_s^{\alpha} - i_s - C(i_s)]$$

such that

$$k_{t+1} = k_t + i_t \quad (\delta = 0) \quad \text{and} \quad C(i_s) = \begin{cases} C \text{ if } i_s \neq 0 \\ 0 \text{ if } i_s = 0 \end{cases}$$



Queen Mary
University of London