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Source: *National Institute Economic Review*, October 2002, No. 182 (October 2002), pp. 72-89

Published by: Cambridge University Press

Stable URL: https://www.jstor.org/stable/23876458

A LONG VIEW OF THE UK BUSINESS CYCLE

Jagjit S. Chadha* and Charles Nolan**

We outline a number of 'stylised' facts on the UK business cycle obtained from analysis of the long-run UK annual dataset. The findings are to some extent standard. Consumption and investment are pro-cyclical, with productivity playing a dominant role in explaining business cycle fluctuations at all horizons. Money neutrality obtains over the long run but there is clear evidence of non-neutrality over the short run, particularly at the business cycle frequencies. Business cycle relationships with the external sector via the real exchange rate and current account are notable. Postwar, the price level is counter-cyclical and real wages are pro-cyclical, as are nominal interest rates. Modern general equilibrium macroeconomic models capture many of these patterns.

1. Introduction

"[T]he business cycle is the phenomenon of a number of important economic aggregates (such as GNP...) being characterised by high pairwise coherences at low business cycle frequencies." Sargent (1987, p.282)

The business cycle has become the unit of account for studying macroeconomic fluctuations.[1] This unit was valued at 6–32 quarters by Burns and Mitchell (1946) and arguably has shown little subsequent tendency to fluctuate in value. Researchers have used the cyclical time unit to study the characteristics of macroeconomic fluctuations around its long-run growth path (see, for example, Backus and Kehoe, 1992). This organising framework has produced a remarkably rich agenda, ranging from business cycle identification and characterisation to the development of small analytical models designed to capture the resultant stylised facts. As a complement to the other articles in this special issue, we undertake an exercise in analysing a number of key relationships at the business cycle frequencies.[2]

This article documents business cycle fluctuations over the past century and a quarter in the United Kingdom. The study of the UK, arguably the best recorded industrialised economy over this long period,[3] allows us to present a study of empirical macroeconomic regularities comprising twenty-two complete (and independent) business cycles. The analysis of this data set contributes to business cycle research in the following respect. The availability of a long-run high quality data set allows us to examine the robustness through time of many key business cycle facts. Most long-run studies of business cycles tend to focus on only a few variables, although they have generally considered a cross-section of countries.[4] Our study, which takes a closer look at an individual country, complements those analyses.

As Granger (1969) and King and Watson (1996) demonstrate, the power spectrum of an economic variable provides an insightful first pass through the data; it demonstrates the importance of business cycles, which we take to be cyclical variations in the data lasting between two and eight years, in line with much of the literature.[5] Analysis in the frequency domain also leads naturally to the use of the bandpass filter to extract the business cycle component of time series.[6] Our analysis is straightforward and is based on two simple nonparametric tools, the spectral density function and time domain correlations, at the business cycle frequencies. We examine the dynamics in the frequency domain using the spectral density function – which represents the Fourier transformation of the autocovariance function. This function examines the unconditional variation of a given variable by frequency, which naturally ranges from the frequency of observation to the range of the sample set. We can also transform the real and imaginary parts of the cross-spectral density in order to analyse both correlation and the lead or lag of the variables with output by frequency.

The article proceeds by outlining the UK annual dataset. Section 3 outlines some issues connected with the identification of the business cycle in the frequency domain. Section 4 outlines the main results of this exercise. The core results are contained in Charts 1–17.

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Charts 1–9 depict the business cycle components of main economic time series against time and two measures of the economic cycle: the business cycle component of output per head and the phases of the economic cycle in terms of expansion (unshaded) and contraction (shaded). The reader is thus encouraged to examine the business cycle correlates through time. Charts 10–17 show the same information for a number of series in the frequency domain. That is, we show what the relationship between output and variables, such as consumption, is at frequencies from two years out to the long run.[7] We also show to what extent the series lag or lead the economic cycle, measured by output per head. Section 5 offers some concluding remarks.

2. The UK dataset

Mitchell (1988) collates most of the macroeconomic series that we use in this paper. This is generally regarded as the best available source, because it gathers together the most reliable data (or estimates) from primary sources and ends in 1980. We overwrite the Mitchell data with Office of National Statistics data to give us more up-to-date information.

Data on real GDP (at factor cost) are taken from Mitchell (p. 837) whose original source is Feinstein (1972) for the period 1855–1948. This GDP series is based on expenditure data, which makes it consistent with the consumers' expenditure and investment data used here. Since 1920 data for the Republic of Ireland have been recorded separately, whereas before they were included in the UK data. Since this break affects all quantity series, we do not adjust the various series for this break. From 1948 onwards we use ONS data and rebase the whole real GDP series to 1990.

The price index series is the RPI series (1987=100) provided by the Bank of England. The real exchange rate series is calculated using the US dollar/sterling exchange rate and US (source: Mitchell, 1988) and UK consumer price indices. Current account data are taken from Mitchell (1988) and supplemented by ONS data where available. For consistency, real consumption and investment (real gross fixed domestic capital formation) data are taken from Mitchell (1988, p. 837) and are treated in a similar way to the GDP data.

Narrow and broad money series start in 1871 and are taken from Capie and Webber (1985). They define narrow money as the monetary base. From 1969 onwards we use data on M0 available from the Bank of England. We then use growth rates of the monetary base before 1969 to project M0 backwards to 1870. Broad money is defined as M3 from 1871 onwards. From 1969 we apply the growth rate of M4 to Capie and Webber's M3 series to give us up-to-date estimates of M3. We use M0 and M3 in order to enable comparison with other OECD countries. Nominal money data are deflated by the RPI series.

The construction of a consistent series for real wages presented us with a number of difficulties, because in the past real wages were estimated from partial information available for some sectors of the economy only (source: Mitchell), whereas ONS data reflect total economy-wide earnings. We proceed as follows. First we take the average real wage rate series from Mitchell (p. 149), which allows for unemployment (indexed to 1850=100).[8] From 1880 onwards we use real wages from Mitchell (p. 150) (indexed to 1914=100).[9] From 1920 the real wage series is calculated as the basic weekly (nominal) wage rate from Mitchell (p. 151) (1956=100) deflated by the RPI series.[10] From 1946 onwards ONS data on total earnings are used and deflated by the RPI data. All component series are then reindexed to 1956=100.

Data on the employed labour force are from Feinstein (op. cit.), whereas Matthews *et al.* (1982) provides data on the capital/labour split in total output.[11] We measure labour productivity as real GDP per employee, and total factor productivity (TFP) growth is then constructed as the difference between output growth and the weighted average of growth in factor inputs. For long interest rates we use the Consol rate (which is the yield on 3 per cent consols until 1888 and 2.5 per cent consols thereafter) and for short rates we use the discount rate on prime bills.[12] Real interest rates are calculated ex-post using a four-year and one-year RPI inflation rate.

2.1 Data quality

The question of temporal stability in sample moments in our dataset opens up the question of the extent to which data problems, specifically measurement error, may play a role in distorting our results.[13] One possibility is that such measurement error biases the sample moments as: (i) the measurement error affects both output and the relevant macroeconomic time series; and (ii) the measurement error might fall with time, thus explaining part of any *fall* in unconditional sample second moments. Dealing with the second point first, although there is likely to be some degradation in the quality of data as the researcher travels to a time before statistical agencies released data[14] – we are fortunate in the UK, insofar as the work of Feinstein (1972), on the main national accounting aggregates and their principal components (note that we use the 'compromise' estimate that averages across available estimates), and Capie and Webber (1985), on monetary series, allows the construction of high quality macroeconomic time series. We would also add, that one would also not expect to find significant deterioration in the quality of financial prices in the distant past.

One way to consider likely measurement errors is to compare different measures of output. We note that Feinstein's (1972) income and expenditure estimates are cyclically similar and Sheffrin (1988) notes that the standard deviation of the two independent measures differ only in the third decimal place. Feinstein's estimates are given as being a "continuous series, consistently defined and measured, over the whole period from 1855 to 1965". There is an extensive discussion in Chapter 1 of Feinstein to which we refer the reader; we interpret the findings as suggesting that the series are appropriate for analysing lower frequency fluctuations over a number of years – that is, of course, precisely our exercise. We cannot, of course, exclude the possibility that measurement error may play an important part in explaining observations, particularly as real quantities seemed to have become less variable through time.[15] We therefore concentrate in this article on understanding key relationships that appear most robust through time.

3. Business cycle measurement

This section rehearses some arguments connected with the measurement of business cycle frequencies. We outline the construction of the some popular filters and then describe how we construct the measurement of the relationship between variables at business cycle frequencies, that is the co-spectra.

3.1 The construction of the approximate band-pass filter

We describe the construction of band-pass filters, which are used to estimate the business cycle components of time series. More details can be found in the original papers by Baxter and King (1999) and Christiano and Fitzgerald (1999). A more rigorous exposition of some of the foundations of spectral analysis can be found in Cox and Miller (1965).[16]

First we outline some important concepts from frequency domain analysis and show that the construction of the band-pass filter can be viewed as a building block in the construction of the spectral representation of an economic variable. Then, we describe the criterion that Baxter and King (BK) and Christiano and Fitzgerald (CF) use to evaluate their approximations to the ideal band-pass filter. We will derive first the BK filter and then use this to construct the recommended filter of CF. It is useful for expositional clarity to consider the issues in this order, although the reader should note that, strictly speaking, the BK filter is derived as a special case of the class of filters constructed by CF.

3.1. The spectral representation of an economic variable

Economists have long recognised the potential attraction of analysis in the frequency domain (Granger, 1969; Sargent, 1987), although it is probably fair to say that the vast majority of empirical work has taken place in the time domain.

A useful point of departure is to note a central result in time-series statistics that any stationary time series can be regarded as the sum of orthogonal sinusoidal components.[17] For example,

$$Y_t = \int_0^{\pi} \cos \omega t \, du(\omega) + \int_0^{\pi} \sin \omega t \, dv(\omega), \tag{1}$$

where $\{Y_t\}_{t=0}^{\infty}$ represents a stationary stochastic real-valued process in discrete time, and $u(\omega)$ and $v(\omega)$ are orthogonal processes defined on the open interval $(0,\pi)$. Under certain fairly weak additional assumptions, the existence of the band-pass filter is implicit in (1) since it implies in effect that we can decompose our stationary time series into components indexed by frequency, ω. We can see this more easily if we re-write (1) in a more general form

$$Y_t = \int_{-\pi}^{\pi} e^{i\omega k} dX(\omega), \tag{2}$$

where $X(\omega)$ is a stochastic process (more specifically a process with orthogonal increments) defined on $[-\pi,\pi]$.[18] We wish to isolate the fluctuations in Y_t which are due to fluctuations corresponding to frequencies in the range $\underline{\omega} < \omega < \overline{\omega}$. That is we wish to calculate $\hat{Y}_t$:

$$\hat{Y}_t = \int_{\underline{\omega}}^{\overline{\omega}} e^{i\omega t} dX(\omega). \tag{3}$$

Canonically, this is calculated by linearly operating on Y_t in the following way

$$\hat{Y}_t = \sum_{h=-\infty}^{\infty} b_h Y_{t-h}, \tag{4}$$

where b_h represent the correct weights for isolating the periodic components of interest. It follows that we need to calculate these weights, which are subject to the requirement that

$$\beta(\omega) = \sum_k b_k e^{i\omega k} = \begin{cases} 1 \text{ if } (\underline{\omega} < \omega < \overline{\omega}) \\ 0 \text{ otherwise.} \end{cases} \tag{5}$$

Applying the inverse Fourier transform to (5) recovers the optimal weights:

$$b_h = \frac{1}{2\pi} \int_{\underline{\omega}}^{\overline{\omega}} e^{i\omega h} d\omega$$

$$= \frac{1}{2\pi} \left[\frac{e^{i\overline{\omega}h} - e^{i\underline{\omega}h}}{ih} \right]. \tag{6}$$

The second equation (6) is the optimal band-pass filter weights. It is easily demonstrated that it represents the difference between two low-pass filters. To see this define the ideal low-pass filter to be that which ensures $\beta(\omega) = 1$ for $|\omega| < \underline{\omega}$), and $\beta(\omega) = 0$ for $|\omega| > \underline{\omega}$) then using this in the first expression in (6) we get that at the zero frequency, $\omega = 0$:

$$b_0 = \frac{1}{2\pi} \int_{-\underline{\omega}}^{\underline{\omega}} e^{i\omega h} d\omega \Rightarrow \frac{1}{2\pi}\Big|_{-\underline{\omega}}^{\underline{\omega}} = \frac{\omega}{\pi}. \tag{7}$$

And for $\omega \neq 0$, the same steps give

$$b_h = \frac{1}{2\pi i h}(e^{i\underline{\omega}h} - e^{-i\underline{\omega}h}), \tag{8}$$

and by applying the Euler relations we simplify to:

$$b_h = \frac{\sin(\underline{\omega}h)}{\pi h}.$$

It follows then that the optimal weights for the band-pass filter can be written as the following sequence of equations:

$$\begin{aligned} b_0 &= \frac{\underline{\omega} - \overline{\omega}}{\pi} \\ b_h &= \frac{\sin(\underline{\omega}h) - \sin(\overline{\omega}h)}{\pi h}, \quad \forall h \geq 1 \end{aligned} \tag{9}$$

The problem with equation (9) is that its construction employs an infinite-order moving average process. As both Baxter and King and Christiano and Fitzgerald point out, in practice some kind of an approximation is needed. In fact, our spectral density calculations are also approximations to the true density.[19]

The issue now is to construct an approximate band-pass filter with desirable properties. There seems, as yet, little agreement as to what these additional properties might be, with BK and CF emphasising different properties as being desirable.[20] These differences can, in principle, result in substantial differences in filter construction. More specifically CF adopt, in effect, a different criterion, or objective, function to BK. We outline the BK filter construction, and then indicate the filter recommended by CF as an extension.

BK adopt a quadratic criterion, which minimises the Euclidean distance between the optimal weights and the actual weights subject to the requirement that the filter return a stationary series. That is, a side-constraint is imposed on the problem such that the filter weights partial out the zero frequency. Formally the Lagrangian for this problem can be written as

$$L = \int_{-\pi}^{\pi} \left[\sum_{h=-\infty}^{\infty} b_h e^{-i\omega h} - \sum_{h=-K}^{K} a_h e^{i\omega h} \right]^2 d\omega - \lambda \left[\sum_{h=-K}^{K} a_h \right]. \tag{10}$$

Note that BK employ a symmetric moving average filter, a choice they justify on the grounds that it avoids phase shift. The first summation term represents the optimal filter weights, with the second and third terms being our choice of weights. The problem proceeds by differentiating (10) with respect to a_h, evaluating the resulting first-order condition for each h and evaluating the relevant integral.

For h=0 we get

$$\frac{\partial L}{\partial a_0} = -2\int[b_0 - a_0]d\omega = \lambda,$$

which simplifies to

$$(b_0 - a_0) = \lambda / 4\pi. \tag{11}$$

For h>0,

$$\frac{\partial L}{\partial a_h} = 2\int[(b_K - a_K)(e^{i\omega K} + e^{-i\omega K})](e^{i\omega K} + e^{-i\omega K})d\omega = 2\lambda.$$

This is straightforward to simplify, and we get for all h>0 that

$$(b_K - a_K) = \lambda / 4\pi. \tag{12}$$

The first-order conditions indicate that, in the absence of any constraint, it is optimal to set the actual weights of the filter equal to the optimal weights. However, for $\lambda \neq 0$ all the (2K+1) first-order conditions have to be altered to ensure a zero response at the zero frequency. It is easy to show that the adjustment factor for each equation results in the following sets of weights:

$$a_0 = b_0 - \frac{\sum_{h=-K}^{K} b_h}{2K+1},$$

$$a_1 = b_1 - \frac{\sum_{h=-K}^{K} b_h}{2K+1}$$

$$\vdots$$

$$\vdots$$

$$a_h = b_h - \frac{\sum_{h=-K}^{K} b_h}{2K+1}. \tag{13}$$

3.2. *The CF filter*

CF argue that the above filter fails to incorporate important information on the time series property of the raw underlying data. They derive formulas for optimal filter weights for a wide class of time series representations of the data. Their recommended filter, which we focus on here, assumes that the data are generated by a pure random walk. Although they note that this assumption is most likely false for most macro time series, they argue that it nevertheless produces a filter that works well in a wide range of circumstances. CF begin by adopting an alternative criterion which incorporates the assumed time series properties of the data:

$$L = \int_{-\pi}^{\pi} \left[\sum_{h=-\infty}^{\infty} b_h e^{-i\omega h} - \sum_{h=-K}^{K} a_h e^{i\omega h} \right]^2 f_y(\omega) d\omega - \lambda \left[\sum_{h=-\infty}^{\infty} b_h \right].$$

We note that the filter is no longer symmetric. In general $k \neq K$, and indeed these lower and upper limits are not constant, so in fact each filtered observations uses all the data. The spectral density function, $f_y(\omega)$ plays a crucial role in raising the filter weights for frequencies where the data have higher spectral mass. As we noted above, the filter recommended by CF, and the filter that we employ in this paper, assumes that the data are generated by a pure random walk. The effects that this has on the calculations of the optimal weights can be seen intuitively by recalling the first-order conditions for the construction of the BK filter.

We note that we have a finite number (2K+1) of parameters to choose. However, we assume that the data are generated by a random walk. Therefore, let x_N denote the final observation in our raw data set, and x_1 denote the first observation. It follows then that:

$$E_t(x_{N+j}) = x_N \quad \forall j \geq 0$$
$$E_t(x_{1-j}) = x_1 \quad \forall j \geq 0$$

In effect, then, we have an infinite number of first-order conditions, where our weights on the first and last observations are calculated using the side constraint, and the weights on our other terms are as in (13), with $K \to \infty$. In other words, $a_1 = b_1$ for all i except b_1 and b_N which represent the (time-varying) weight on the initial data point and the final data point, respectively. We then get that, using our first order condition with respect to the undetermined multiplier that,

$$(\text{i.e., } b_0 + 2\sum_{i=1}^{\infty} b_i = 0),$$

$$b_N = -\frac{1}{2}b_0 - \sum_{i=1}^{N-1} b_i,$$

and

$$b_1 = -(b_2 + b_3 + \ldots b_0 + \ldots b_N).$$

3.3 Cross-spectrum

We can define the cross-spectrum between two series as:

$$\varpi_{12}(\alpha) = \sum_{s=-\infty}^{\infty} \rho_{(12)s} e^{i\alpha s}, \tag{14}$$

with the corresponding integrated spectral function $W(\alpha)$ defined over the range 0 to π. Solving for the cross correlation we find that

$$\rho_{(12)s} = \frac{1}{\pi} \int_{-\pi}^{\pi} \varpi_{12}(\alpha) e^{-i\alpha s} \partial\alpha. \tag{15}$$

In univariate settings, the sine terms cancel as $\rho_k = \rho_{-k}$ and the spectral density is real. Hence,

$$\varpi_{12}(\alpha) = 1 + \sum_{1}^{\infty} \{\rho_{(12)s} \cos s\alpha + \rho_{(12)-s} \cos s\alpha\} + i\left\{\sum_{1}^{\infty} \rho_{(12)s} \sin s\alpha + \rho_{(12)-s} \sin s\alpha\right\}. \tag{16}$$

From (16) we can see that the cross-spectra has an imaginary and real component. The first two terms on the right hand side, $c(\alpha)$, are the co-spectra and the final term, $q(\alpha)$, is the spectral density. The sum of the squares of these two terms is the amplitude, which when standardised by the spectral densities of each separate series, $\varpi_i(\alpha)$, is called the coherence:

$$C(\alpha) = \frac{c^2(\alpha) + q^2(\alpha)}{\varpi_1(\alpha)\varpi_2(\alpha)} \tag{17}$$

The coherence measures the degree to which the series vary together and can be thought of as the squared correlation coefficient. The gain diagram plots ordinate $R_{12}^2(\alpha)$ against α as abscissa, where

$$R_{12}^2(\alpha) = \frac{\varpi_1(\alpha)}{\varpi_2(\alpha)} C(\alpha). \tag{18}$$

The gain is analogous to a regression coefficient. Finally, the phase diagram plots $\psi(\alpha)$ as against α as abscissa, with the phase measuring the lead or lag in the relationship at each frequency:

$$\psi(\alpha) = \arctan \frac{q(\alpha)}{c(\alpha)}. \tag{19}$$

4. Business cycle facts

4.1 Duration of the UK business cycle

Business cycles are typically represented in industrialised economies as periodic fluctuations across a range of macro-aggregates with duration of some eight to 32 quarters. Quantitative measurement of the business cycle (or cyclical time unit) therefore requires the isolation of this component of aggregate fluctuations.[21] But first we ask whether the UK evidence suggests that business cycles can be characterised by a similar duration of periodic fluctuations. The business cycle dates underpinning the lengths are derived from the work of several researchers, closely connected with the NBER in the US and the National Institute of Economic and Social Research in the UK. Although some uncertainty is attached to any particularly dating methodology, we strongly suspect the typical duration will be reasonably insensitive to alternate dating strategies.[22]

Table 1 shows averages for the duration of complete UK business cycles since 1871 and clearly shows that business cycle duration is entirely consistent with that definition.[23] There is some evidence of changes in

Table 1. Average length in months of UK business cycles (from 1871 onwards)

Average, all cycles:	Peak to peak	Trough to trough
1871–1997	61.48	62.81
1871–1913	79.71	78.43
1946–1997	60.75	55.63
Average peacetime cycles:		
1871–1913	79.71	78.43
1919–1939	45.4	46.6
1946–1997	60.75	55.63
Standard error, all cycles:		
1871–1997	28.80	27.80
1871–1913	32.93	33.47
1946–1997	26.54	10.68
Standard error, peacetime cycles:		
1871–1913	32.93	33.47
1919–1939	20.94	32.75
1946–1997	26.54	10.68

Source: Moore and Zarnowitz (1986) supplemented by Artis *et al.* (1995) and Dow (1998). See Annex A in Chadha, Janssen and Nolan (2000b) for table of dates.

Table 2. Correlation matrix

	Output	Cons.	Invest.	M0	M3	Real M0	Real M3	Short	Long	Real Sh.	Real Lo.	RER	Wages	Prices	CA	TFP
Output	2.23															
Cons.	0.16	1.99														
Invest.	0.23**	0.48	8.01													
M0	0.01	0.51**	0.31**	2.28												
M3	–0.00	0.43**	0.60**	0.60**	2.46											
Real M0	0.09	0.52**	0.22**	0.62**	0.19**	3.39										
Real M3	0.09	0.54**	0.22**	0.42**	0.56**	0.79	3.11									
Short	–0.04	0.08	0.03	0.08	0.22**	–0.09	0.01	0.99								
Long	–0.22**	–0.17	–0.15	0.03	0.15	–0.16	–0.07	0.62**	0.43							
Real Sh.	–0.13	–0.03	0.09	–0.01	–0.05	0.48**	0.49**	–0.06	–0.10	3.31						
Real Lo.	–0.47**	–0.32**	–0.04	–0.05	0.15	–0.37**	–0.25**	0.03	0.22**	0.28**	1.49					
RER	0.20**	0.04	0.19**	0.10	0.05	0.31**	0.30**	0.08	0.04	0.19*	–0.17	6.43				
Wages	–0.05	0.42**	0.31**	0.44**	0.32**	0.37**	0.33**	0.28**	0.15	–0.01	–0.17	–0.15	2.07			
Prices	–0.11	–0.23**	–0.01	0.07	0.27**	–0.74**	–0.65**	0.19*	0.23**	–0.61**	0.43**	–0.31**	–0.09	2.67		
CA	–0.24**	–0.11	–0.04	–0.15	–0.18	–0.16	–0.21**	–0.03	0.03	0.15	0.33**	0.07	–0.06	0.08	8.92	
TFP	0.90**	–0.02	0.14	–0.14	–0.16	0.05	0.02	–0.23**	–0.27**	–0.01	–0.37**	0.15	–0.23**	–0.18	–0.22**	2.00

Notes: a) the data are band-pass filtered, b) the diagonal corresponds to the variable's standard deviation, c) * indicates significant at 5 per cent and ** at 1 per cent, using a Student's t-distribution where $t = r\sqrt{n-2} / \sqrt{1-r^2}$, with r is the sample correlation coefficient and n the number of observations. d) Data is 1871–1997.

Table 3. Correlations between filters and correlations with output

Variable	BK–CF	BK–HP	CF–HP	BK	CF	HP
Output	**0.94**	0.85	0.83	2.358	2.180	2.431
Cons.	**0.94**	0.90	0.82	0.16	0.18	0.05
Invest.	**0.94**	0.89	0.82	0.23	0.26	0.08
M0	**0.86**	0.70	0.68	0.01	0.02	0.00
M3	**0.84**	0.76	0.76	0.00	0.02	–0.06
Real M0	**0.95**	0.88	0.84	0.09	0.07	–0.04
Real M3	**0.91**	0.82	0.82	0.09	0.08	–0.09
Short	**0.94**	0.92	**0.94**	–0.04	–0.05	0.02
Long	**0.93**	**0.93**	0.76	–0.22	–0.21	–0.15
Real Sh.	**0.96**	0.95	0.93	–0.13	–0.09	–0.21
Real Lo.	**0.90**	0.87	0.76	–0.47	–0.38	–0.46
RER	**0.94**	0.91	0.85	0.20	0.26	0.13
Wages	**0.96**	0.89	0.90	–0.05	–0.01	–0.12
Prices	**0.88**	0.79	0.74	–0.11	–0.08	0.04
CA	**0.96**	0.92	0.94	–0.24	–0.17	–0.31
TFP	**0.97**	0.92	0.88	0.90	0.73	0.90

Notes:a) the highest correlation in columns two–four is given in bold, b) in columns three and four, * denotes that the sample correlation coefficient is outside the 95 per cent confidence limit of the BK–CF correlation coefficient; and c) the correlations given here are the full sample.

business cycle length in sub-periods but this is weak. There is, perhaps more interesting dispute on the correct dates to ascribe to specific peaks and troughs. It seems that the average UK business cycle, from peak to peak or from trough to trough, lasts some 62 months, with a standard error of some 28 months, implying that the business cycle can be thought to vary, on UK evidence, in the region of 11–30 quarters.[24]

4.2 Business cycle moments

Quantitative macroeconomic models are often constructed to capture key aspects of business cycle fluctuations. We therefore now analyse how the cyclical components of these variables interact contemporaneously with one another and with output. Along with the familiar filter due to Hodrick and Prescott, we use the two recently developed versions of the band-pass filter outlined above as a natural complement to our frequency-domain characterisation of the data.

First, we use the Baxter-King band-pass filter. This is a two-sided symmetric filter where the lag/lead length needs to be chosen. As Baxter and King note there is a trade-off here, since a longer lag length approximates the optimal filter more closely, while it shortens the cyclical series obtained. We experimented with a number of lag/lead lengths and found that the results were virtually identical after a lag/lead of six years. With respect to the Christiano and Fitzgerald filter we use their recommended filter which assumes the underlying data is well-characterised by a pure random walk. We have also extracted the cyclical components of the variables using the Hodrick-Prescott filter with the smoothing parameter set to 7 and 100 (see Harvey and Jaeger, 1993).

Table 2 and Charts 1–9 should be read in conjunction with one another. Chart 1 plots the detrended output per head series (dotted line) against time with periods of expansion in light relief and periods of contraction in dark relief. The final (solid) line is the detrended series for consumption per head. We note clear procyclicality and relative smoothness in consumption. The relatively low whole sample correlation seems driven by some unusual interwar patterns.

We note here as well that there is very little difference across filters for the series. The first three columns of Table 3 show the correlations between the business cycle components captured by the three filters – in every case the correlation is highly positive and significant. We therefore concentrate on showing the results for the Baxter-King filter in the remaining charts.

Chart 2 illustrates the high volatility of investment along with its basic procyclicality. Chart 3 shows the very close mapping between TFP and output. Chart 4 suggests that procyclical money balances are very much a postwar phenomenon, as are countercyclical prices, see Chart 5. Similarly, real wages, Chart 6, seem to move from being counter-procyclical with time. Short nominal rates, Chart 7, have only a limited systematic relationship with the cycle, whereas the current account, Chart 8, and the real exchange rate, Chart 9, seem robustly related to the economic cycle, negatively and positively, respectively.

Arguably the real quantities seem more robustly related to the economic cycle, whereas those macroeconomic indicators related to the price level, apart from the real exchange rate, seem, in a number of cases, to change its relationship through time (particularly postwar), for example, real wages, the price level and real money balances.[25]

4.3 Co-spectra

A natural extension of the analysis in the previous sections is to ask whether the significant time series correlations we uncover remain significant when we examine the correlations among the raw data in the frequency domain. The cross-spectra indicate the importance of the relationship between each variable and output across *all* frequencies.

Charts 10–17 plot the co-spectra of the time series. The coherence measures the degree to which the series vary together and can be thought of as the squared correlation coefficient and lies between 0 and 1. The gain is analogous to the value of a regression coefficient and therefore is not so bounded. Finally, the phase diagram plots $\psi(\alpha)$ (eqn. 19) as against α as abscissa, with the phase measuring the lead or lag in the relationship at each frequency where positive values indicate, for example, output leads, and negative values output lags. The extreme right-hand side of each plot corresponds to half the highest frequency of the data, in this case two years, and the half-way point of each plot represents quarter a cycle per year i.e. four years. Our notion of business cycle coherence therefore corresponds broadly to the section of the plots lying between 0 and 0.5 cycles per year.

These charts provide a useful representation of the association found in macroeconomic aggregates with reference to the cycle. Chart 10 illustrates the high cross-spectra of consumption with output across the frequency range. Note that peak coherence and gain occur at just over four years, i.e. just to the left of 0.5. Output seems to lead consumption at the higher business cycle frequencies, that is up to 0.125, but subsequently it is consumption that leads output. Chart 11 suggests that investment is most closely related to output at around four years but at lower frequencies, over eight years and more, investment seems to provide an important lead for output. Chart 12 shows very clearly why TFP shocks perform an admirable role in explaining the business cycle in the modern literature. Chart 13 demonstrates both the long-run neutrality of money, with both coherence and gain heading for 1 at the trend frequency, and the existence of some price stickiness, with money to some extent leading prices at the higher business cycle frequencies of 2–3 years.

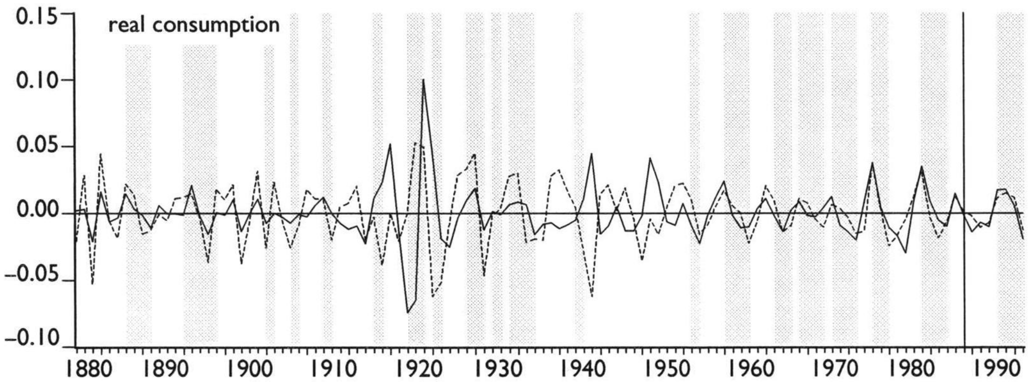
Chart 14 shows a significant coherence between output and real wages at higher frequencies with the gain increasing with frequency up to five years. Output leads at lower frequencies though there is some (noisy) evidence of wages leading at higher frequencies. Chart 15 examines interest rates and the cycle and finds significant coherences across the business cycle but the gain suggests relatively low response of output to the interest rate and at the business cycle frequency there is strong evidence of an output lead except at 2–4 years, when interest rates seem to lead output: this corresponds fairly closely to the horizon at which monetary policy may operate best. Charts 16 and 17 illustrate the open economy aspect. The current account not only has considerable coherence with the business cycle at low business cycle frequencies, but also has a significant lead over output. Finally, fluctuations in international relative prices seem closely associated with longer run output fluctuations.

5. Concluding remarks

Robert Lucas (1977) wrote: 'There is...no need to qualify [business cycle] observations by restricting them to particular countries or time periods; they appear to be regularities common to all decentralised market economies.' Nevertheless, establishing facts for small analytical models represents an important agenda for macroeconomic research. Much modern theory considers the systematic action of agents and policymakers in response to economic structure and the uncertainty induced by shocks. To the extent that these characterisations of the world bear some resemblance to reality, stylised facts are required.[26]

This article has collected a number of such facts. A key finding is that consumption, investment and the current account have significant leads for output fluctuations at business cycle frequencies. For instance, the current account is found to be consistently countercyclical and with important leads over output.[27] Measured total factor productivity has significant explanatory power over output fluctuations at all frequencies.[28] Money neutrality obtains in the long run but that masks an important non-neutrality at business cycle frequencies. But there is some evidence of temporal instability as real wages are acyclical over the full sample period and this conceals prewar countercyclicality and postwar procyclicality.[29] Similarly, the countercyclicality of prices would seem to be a post-war phenomenon; prior to 1914 prices seem procyclical.[30] The temporal stability of these results, and particularly the behaviour of the price level, are an obvious avenue for further analysis.[31] But one fact emerges very clearly and that is productivity (see Charts 3 and 12) seems central to the process of understanding economic fluctuations, as well as long-run growth (see Prescott, 2002).

Chart 1A. Output (dotted) with consumption –Baxter-King



Note: The last complete business cycle identified by these methods was completed in 1989. This is indicated by the vertical line on charts 1A–9.

Chart 1B. Output (dotted) with consumption – Christiano-Fitzgerald

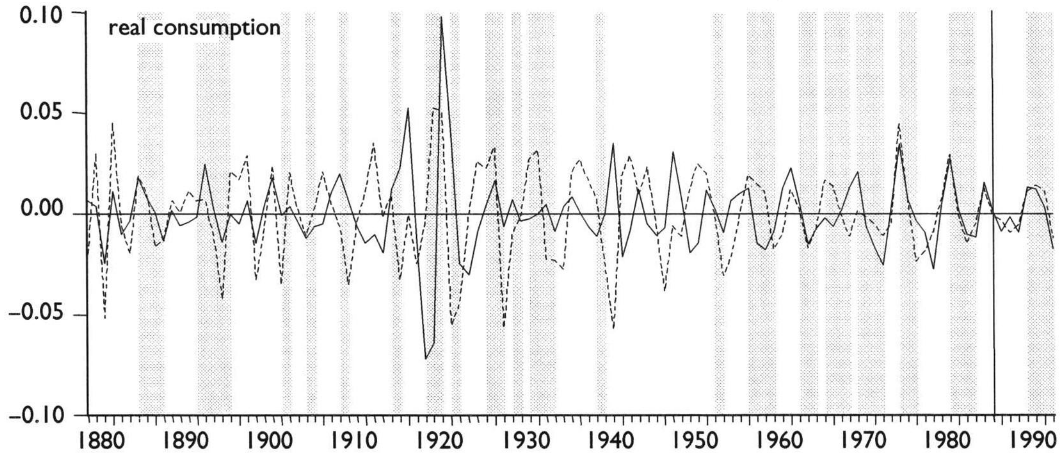


Chart 1C. Output (dotted) with consumption – Hodrick-Prescott

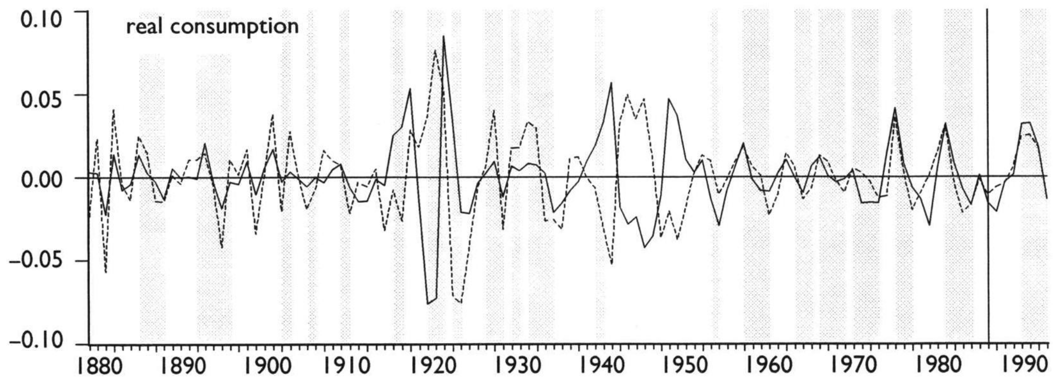


Chart 2. Output (dotted) with investment – Baxter-King

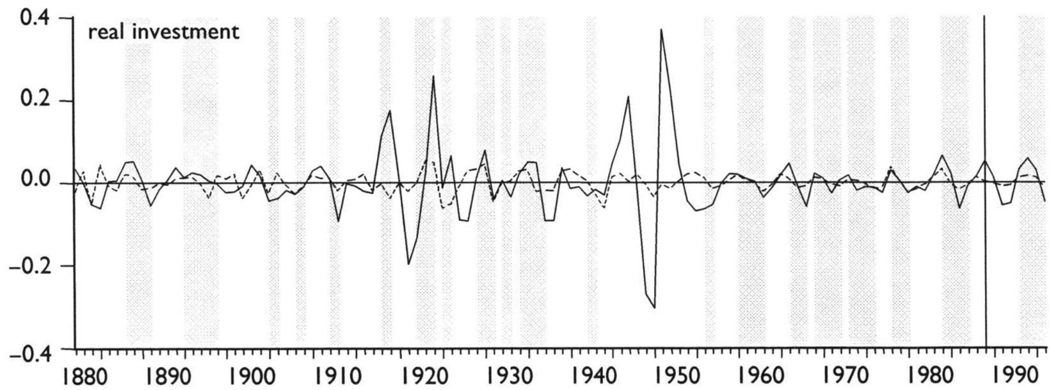


Chart 3. Output (dotted) with TFP – Baxter-King

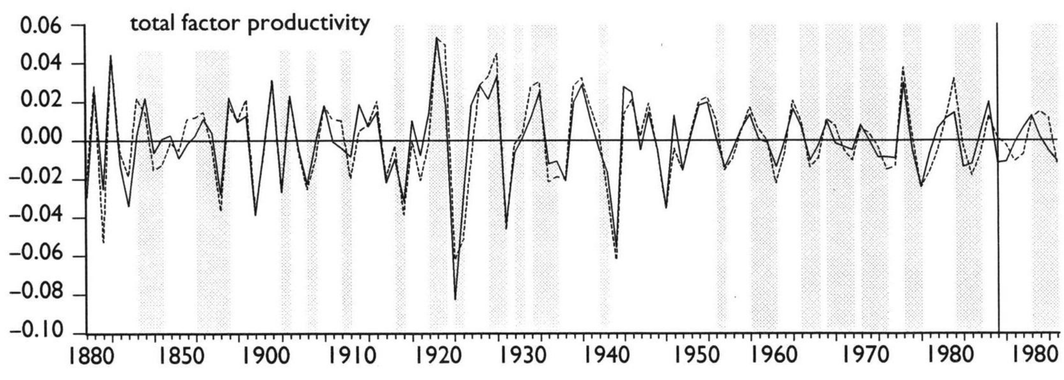


Chart 4. Output (dotted) with real M0 – Baxter-King

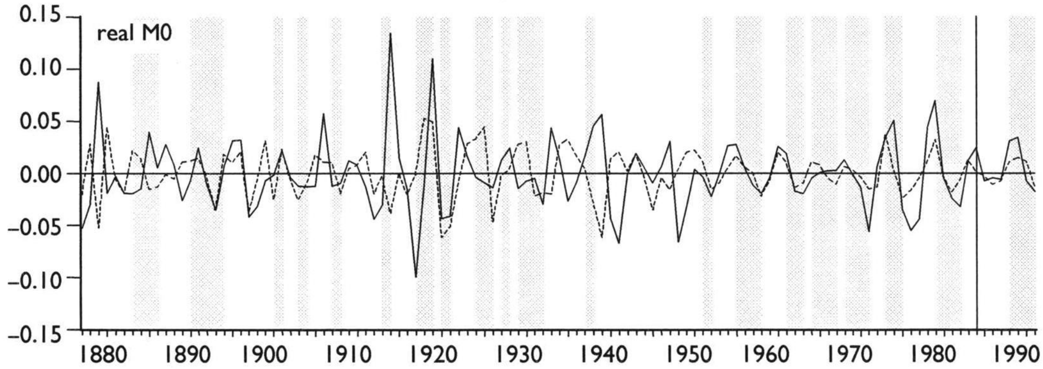


Chart 5. Output (dotted) with price level – Baxter-King

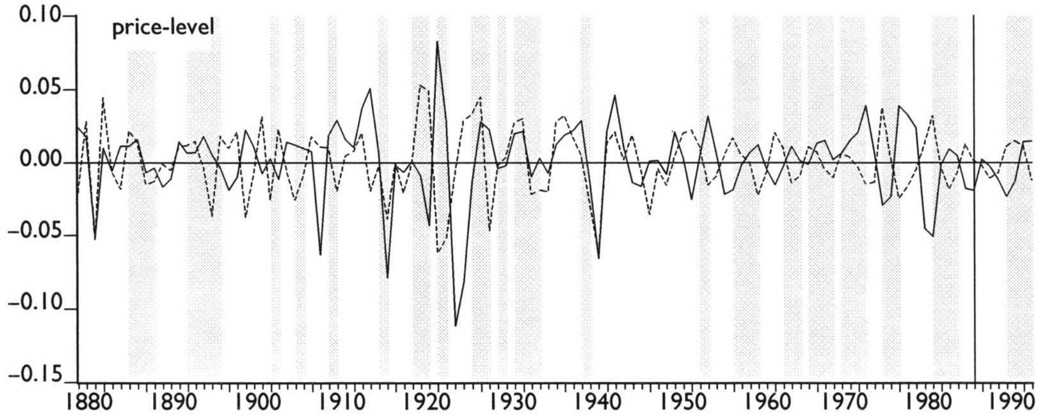


Chart 6. Output (dotted) with real wages – Baxter-King

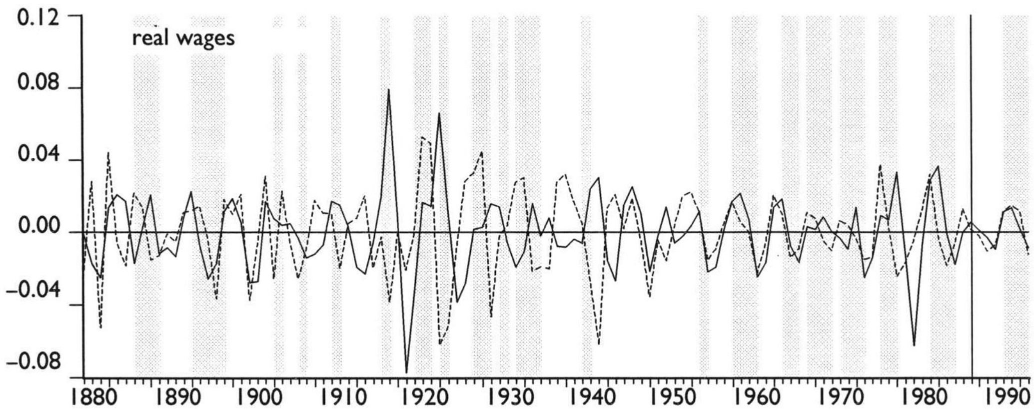


Chart 7. Output (dotted) with short rates – Baxter-King

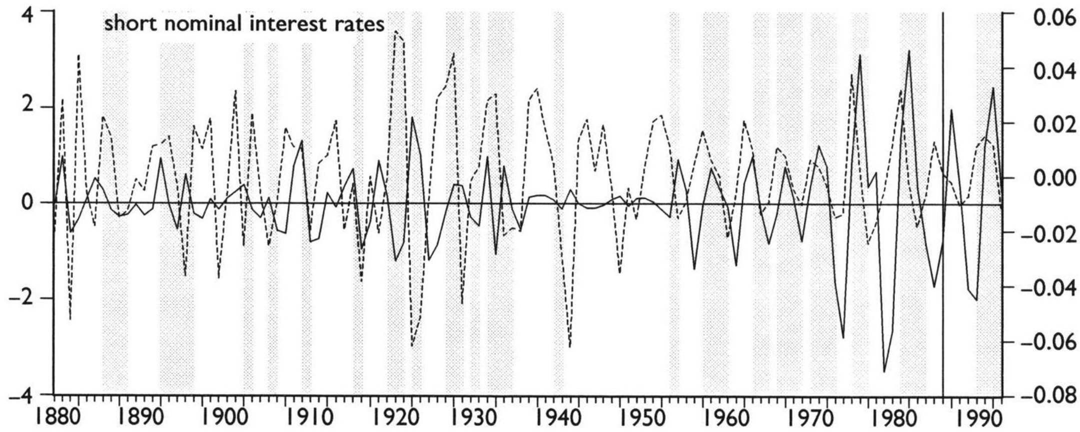


Chart 8. Output (dotted) with the current account – Baxter-King

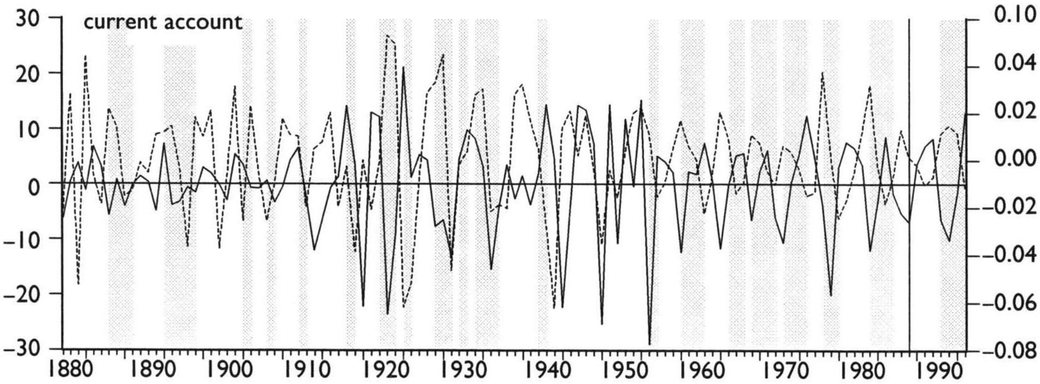


Chart 9. Output (dotted) with the real exchange rate – Baxter-King

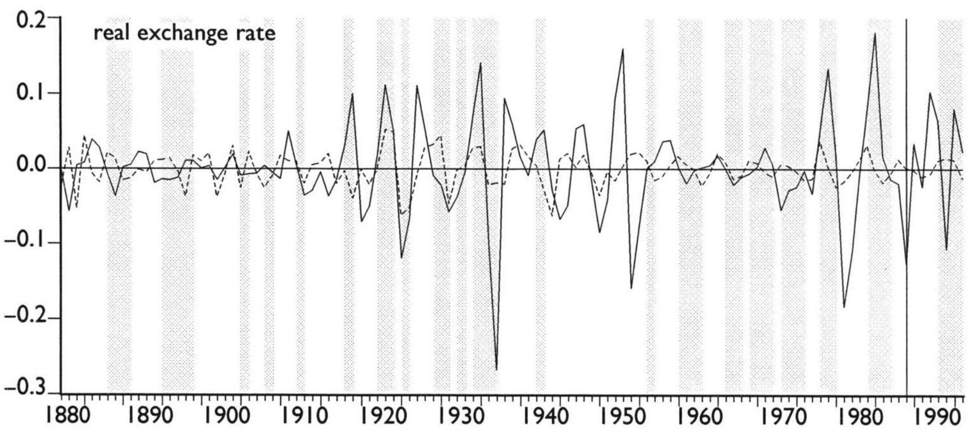


Chart 10. Cross-spectra of output with consumption (1871–1998)

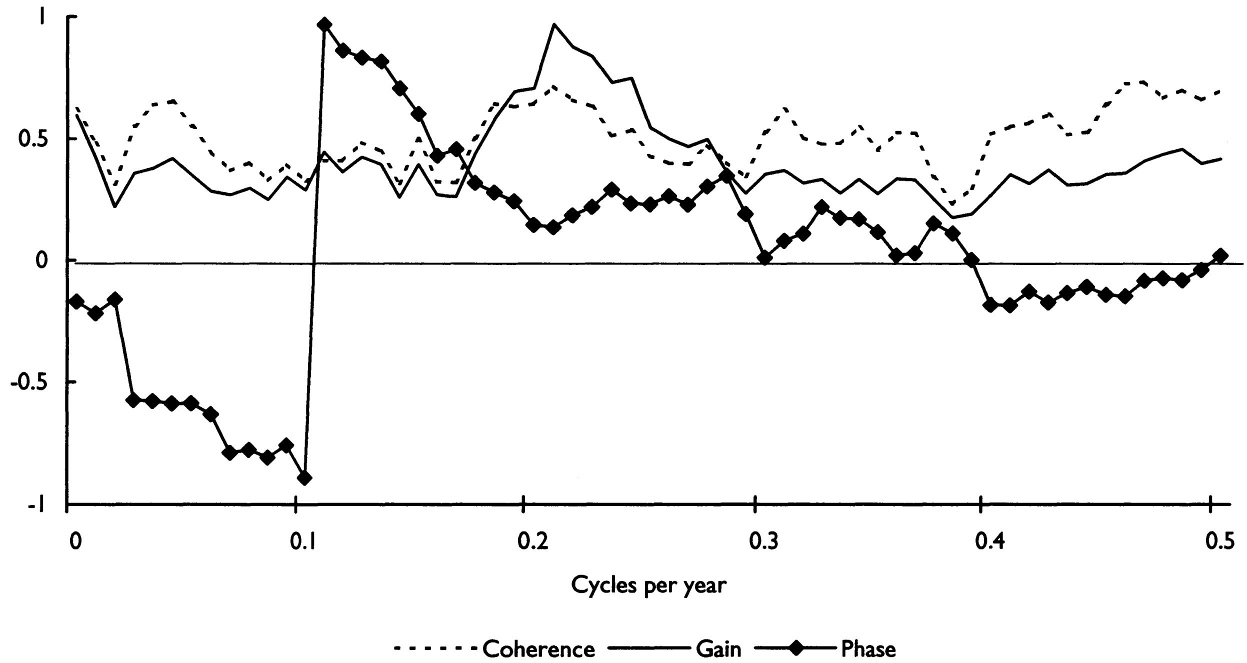


Chart 11. Cross-spectra of output with investment (1871–1998)

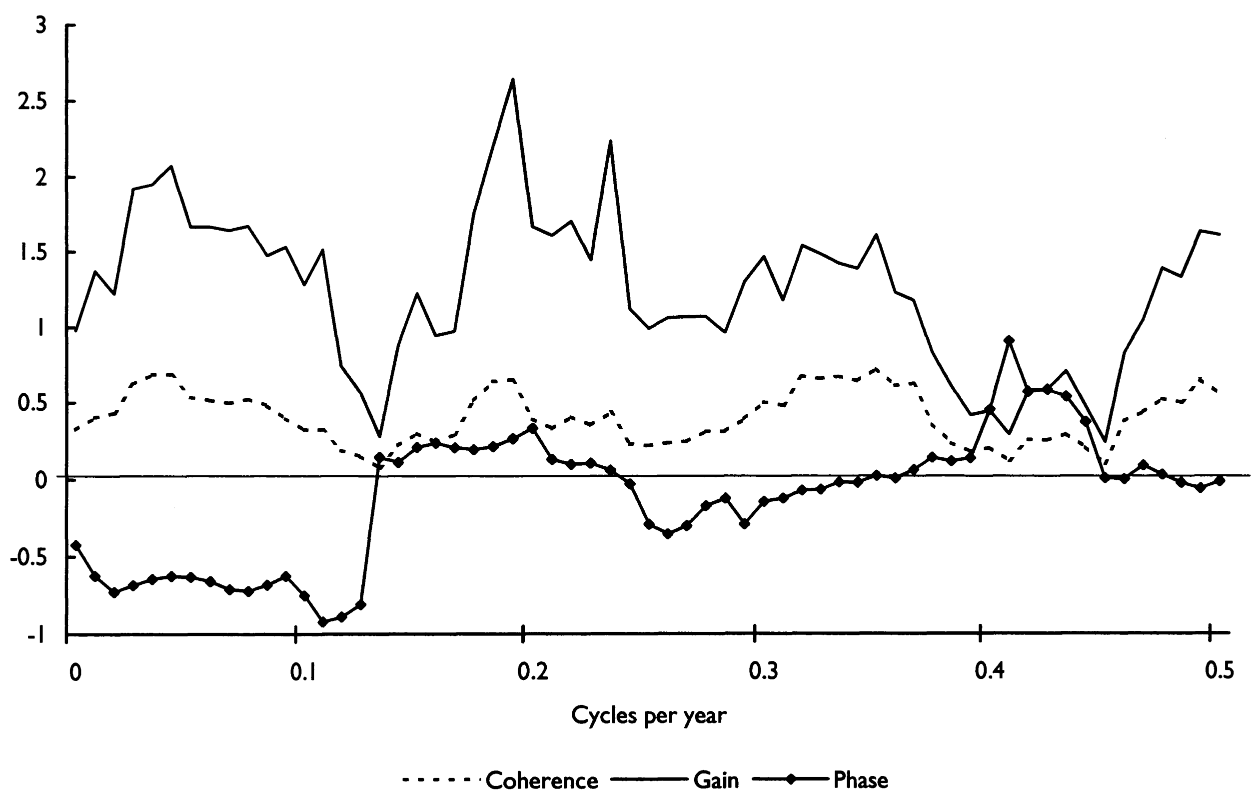


Chart 12. Cross-spectra of output with TFP (1871–1998)

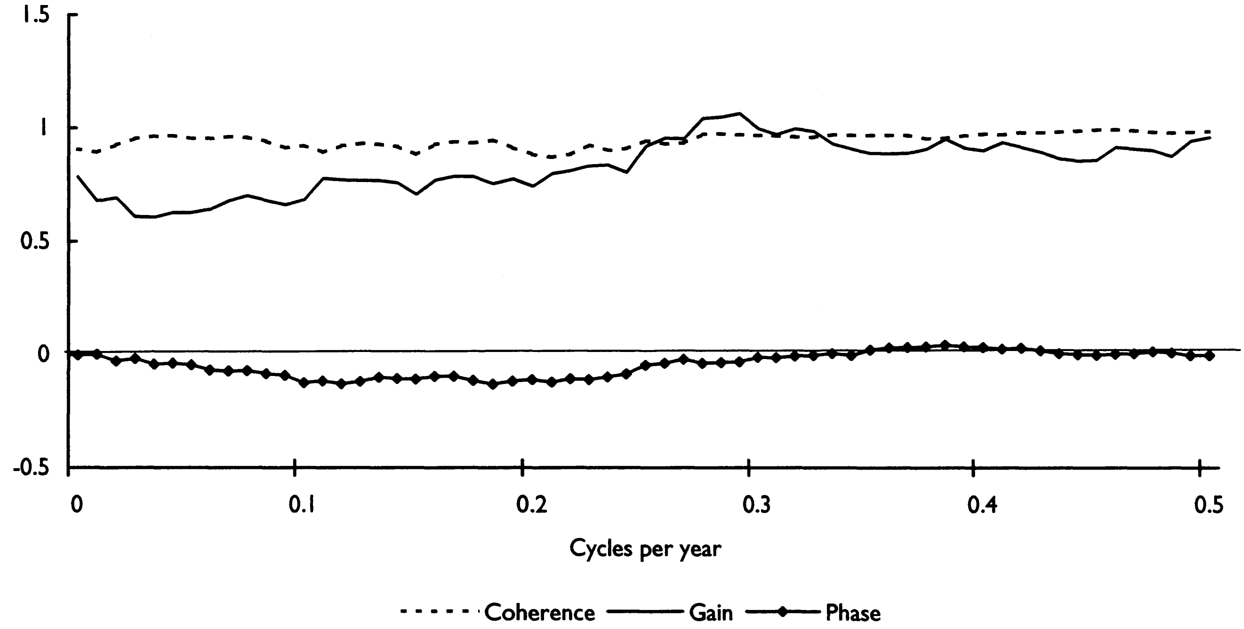


Chart 13. Cross-spectra of money with prices (1871–1998)

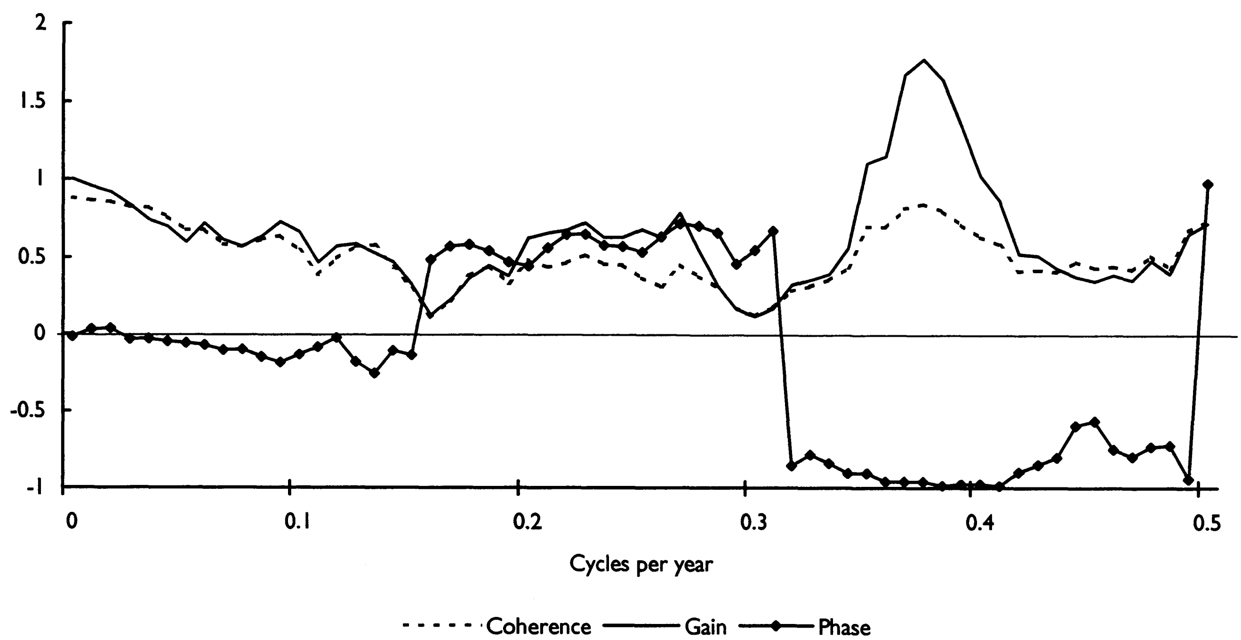


Chart 14. Cross-spectra of output with real wages (1871–1998)

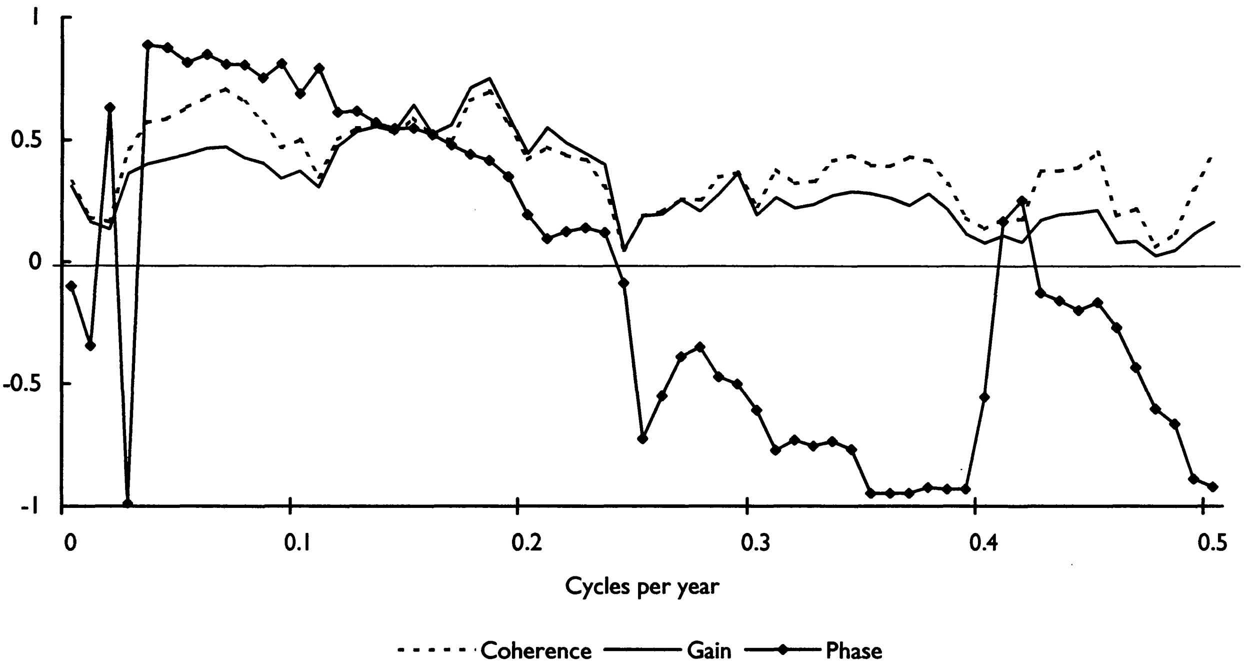


Chart 15. Cross-spectra of output with interest rates (1871–1998)

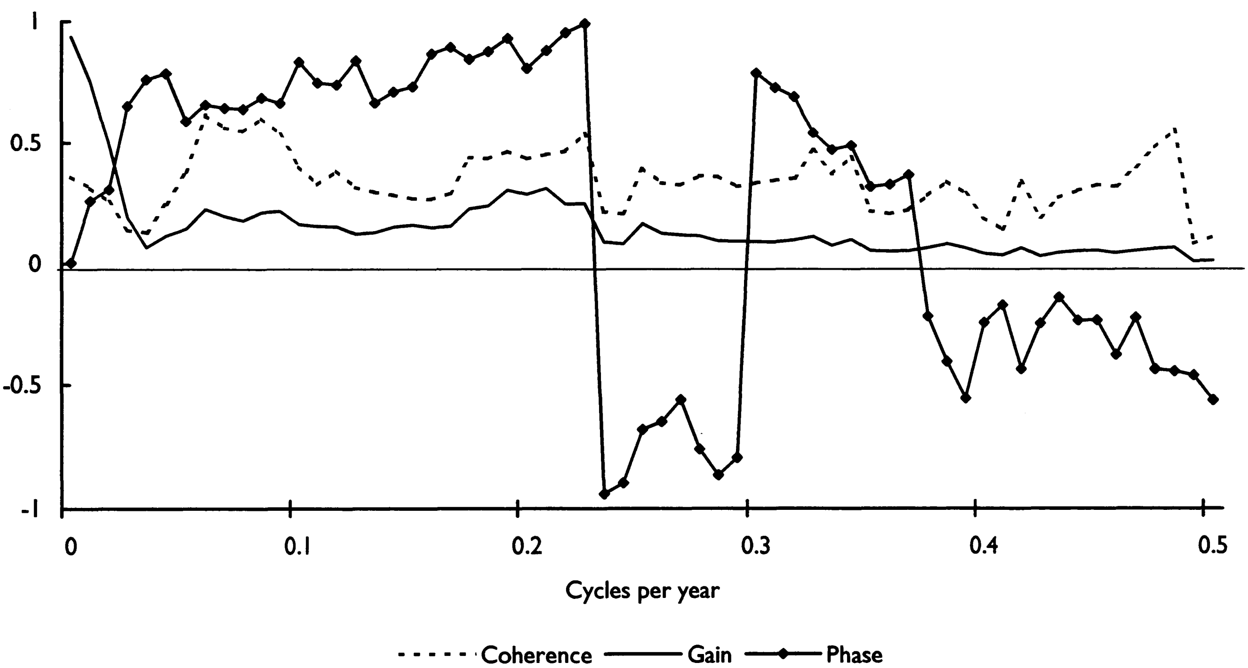


Chart 16. Cross-spectra of output with the current account (1871–1998)

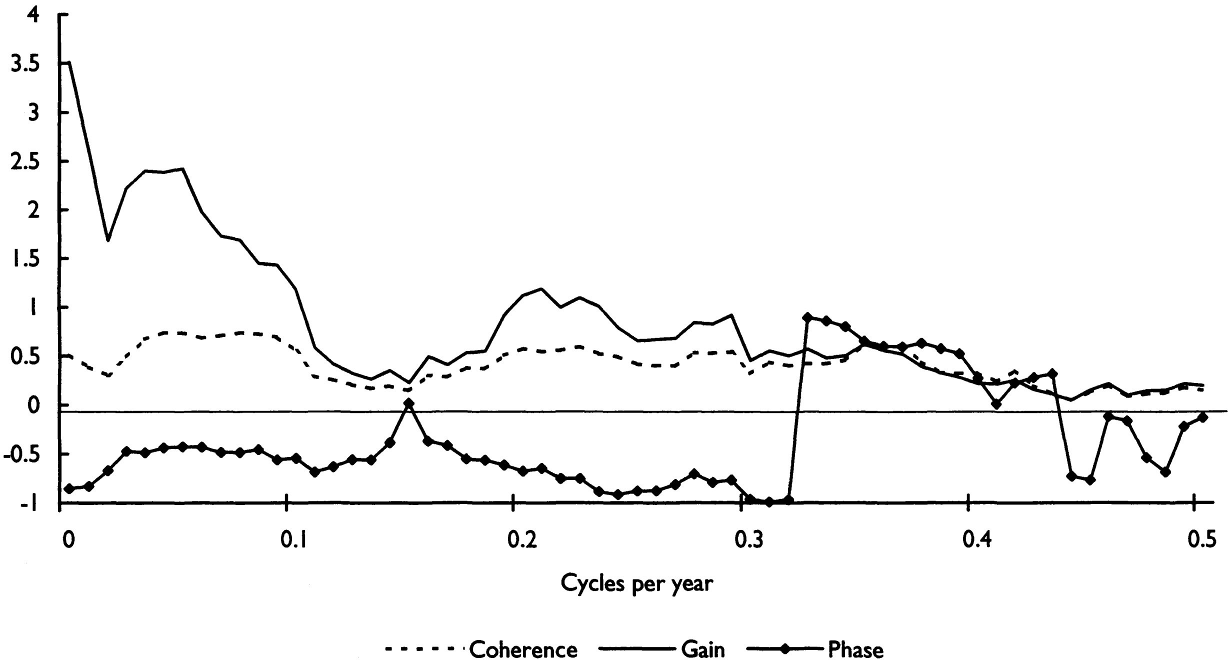
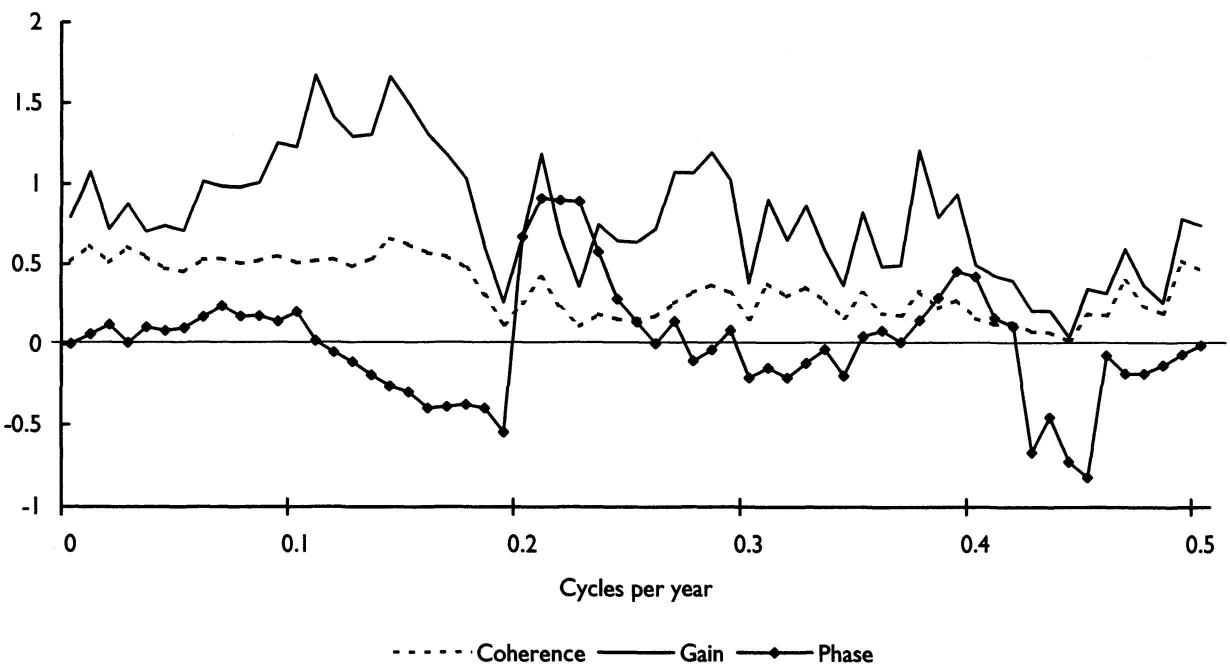


Chart 17. Cross-spectra of output with the real exchange rate (1871–1998)



NOTES

1 See Cooley (1995).
2 This note draws heavily on Chadha, Janssen and Nolan (2000a,b, 2001), Chadha and Nolan (2002a,b,c) and see Altug, Chadha and Nolan (forthcoming 2003) for a number of examples of small theoretical models.
3 See Feinstein (1972), Matthews *et al* (1982) and Mitchell (1988) for the sources of the UK series.
4 See most notably, Backus and Kehoe (1992) who particularly consider the cyclical and temporal behaviour of the aggregate price level across countries.
5 As well as providing a pass-through the data, the spectral density results provide additional criteria for testing the artificial economy that is generated by general equilibrium models.
6 We employ therefore the band-pass filters recently developed by Baxter and King (1999) and by Christiano and Fitzgerald (1999), as well as the more widely used filter recommended by Hodrick and Prescott (1997), though in this article we rely mainly on the Baxter-King filter.
7 The coherence measures the squared correlation coefficient at any given frequency, the gain the regression coefficient and the phase the extent to which the first named series leads the second. Note that the highest frequency corresponds to 1 on the abscissa and the lowest, or trend frequency, to 0.
8 The original source for these data is Wood (1909), cited in Mitchell (1988).
9 Mitchell's data are taken from Bowley (1937), cited in Mitchell (1988), who estimates economy-wide wages using partial information about wages in some industries.
10 The Department of Employment and Productivity (1971), cited in Mitchell (1988), collected original data.
11 These data are not annual, but only available as averages for six subperiods.
12 Source: Homer and Sylla (1987).
13 The US debate on the difference between pre and postwar cycles has been almost left incontestable following the convincing attack by Romer (1989) on the quality of data sources.
14 See Table 1.9 in Feinstein (1972) for a description of the reliability of the component series.
15 See Chadha, Janssen and Nolan, 2000b for more description of this point.
16 Although we will provide a fair amount of detail on this, our exposition proceeds at a somewhat informal level, and we deliberately sidestep several important technical issues. Hopefully this approach will aid intuition at minimal cost in terms of lack of rigour.
17 We are here referring to the spectral representation theorem, which holds for all, complex and real-valued, functions. Cox and Miller (1965) derive the spectral representation theorem (Chapter 8).
18 We note, as an aside, that because of these properties it is, in principle, straightforward to decompose the variance of our stationary time series by frequency. That is:

$$\text{var}(Y_t) = \text{var}\left[\int_{-\pi}^{\pi} dX(\varpi)\right] = F(\varpi).$$

In other words, $F(\omega)$ is the spectral distribution function, the proportion of the variance produced by frequencies in the range $(0, \omega)$. The power spectral density function is then given by $f(\varpi) = dF(\varpi) / d\varpi$, which forms the basis of our calculations of the power spectrum.
19 In fact, frequency domain analysis more generally tends to be data intensive and severely limits our ability to split the data into many sub-samples.
20 BK and CF both contain detailed and important discussions as to these desirable properties. We do not cover these arguments.
21 See Neftci (1986) for an introduction to this issue.
22 See Artis *et al.* (1995), Dow (1998) and Moore and Zarnowitz (1986).
23 The sources of the dates are given in Chadha, Janssen and Nolan (2000b).
24 The resulting period encompassing one standard deviation of the business cycle, 11 to 30 quarters, corresponds almost exactly to our chosen frequency for annual data of 2 to 8 years.
25 When we say postwar, we mean the period following WWII and prewar refers to the period priot to WWI.
26 See Chadha and Nolan (2002a) for an example of an exercise in understanding the implications for the design of monetary policy.
27 See Chadha, Janssen and Nolan (2001).
28 Though note that there is clear lead and lag information for output from TFP.
29 Note that if we corrected the real wage index to measure real wage per unit of efficiency the procyclicality is likely to be stronger, see Solon *et al.* (1994) on this point.
30 There is also some evidence of price stickiness, in the sense that both money and prices display similar spectral densities.
31 See Chadha, Janssen and Nolan (2000a,b)

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