We consider the 2-star model

$$P(G) = \frac{1}{Z} e^{-H(G)} \quad \text{where}$$

$$H(G) = -\beta \sum_{i<j} a_{ij} - \gamma \sum_{i} \sum_{\substack{j,l \\ j\neq l \\ j,l \neq i}} \underline{a_{ij} a_{il}}$$

We consider the MF approximation

$$\boxed{a_{ij} a_{il} \simeq a_{ij} \langle a_{il} \rangle + \langle a_{ij} \rangle a_{il} - \langle a_{il} \rangle \langle a_{il} \rangle}$$

If we do the average of both side

$$\langle a_{ij} a_{il} \rangle \simeq \langle a_{ij} \rangle \langle a_{il} \rangle + \langle a_{ij} \rangle \langle a_{il} \rangle$$
$$- \langle a_{il} \rangle \langle a_{il} \rangle = \langle a_{ij} \rangle \langle a_{il} \rangle$$

"the average of the product is equal to the product of the average" – <u>We neglect correlation</u>

The MF Hamiltonian is given by

$$H_{MF} = -\beta \sum_{i<j} a_{ij} - \gamma \sum_{i=1}^{N} \sum_{\substack{j,l \\ j\neq l \\ j,l\neq i}} \Big[a_{ij} \underbrace{\langle a_{jl}\rangle}_{p} + \underbrace{\langle a_{ij}\rangle}_{p} a_{il} - \langle a_{ij}\rangle\langle a_{il}\rangle \Big]$$

We put $\langle a_{ij}\rangle = p \quad \forall (i,j)$

$$H_{MF} = -\beta \sum_{i<j} a_{ij} - \gamma \Big[\sum_{i,j} a_{ij} \overbrace{\sum_{\substack{l\neq i \\ l\neq j}} \underbrace{\langle a_{jl}\rangle}_{=p}}^{(N-2)p} +$$

$$+ \sum_{j,l} a_{jl} \overbrace{\sum_{\substack{i\neq j \\ i\neq l}} \underbrace{\langle a_{il}\rangle}_{=p}}^{(N-2)p} \Big] + C =$$

$$= -\beta \sum_{i<j} a_{ij} - \gamma (N-2) p \Big[\sum_{ij} a_{ij} + \sum_{ij} a_{ij} \Big]$$

$$+ C' = -\beta \sum_{i<j} a_{ij} - 4\gamma (N-2) p \sum_{i<j} a_{ij} + C'$$

For $N \gg 1$ $\quad N-2 \simeq N$

$$H_{MF}(G) \simeq \underbrace{\left(-\beta - 4\gamma N p\right)}\ \underline{\sum_{i<j} a_{ij}} + \underbrace{C'}$$

Random graph

$$H(G) = -\underline{\lambda} \sum_{i<j} a_{ij}$$

Probability of a network G in the MF approx.

$$P(G) = \frac{1}{Z} e^{-H_{MF}(G)}$$

where $Z = \sum_{G} e^{-H_{MF}(G)}$

$$P(G) = \frac{e^{\left[\beta + 4\gamma N p\right] \sum_{i<j} a_{ij} - \cancel{C}}}{\sum_{G} e^{(\beta + 4\gamma N p) \sum_{i<j} a_{ij} - \cancel{C}}}$$

$$H'_{MF} = \left[\beta + 4\gamma N p\right] \sum_{i<j} a_{ij}$$

$$Z'_{MF} = \sum_{G} e^{-H'_{MF}(G)}$$

$$\boxed{P(G) = \frac{e^{-H'_{MF}(G)}}{Z'_{MF}} = \frac{e^{[\beta + 4\gamma N p] \sum_{i<j} a_{ij}}}{Z'_{MF}}}$$

Probability of a link

$$\langle a_{ij} \rangle = p = \frac{e^{[\beta + 4\gamma N p]}}{1 + e^{[\beta + 4\gamma N p]}}$$