CHAPTER 10

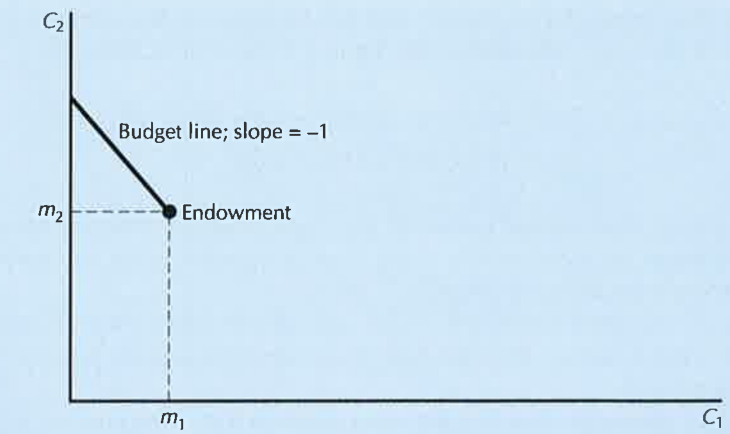
INTERTEMPORAL CHOICE

In this chapter we continue our examination of consumer behavior by considering the choices involved in saving and consuming over time. Choices of consumption over time are known as **intertemporal choices**.

10.1 The Budget Constraint

Let us imagine a consumer who chooses how much of some good to consume in each of two time periods. We will usually want to think of this good as being a composite good, as described in Chapter 2, but you can think of it as being a specific commodity if you wish. We denote the amount of consumption in each period by (c_1, c_2) and suppose that the prices of consumption in each period are constant at 1. The amount of money the consumer will have in each period is denoted by (m_1, m_2).

Suppose initially that the only way the consumer has of transferring money from period 1 to period 2 is by saving it without earning interest. Furthermore let us assume for the moment that he has no possibility of



Budget constraint. This is the budget constraint when the rate of interest is zero and no borrowing is allowed. The less the individual consumes in period 1, the more he can consume in period 2.

Figure 10.1

borrowing money, so that the most he can spend in period 1 is m_1. His budget constraint will then look like the one depicted in Figure 10.1.

We see that there will be two possible kinds of choices. The consumer could choose to consume at (m_1, m_2), which means that he just consumes his income each period, or he can choose to consume less than his income during the first period. In this latter case, the consumer is saving some of his first-period consumption for a later date.

Now, let us allow the consumer to borrow and lend money at some interest rate r. Keeping the prices of consumption in each period at 1 for convenience, let us derive the budget constraint. Suppose first that the consumer decides to be a saver so his first period consumption, c_1, is less than his first-period income, m_1. In this case he will earn interest on the amount he saves, $m_1 - c_1$, at the interest rate r. The amount that he can consume next period is given by

$$\begin{aligned} c_2 &= m_2 + (m_1 - c_1) + r(m_1 - c_1) \\ &= m_2 + (1 + r)(m_1 - c_1). \end{aligned} \tag{10.1}$$

This says that the amount that the consumer can consume in period 2 is his income plus the amount he saved from period 1, plus the interest that he earned on his savings.

Now suppose that the consumer is a borrower so that his first-period consumption is greater than his first-period income. The consumer is a borrower if $c_1 > m_1$, and the interest he has to *pay* in the second period will be $r(c_1 - m_1)$. Of course, he also has to pay back the amount that he borrowed, $c_1 - m_1$. This means his budget constraint is given by

$$\begin{aligned} c_2 &= m_2 - r(c_1 - m_1) - (c_1 - m_1) \\ &= m_2 + (1+r)(m_1 - c_1), \end{aligned}$$

which is just what we had before. If $m_1 - c_1$ is positive, then the consumer earns interest on this savings; if $m_1 - c_1$ is negative, then the consumer pays interest on his borrowings.

If $c_1 = m_1$, then necessarily $c_2 = m_2$, and the consumer is neither a borrower nor a lender. We might say that this consumption position is the "Polonius point."[1]

We can rearrange the budget constraint for the consumer to get two alternative forms that are useful:

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2 \tag{10.2}$$

and

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}. \tag{10.3}$$

Note that both equations have the form

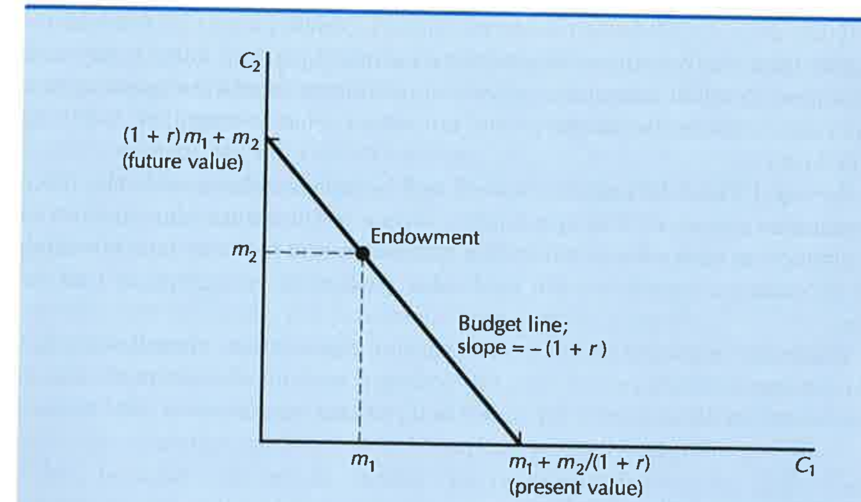
$$p_1 x_1 + p_2 x_2 = p_1 m_1 + p_2 m_2.$$

In equation (10.2), $p_1 = 1 + r$ and $p_2 = 1$. In equation (10.3), $p_1 = 1$ and $p_2 = 1/(1+r)$.

We say that equation (10.2) expresses the budget constraint in terms of **future value** and that equation (10.3) expresses the budget constraint in terms of **present value**. The reason for this terminology is that the first budget constraint makes the price of future consumption equal to 1, while the second budget constraint makes the price of present consumption equal to 1. The first budget constraint measures the period-1 price *relative* to the period-2 price, while the second equation does the reverse.

The geometric interpretation of present value and future value is given in Figure 10.2. The present value of an endowment of money in two periods is the amount of money in period 1 that would generate the same budget set as the endowment. This is just the horizontal intercept of the budget line, which gives the maximum amount of first-period consumption possible.

1 "Neither a borrower, nor a lender be; For loan oft loses both itself and friend, And borrowing dulls the edge of husbandry." *Hamlet*, Act I, scene *iii*; Polonius giving advice to his son.



Present and future values. The vertical intercept of the budget line measures future value, and the horizontal intercept measures the present value.

Figure 10.2

Examining the budget constraint, this amount is $\bar{c}_1 = m_1 + m_2/(1+r)$, which is the present value of the endowment.

Similarly, the vertical intercept is the maximum amount of second-period consumption, which occurs when $c_1 = 0$. Again, from the budget constraint, we can solve for this amount $\bar{c}_2 = (1+r)m_1 + m_2$, the future value of the endowment.

The present-value form is the more important way to express the intertemporal budget constraint since it measures the future relative to the present, which is the way we naturally look at it.

It is easy from any of these equations to see the form of this budget constraint. The budget line passes through (m_1, m_2), since that is always an *affordable* consumption pattern, and the budget line has a slope of $-(1+r)$.

10.2 Preferences for Consumption

Let us now consider the consumer's preferences, as represented by his indifference curves. The shape of the indifference curves indicates the consumer's tastes for consumption at different times. If we drew indifference curves with a constant slope of -1, for example, they would represent tastes of a consumer who didn't care whether he consumed today or tomorrow. His marginal rate of substitution between today and tomorrow is -1.

If we drew indifference curves for perfect complements, this would indicate that the consumer wanted to consume equal amounts today and tomorrow. Such a consumer would be unwilling to substitute consumption from one time period to the other, no matter what it might be worth to him to do so.

As usual, the intermediate case of well-behaved preferences is the more reasonable situation. The consumer is willing to substitute some amount of consumption today for consumption tomorrow, and how much he is willing to substitute depends on the particular pattern of consumption that he has.

Convexity of preferences is very natural in this context, since it says that the consumer would rather have an "average" amount of consumption each period rather than have a lot today and nothing tomorrow or vice versa.

10.3 Comparative Statics

Given a consumer's budget constraint and his preferences for consumption in each of the two periods, we can examine the optimal choice of consumption (c_1, c_2). If the consumer chooses a point where $c_1 < m_1$, we will say that she is a **lender**, and if $c_1 > m_1$, we say that she is a **borrower**. In Figure 10.3A we have depicted a case where the consumer is a borrower, and in Figure 10.3B we have depicted a lender.

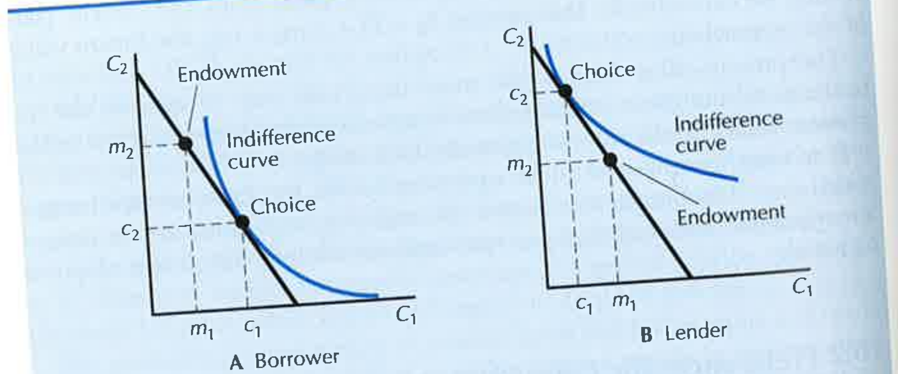


Figure 10.3

Borrower and lender. Panel A depicts a borrower, since $c_1 > m_1$, and panel B depicts a lender, since $c_1 < m_1$.

Let us now consider how the consumer would react to a change in the

interest rate. From equation (10.1) we see that increasing the rate of interest must tilt the budget line to a steeper position: for a given reduction in c_1 you will get more consumption in the second period if the interest rate is higher. Of course the endowment always remains affordable, so the tilt is really a pivot around the endowment.

We can also say something about how the choice of being a borrower or a lender changes as the interest rate changes. There are two cases, depending on whether the consumer is initially a borrower or initially a lender. Suppose first that he is a lender. Then it turns out that if the interest rate increases, the consumer must remain a lender.

This argument is illustrated in Figure 10.4. If the consumer is initially a lender, then his consumption bundle is to the left of the endowment point. Now let the interest rate increase. Is it possible that the consumer shifts to a new consumption point to the *right* of the endowment?

No, because that would violate the principle of revealed preference: choices to the right of the endowment point were available to the consumer when he faced the original budget set and were rejected in favor of the chosen point. Since the original optimal bundle is still available at the new budget line, the new optimal bundle must be a point *outside* the old budget set—which means it must be to the left of the endowment. The consumer must remain a lender when the interest rate increases.

There is a similar effect for borrowers: if the consumer is initially a borrower, and the interest rate declines, he or she will remain a borrower. (You might sketch a diagram similar to Figure 10.4 and see if you can spell out the argument.)

Thus if a person is a lender and the interest rate increases, he will remain a lender. If a person is a borrower and the interest rate decreases, he will remain a borrower. On the other hand, if a person is a lender and the interest rate decreases, he may well decide to switch to being a borrower; similarly, an increase in the interest rate may induce a borrower to become a lender. Revealed preference tells us nothing about these last two cases.

Revealed preference can also be used to make judgments about how the consumer's welfare changes as the interest rate changes. If the consumer is initially a borrower, and the interest rate rises, but he decides to remain a borrower, then he must be worse off at the new interest rate. This argument is illustrated in Figure 10.5; if the consumer remains a borrower, he must be operating at a point that was affordable under the old budget set but was rejected, which implies that he must be worse off.

10.4 The Slutsky Equation and Intertemporal Choice

The Slutsky equation can be used to decompose the change in demand due to an interest rate change into income effects and substitution effects, just

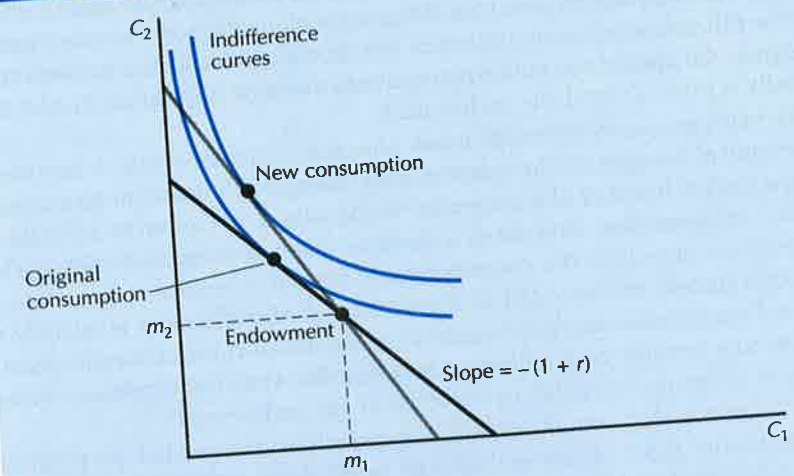


Figure 10.4

If a person is a lender and the interest rate rises, he or she will remain a lender. Increasing the interest rate pivots the budget line around the endowment to a steeper position; revealed preference implies that the new consumption bundle must lie to the left of the endowment.

as in Chapter 9. Suppose that the interest rate rises. What will be the effect on consumption in each period?

This is a case that is easier to analyze by using the future-value budget constraint, rather than the present-value constraint. In terms of the future-value budget constraint, raising the interest rate is just like raising the price of consumption today as compared to consumption tomorrow. Writing out the Slutsky equation we have

$$\underset{(?)}{\frac{\Delta c_1^t}{\Delta p_1}} = \underset{(-)}{\frac{\Delta c_1^s}{\Delta p_1}} + \underset{(?)}{(m_1 - c_1)} \underset{(+)}{\frac{\Delta c_1^m}{\Delta m}}.$$

The substitution effect, as always, works opposite the direction of price. In this case the price of period-1 consumption goes up, so the substitution effect says the consumer should consume less first period. This is the meaning of the minus sign under the substitution effect. Let's assume that consumption this period is a normal good, so that the very last term—how consumption changes as income changes—will be positive. So we put a plus sign under the last term. Now the sign of the whole expression will depend on the sign of $(m_1 - c_1)$. If the person is a borrower, this term will be negative and the whole expression will therefore unambiguously be

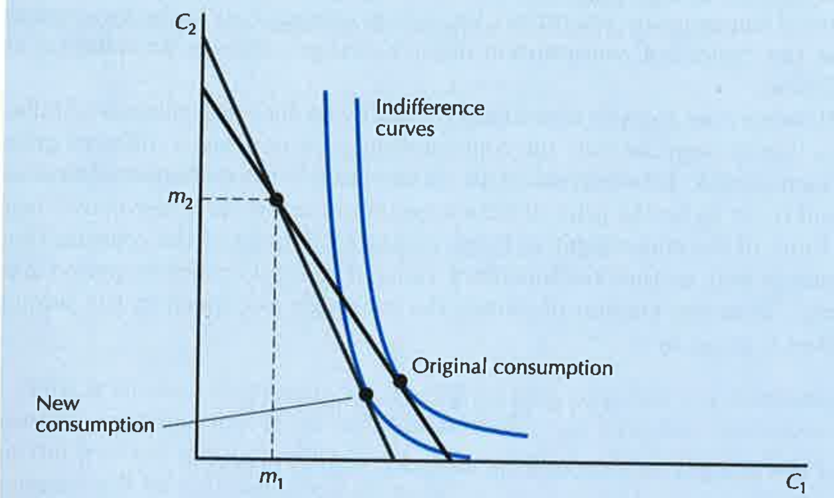


Figure 10.5

A borrower is made worse off by an increase in the interest rate. When the interest rate facing a borrower increases and the consumer chooses to remain a borrower, he or she is certainly worse off.

negative—for a borrower, an increase in the interest rate must lower today's consumption.

Why does this happen? When the interest rate rises, there is always a substitution effect towards consuming less today. For a borrower, an increase in the interest rate means that he will have to pay more interest tomorrow. This effect induces him to borrow less, and thus consume less, in the first period.

For a lender the effect is ambiguous. The total effect is the sum of a negative substitution effect and a positive income effect. From the viewpoint of a lender an increase in the interest rate may give him so much extra income that he will want to consume even more first period.

The effects of changing interest rates are not terribly mysterious. There is an income effect and a substitution effect as in any other price change. But without a tool like the Slutsky equation to separate out the various effects, the changes may be hard to disentangle. With such a tool, the sorting out of the effects is quite straightforward.

10.5 Inflation

The above analysis has all been conducted in terms of a general "consumption" good. Giving up Δc units of consumption today buys you $(1+r)\Delta c$ units of consumption tomorrow. Implicit in this analysis is the assumption that the "price" of consumption doesn't change—there is no inflation or deflation.

However, the analysis is not hard to modify to deal with the case of inflation. Let us suppose that the consumption good now has a different price in each period. It is convenient to choose today's price of consumption as 1 and to let p_2 be the price of consumption tomorrow. It is also convenient to think of the endowment as being measured in units of the consumption goods as well, so that the monetary value of the endowment in period 2 is $p_2 m_2$. Then the amount of money the consumer can spend in the second period is given by

$$p_2 c_2 = p_2 m_2 + (1+r)(m_1 - c_1),$$

and the amount of consumption available second period is

$$c_2 = m_2 + \frac{1+r}{p_2}(m_1 - c_1).$$

Note that this equation is very similar to the equation given earlier—we just use $(1+r)/p_2$ rather than $1+r$.

Let us express this budget constraint in terms of the rate of inflation. The inflation rate, π, is just the rate at which prices grow. Recalling that $p_1 = 1$, we have

$$p_2 = 1 + \pi,$$

which gives us

$$c_2 = m_2 + \frac{1+r}{1+\pi}(m_1 - c_1).$$

Let's create a new variable ρ, the **real interest rate**, and define it by[2]

$$1 + \rho = \frac{1+r}{1+\pi}$$

so that the budget constraint becomes

$$c_2 = m_2 + (1+\rho)(m_1 - c_1).$$

One plus the real interest rate measures how much extra *consumption* you can get in period 2 if you give up some *consumption* in period 1. That is why it is called the *real* rate of interest: it tells you how much extra consumption you can get, not how many extra dollars you can get.

2 The Greek letter ρ, rho, is pronounced "row."

The interest rate on dollars is called the **nominal** rate of interest. As we've seen above, the relationship between the two is given by

$$1 + \rho = \frac{1+r}{1+\pi}.$$

In order to get an explicit expression for ρ, we write this equation as

$$\begin{aligned}\rho &= \frac{1+r}{1+\pi} - 1 = \frac{1+r}{1+\pi} - \frac{1+\pi}{1+\pi} \\ &= \frac{r-\pi}{1+\pi}.\end{aligned}$$

This is an exact expression for the real interest rate, but it is common to use an approximation. If the inflation rate isn't too large, the denominator of the fraction will be only slightly larger than 1. Thus the real rate of interest will be approximately given by

$$\rho \approx r - \pi,$$

which says that the real rate of interest is just the nominal rate minus the rate of inflation. (The symbol $\approx$ means "approximately equal to.") This makes perfectly good sense: if the interest rate is 18 percent, but prices are rising at 10 percent, then the real interest rate—the extra consumption you can buy next period if you give up some consumption now—will be roughly 8 percent.

Of course, we are always looking into the future when making consumption plans. Typically, we know the nominal rate of interest for the next period, but the rate of inflation for next period is unknown. The real interest rate is usually taken to be the current interest rate minus the *expected* rate of inflation. To the extent that people have different estimates about what the next year's rate of inflation will be, they will have different estimates of the real interest rate. If inflation can be reasonably well forecast, these differences may not be too large.

10.6 Present Value: A Closer Look

Let us return now to the two forms of the budget constraint described earlier in section 10.1 in equations (10.2) and (10.3):

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

and

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}.$$

Consider just the right-hand sides of these two equations. We said that the first one expresses the value of the endowment in terms of future value and that the second one expresses it in terms of present value.

Let us examine the concept of future value first. If we can borrow and lend at an interest rate of r, what is the future equivalent of \$1 today? The answer is $(1+r)$ dollars. That is, \$1 today can be turned into $(1+r)$ dollars next period simply by lending it to the bank at an interest rate r. In other words, $(1+r)$ dollars next period is equivalent to \$1 today since that is how much you would have to pay next period to purchase—that is, borrow—\$1 today. The value $(1+r)$ is just the price of \$1 today, relative to \$1 next period. This can be easily seen from the first budget constraint: it is expressed in terms of future dollars—the second-period dollars have a price of 1, and first-period dollars are measured relative to them.

What about present value? This is just the reverse: everything is measured in terms of today's dollars. How much is a dollar next period worth in terms of a dollar today? The answer is $1/(1+r)$ dollars. This is because $1/(1+r)$ dollars can be turned into a dollar next period simply by saving it at the rate of interest r. The *present value* of a dollar to be delivered next period is $1/(1+r)$.

The concept of present value gives us another way to express the budget for a two-period consumption problem: a consumption plan is affordable if *the present value of consumption equals the present value of income.*

The idea of present value has an important implication that is closely related to a point made in Chapter 9: if the consumer can freely buy and sell goods at constant prices, then the consumer would always prefer a higher-valued endowment to a lower-valued one. In the case of intertemporal decisions, this principle implies that *if a consumer can freely borrow and lend at a constant interest rate, then the consumer would always prefer a pattern of income with a higher present value to a pattern with a lower present value.*

This is true for the same reason that the statement in Chapter 9 was true: an endowment with a higher value gives rise to a budget line that is farther out. The new budget set contains the old budget set, which means that the consumer would have all the consumption opportunities she had with the old budget set plus some more. Economists sometimes say that an endowment with a higher present value **dominates** one with a lower present value in the sense that the consumer can have larger consumption in *every* period by selling the endowment with the higher present value that she could get by selling the endowment with the lower present value.

Of course, if the present value of one endowment is higher than another, then the future value will be higher as well. However, it turns out that the present value is a more convenient way to measure the purchasing power of an endowment of money over time, and it is the measure to which we will devote the most attention.

10.7 Analyzing Present Value for Several Periods

Let us consider a three-period model. We suppose that we can borrow or lend money at an interest rate r each period and that this interest rate will remain constant over the three periods. Thus the price of consumption in period 2 in terms of period-1 consumption will be $1/(1+r)$, just as before.

What will the price of period-3 consumption be? Well, if I invest \$1 today, it will grow into $(1+r)$ dollars next period; and if I leave this money invested, it will grow into $(1+r)^2$ dollars by the third period. Thus if I start with $1/(1+r)^2$ dollars today, I can turn this into \$1 in period 3. The price of period-3 consumption relative to period-1 consumption is therefore $1/(1+r)^2$. Each extra dollar's worth of consumption in period 3 costs me $1/(1+r)^2$ dollars today. This implies that the budget constraint will have the form

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} = m_1 + \frac{m_2}{1+r} + \frac{m_3}{(1+r)^2}.$$

This is just like the budget constraints we've seen before, where the price of period-t consumption in terms of today's consumption is given by

$$p_t = \frac{1}{(1+r)^{t-1}}.$$

As before, moving to an endowment that has a higher present value at these prices will be preferred by any consumer, since such a change will necessarily shift the budget set farther out.

We have derived this budget constraint under the assumption of constant interest rates, but it is easy to generalize to the case of changing interest rates. Suppose, for example, that the interest earned on savings from period 1 to 2 is r_1, while savings from period 2 to 3 earn r_2. Then \$1 in period 1 will grow to $(1+r_1)(1+r_2)$ dollars in period 3. The present value of \$1 in period 3 is therefore $1/(1+r_1)(1+r_2)$. This implies that the correct form of the budget constraint is

$$c_1 + \frac{c_2}{1+r_1} + \frac{c_3}{(1+r_1)(1+r_2)} = m_1 + \frac{m_2}{1+r_1} + \frac{m_3}{(1+r_1)(1+r_2)}.$$

This expression is not so hard to deal with, but we will typically be content to examine the case of constant interest rates.

Table 10.1 contains some examples of the present value of \$1 T years in the future at different interest rates. The notable fact about this table is how quickly the present value goes down for "reasonable" interest rates. For example, at an interest rate of 10 percent, the value of \$1 20 years from now is only 15 cents.

Table 10.1

The present value of \$1 t years in the future.

Rate	1	2	5	10	15	20	25	30
.05	.95	.91	.78	.61	.48	.37	.30	.23
.10	.91	.83	.62	.39	.24	.15	.09	.06
.15	.87	.76	.50	.25	.12	.06	.03	.02
.20	.83	.69	.40	.16	.06	.03	.01	.00

10.8 Use of Present Value

Let us start by stating an important general principle: *present value is the only correct way to convert a stream of payments into today's dollars.* This principle follows directly from the definition of present value: the present value measures the value of a consumer's endowment of money. As long as the consumer can borrow and lend freely at a constant interest rate, an endowment with higher present value can always generate *more* consumption in every period than an endowment with lower present value. Regardless of your own tastes for consumption in different periods, you should always prefer a stream of money that has a higher present value to one with lower present value—since that always gives you more consumption possibilities in every period.

This argument is illustrated in Figure 10.6. In this figure, (m_1', m_2') is a worse consumption bundle than the consumer's original endowment, (m_1, m_2), since it lies beneath the indifference curve through her endowment. Nevertheless, the consumer would prefer (m_1', m_2') to (m_1, m_2) if she is able to borrow and lend at the interest rate r. This is true because with the endowment (m_1', m_2') she can afford to consume a bundle such as (c_1, c_2), which is unambiguously better than her current consumption bundle.

One very useful application of present value is in valuing the income streams offered by different kinds of investments. If you want to compare two different investments that yield different streams of payments to see which is better, you simply compute the two present values and choose the larger one. The investment with the larger present value always gives you more consumption possibilities.

Sometimes it is necessary to purchase an income stream by making a stream of payments over time. For example, one could purchase an apartment building by borrowing money from a bank and making mortgage payments over a number of years. Suppose that the income stream (M_1, M_2) can be purchased by making a stream of payments (P_1, P_2).

In this case we can evaluate the investment by comparing the present

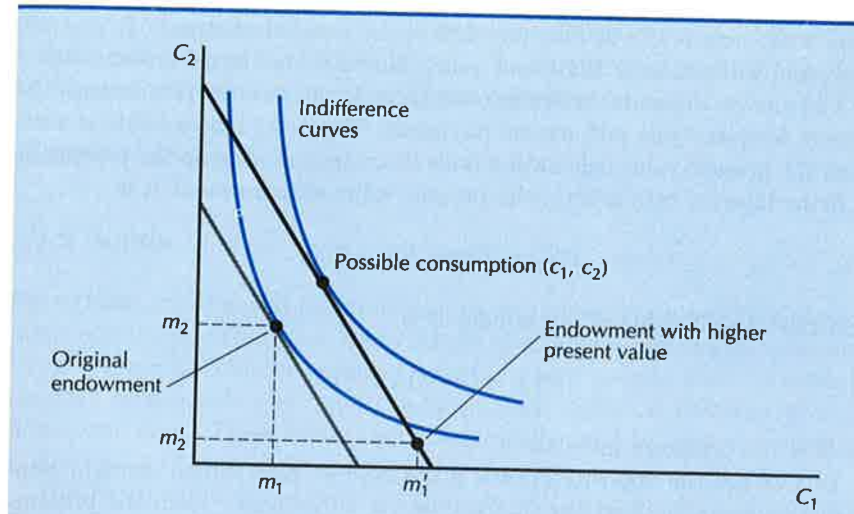


Figure 10.6

Higher present value. An endowment with higher present value gives the consumer more consumption possibilities in each period if she can borrow and lend at the market interest rates.

value of the income stream to the present value of the payment stream. If

$$M_1 + \frac{M_2}{1+r} > P_1 + \frac{P_2}{1+r}, \tag{10.4}$$

the present value of the income stream exceeds the present value of its cost, so this is a good investment—it will increase the present value of our endowment.

An equivalent way to value the investment is to use the idea of **net present value**. In order to calculate this number we calculate at the *net* cash flow in each period and then discount this stream back to the present. In this example, the net cash flow is $(M_1 - P_1, M_2 - P_2)$, and the net present value is

$$NPV = M_1 - P_1 + \frac{M_2 - P_2}{1+r}.$$

Comparing this to equation (10.4) we see that the investment should be purchased if and only if the net present value is positive.

The net present value calculation is very convenient since it allows us to add all of the positive and negative cash flows together in each period and then discount the resulting stream of cash flows.

EXAMPLE: Valuing a Stream of Payments

Suppose that we are considering two investments, A and B. Investment A pays \$100 now and will also pay \$200 next year. Investment B pays \$0 now, and will generate \$310 next year. Which is the better investment?

The answer depends on the interest rate. If the interest rate is zero, the answer is clear—just add up the payments. For if the interest rate is zero, then the present-value calculation boils down to summing up the payments.

If the interest rate is zero, the present value of investment A is

$$PV_A = 100 + 200 = 300,$$

and the present value of investment B is

$$PV_B = 0 + 310 = 310,$$

so B is the preferred investment.

But we get the opposite answer if the interest rate is high enough. Suppose, for example, that the interest rate is 20 percent. Then the present-value calculation becomes

$$PV_A = 100 + \frac{200}{1.20} = 266.67$$
$$PV_B = 0 + \frac{310}{1.20} = 258.33.$$

Now A is the better investment. The fact that A pays back more money earlier means that it will have a higher present value when the interest rate is large enough.

EXAMPLE: The True Cost of a Credit Card

Borrowing money on a credit card is expensive: many companies quote yearly interest charges of 15 to 21 percent. However, because of the way these finance charges are computed, the true interest rate on credit card debt are much higher than this.

Suppose that a credit card owner charges a \$2000 purchase on the first day of the month and that the finance charge is 1.5 percent a month. If the consumer pays the entire balance by the end of the month, he does not have to pay the finance charge. If the consumer pays none of the \$2,000, he has to pay a finance charge of \$2000 × .015 = \$30 at the beginning of the next month.

What happens if the consumer pays \$1,800 towards the \$2000 balance on the last day of the month? In this case, the consumer has borrowed only \$200, so the finance charge should be \$3. However, many credit card companies charge the consumers much more than this. The reason is that many companies base their charges on the "average monthly balance," even if part of that balance is paid by the end of the month. In this example, the average monthly balance would be about \$2000 (30 days of the \$2000 balance and 1 day of the \$200 balance). The finance charge would therefore be slightly less than \$30, even though the consumer has only borrowed \$200. Based on the actual amount of money borrowed, this is an interest rate of 15 percent a month!

10.9 Bonds

Securities are financial instruments that promise certain patterns of payment schedules. There are many kinds of financial instruments because there are many kinds of payment schedules that people want. Financial markets give people the opportunity to trade different patterns of cash flows over time. These cash flows are typically used to finance consumption at some time or other.

The particular kind of security that we will examine here is a **bond**. Bonds are issued by governments and corporations. They are basically a way to borrow money. The borrower—the agent who issues the bond—promises to pay a fixed number of dollars x (the **coupon**) each period until a certain date T (the **maturity date**), at which point the borrower will pay an amount F (the **face value**) to the holder of the bond.

Thus the payment stream of a bond looks like $(x, x, x, \ldots, F)$. If the interest rate is constant, the present discounted value of such a bond is easy to compute. It is given by

$$PV = \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \cdots + \frac{F}{(1+r)^T}.$$

Note that the present value of a bond will decline if the interest rate increases. Why is this? When the interest rate goes up the price now for \$1 delivered in the future goes down. So the future payments of the bond will be worth less now.

There is a large and developed market for bonds. The market value of outstanding bonds will fluctuate as the interest rate fluctuates since the present value of the stream of payments represented by the bond will change.

An interesting special kind of a bond is a bond that makes payments forever. These are called **consols** or **perpetuities**. Suppose that we consider a consol that promises to pay \$$x$ dollars a year forever. To compute the value of this consol we have to compute the infinite sum:

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \cdots.$$

The trick to computing this is to factor out $1/(1+r)$ to get

$$PV = \frac{1}{1+r}\left[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \cdots\right].$$

But the term in the brackets is just x plus the present value! Substituting and solving for PV:

$$PV = \frac{1}{(1+r)}[x + PV]$$
$$= \frac{x}{r}.$$

This wasn't hard to do, but there is an easy way to get the answer right off. How much money, V, would you need at an interest rate r to get x dollars forever? Just write down the equation

$$Vr = x,$$

which says that the interest on V must equal x. But then the value of such an investment is given by

$$V = \frac{x}{r}.$$

Thus it must be that the present value of a consol that promises to pay x dollars forever must be given by x/r.

For a consol it is easy to see directly how increasing the interest rate reduces the value of a bond. Suppose, for example, that a consol is issued when the interest rate is 10 percent. Then if it promises to pay \$10 a year forever, it will be worth \$100 now—since \$100 would generate \$10 a year in interest income.

Now suppose that the interest rate goes up to 20 percent. The value of the consol must fall to \$50, since it only takes \$50 to earn \$10 a year at a 20 percent interest rate.

The formula for the consol can be used to calculate an approximate value of a long-term bond. If the interest rate is 10 percent, for example, the value of \$1 30 years from now is only 6 cents. For the size of interest rates we usually encounter, 30 years might as well be infinity.

EXAMPLE: Installment Loans

Suppose that you borrow \$1000 that you promise to pay back in 12 monthly installments of \$100 each. What rate of interest are you paying?

At first glance it seems that your interest rate is 20 percent: you have borrowed \$1000, and you are paying back \$1200. But this analysis is incorrect. For you haven't really borrowed \$1000 for an entire year. You have borrowed \$1000 for a month, and then you pay back \$100. Then you only have borrowed \$900, and you owe only a month's interest on the \$900. You borrow that for a month and then pay back another \$100. And so on.

The stream of payments that we want to value is

$$(1000, -100, -100, \ldots, -100).$$

We can find the interest rate that makes the present value of this stream equal to zero by using a calculator or a computer. The actual interest rate that you are paying on the installment loan is about 35 percent!

10.10 Taxes

In the United States, interest payments are taxed as ordinary income. This means that you pay the same tax on interest income as on labor income. Suppose that your marginal tax bracket is t, so that each *extra* dollar of income, Δm, increases your tax liability by $t\Delta m$. Then if you invest X dollars in an asset, you'll receive an interest payment of rX. But you'll also have to pay taxes of trX on this income, which will leave you with only $(1-t)rX$ dollars of after-tax income. We call the rate $(1-t)r$ the **after-tax interest rate**.

What if you decide to borrow X dollars, rather than lend them? Then you'll have to make an interest payment of rX. In the United States, some interest payments are tax deductible and some are not. For example, the interest payments for a mortgage are tax deductable, but interest payments on ordinary consumer loans are not. On the other hand, businesses can deduct most kinds of the interest payments that they make.

If a particular interest payment is tax deductible, you can subtract your interest payment from your other income and only pay taxes on what's left. Thus the rX dollars you pay in interest will reduce your tax payments by trX. The total cost of the X dollars you borrowed will be $rX - trX = (1-t)rX$.

Thus the after-tax interest rate is the same whether you are borrowing or lending, for people in the same tax bracket. The tax on saving will reduce the amount of money that people want to save, but the subsidy on borrowing will increase the amount of money that people want to borrow.

EXAMPLE: Scholarships and Savings

Many students in the United States receive some form of financial aid to help defray college costs. The amount of financial aid a student receives depends on many factors, but one important factor is the family's ability to pay for college expenses. Most U.S. colleges and universities use a standard measure of ability to pay calculated by the College Entrance Examination Board (CEEB).

If a student wishes to apply for financial aid, his or her family must fill out a questionnaire describing their financial circumstances. The CEEB uses the information on the income and assets of the parents to construct a measure of "adjusted available income." The fraction of their adjusted available income that parents are expected to contribute varies between 22 and 47 percent, depending on income. In 1985, parents with a total before-tax income of around \$35,000 dollars were expected to contribute about \$7000 toward college expenses.

Each additional dollar of assets that the parents accumulate increases their expected contribution and decreases the amount of financial aid that their child can hope to receive. The formula used by the CEEB effectively imposes a tax on parents who save for their children's college education. Martin Feldstein, President of the National Bureau of Economic Research (NBER) and Professor of Economics at Harvard University, calculated the magnitude of this tax.[3]

Consider the situation of some parents contemplating saving an additional dollar just as their daughter enters college. At a 6 percent rate of interest, the future value of a dollar 4 years from now is $1.26. Since federal and state taxes must be paid on interest income, the dollar yields $1.19 in after-tax income in 4 years. However, since this additional dollar of savings increases the total assets of the parents, the amount of aid received by the daughter goes down during *each* of her four college years. The effect of this "education tax" is to reduce the future value of the dollar to only 87 cents after 4 years. This is equivalent to an income tax of 150 percent!

Feldstein also examined the savings behavior of a sample of middle-class households with pre-college children. He estimates that a household with income of $40,000 a year and two college-age children saves about 50 percent less than they would otherwise due to the combination of federal, state, and "education" taxes that they face.

10.11 Choice of the Interest Rate

In the above discussion, we've talked about "the interest rate." In real life there are many interest rates: there are nominal rates, real rates, before-tax rates, after-tax rates, short-term rates, long-term rates, and so on. Which is the "right" rate to use in doing present-value analysis?

The way to answer this question is to think about the fundamentals. The idea of present discounted value arose because we wanted to be able to convert money at one point in time to an equivalent amount at another point in time. "The interest rate" is the return on an investment that allows us to transfer funds in this way.

If we want to apply this analysis when there are a variety of interest rates available, we need to ask which one has the properties most like the stream of payments we are trying to value. If the stream of payments is not taxed, we should use an after-tax interest rate. If the stream of payments will continue for 30 years, we should use a long-term interest rate. If the stream of payments is risky, we should use the interest rate on an investment with similar risk characteristics. (We'll have more to say later about what this last statement actually means.)

[3] Martin Feldstein, "College Scholarship Rules and Private Savings," NBER Working Paper 4032, March 1992.

The interest rate measures the **opportunity cost** of funds—the value of alternative uses of your money. So every stream of payments should be compared to your best alternative that has similar characteristics in terms of tax treatment, risk, and liquidity.

Summary

1. The budget constraint for intertemporal consumption can be expressed in terms of present value or future value.

2. The comparative statics results derived earlier for general choice problems can be applied to intertemporal consumption as well.

3. The real rate of interest measures the extra consumption that you can get in the future by giving up some consumption today.

4. A consumer who can borrow and lend at a constant interest rate should always prefer an endowment with a higher present value to one with a lower present value.

REVIEW QUESTIONS

1. How much is $1 million to be delivered 20 years in the future worth today if the interest rate is 20 percent?

2. As the interest rate rises, does the intertemporal budget constraint become steeper or flatter?

3. Would the assumption that goods are perfect substitutes be valid in a study of intertemporal food purchases?

4. A consumer, who is initially a lender, remains a lender even after a decline in interest rates. Is this consumer better off or worse off after the change in interest rates? If the consumer becomes a borrower after the change is he better off or worse off?

5. What is the present value of $100 one year from now if the interest rate is 10%? What is the present value if the interest rate is 5%?