Queen Mary
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Macro for Policy

Endogenous Growth Theory (part 1)

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Agenda

1. The Solow model in continuous time
2. Endogenous growth model 1: AK model (learning-by-doing)
3. Endogenous growth model 2: Romer's R&D model
4. Endogenous growth model 3: Schumpeterian growth model

Summary of the Solow model

- Continuous time
- Growth rates

Try to compare the predictions of the Solow model to those of AK model we will study next

Key ingredients

- ***The production function:*** $\boldsymbol{Y = K^{\alpha}(AL)^{1-\alpha}}$

- ***Technological progress and population (labor force) growth:*** $\boldsymbol{\dot{A} = gA}$
- $\boldsymbol{\dot{L} = nL}$

The Solow model in continuous time

- *Capital accumulation:* $\dot{K} = sY - \delta K$

- *Capital accumulation per efficiency unit of labor:* $\dot{k} = (K/AL)^{\cdot} = \frac{AL\dot{K} - \dot{A}LK - A\dot{L}K}{(AL)^2}$

$$= \frac{sk^{\alpha}(AL)^{1-\alpha} - \delta K}{AL} - \frac{\dot{A}LK}{ALAL} - \frac{A\dot{L}K}{ALAL}$$

$$= sk^{\alpha}(AL)^{-\alpha} - \delta k - gk - nk$$

$$= sk^{\alpha} - (\delta + g + n)k$$

Additional terms have been added (colored in red).

- *The growth rate of capital per efficiency unit of labor:* $\frac{\dot{k}}{k} = sk^{\alpha-1} - (\delta + g + n)$

$$k = \frac{K}{AL} \rightarrow g_K = g_A + g_L = g + n$$

- *The growth rate of output per worker:* $g_y = g_Y - g_L = g + n - n = g$

Recap: The Solow Growth Model

- A fraction of resources is allocated to capital accumulation (*consumption –v- savings*).
- Capital accumulation alone does not lead to sustained growth (*diminishing returns*).
- In the model, factors like technological progress drives economic growth (*exogenous*).

Recap: The Solow Growth Model

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Moving forward:

1. Idea-wise (what drives technical progress?)

2. Technical issues (how to avoid diminishing returns?)

New Growth Theory (Endogenous Growth)

- Learning by doing (Romer, 1986)
- Human capital model (Lucas, 1988)
- Models of innovation (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992)

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We'll look at:

1. Learning-by-doing ("AK model")

2. R&D model

AK model

- Workers accumulate knowledge during production.
- Externalities from production at the individual level lead to technical progress at the aggregate level.

→ Learning-by-doing is the source of technological progress

Key ingredients

- ***The production function:*** $\boldsymbol{Y = \bar{A} K^{\alpha} L^{1-\alpha}}$

- ***The stock of knowledge is a function of the stock of capital per worker:*** $\boldsymbol{\bar{A} = A(\frac{K}{L})^{\phi}}$

AK model: capital accumulation

- *Capital accumulation :* $\dot{K} = s\bar{A}K^{\alpha}L^{1-\alpha} - \delta K$

- *Capital accumulation per worker :* $\dot{k} = (\dot{K/L}) = \frac{L\dot{k} - \dot{L}K}{L^2} = \frac{s\bar{A}K^{\alpha}L^{1-\alpha} - \delta K}{L} - \frac{\dot{L}}{L}\frac{K}{L} = s\bar{A}k^{\alpha} - (\delta + n)k$

- *The growth rate of capital per worker:* $\frac{\dot{k}}{k} = \frac{s\bar{A}k^{\alpha} - (\delta+n)k}{k} = s\bar{A}k^{\alpha-1} - (\delta + n)$

  $= sAk^{\phi}k^{\alpha-1} - (\delta + n)$

"K" has been removed from here.

AK model: long-run growth

- *(again) The growth rate of capital per worker:* $g_k = \frac{\dot{k}}{k} = sAk^{\phi+\alpha-1} - (n+\delta)$

- *The growth rate of output per worker:* $Y = \bar{A}K^{\alpha}L^{1-\alpha} = AK^{\phi+\alpha}L^{1-\phi-\alpha} = AK$

$$y = \frac{Y}{L} = \frac{AK}{L} = Ak \rightarrow g_y = g_k = sAk^{\phi+\alpha-1} - (n+\delta)$$
$$= sA - (n+\delta)$$

called the AK model…

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