

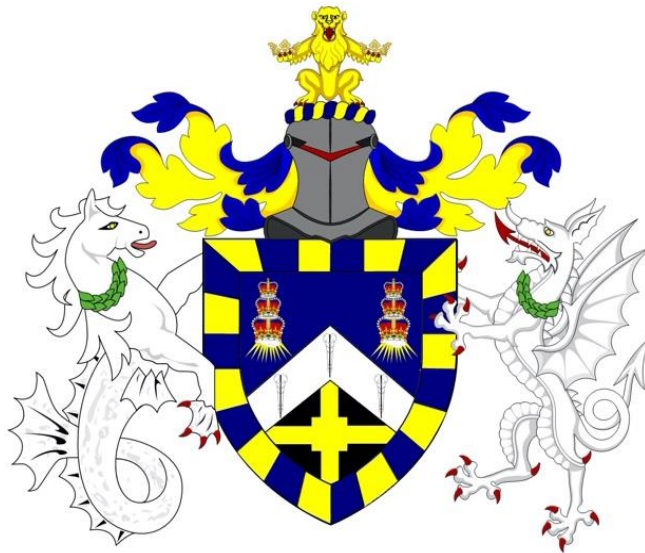
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Signal Processing in Financial Time Series:

An Analysis of Nord Pool Extreme Value

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Declaration of original work

This declaration is made on May 6th, 2021.

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Abstracts

This thesis could be divided into three main sections. The theoretical part of the algorithm, examples of the method application, and Nordic electricity market data analysis.

The first part describes the derivation of the Hilbert transform and the calculation of the instantaneous Amplitude by means of it. The second part demonstrates, by means of enumerative examples, how to analyze instantaneous amplitude images calculated from complex signals. The third section begins with a brief background on the Nordic electricity price market and uses R as well as Matlab to process and analyze the data from the Nordic electricity price market in extreme values.

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1.0 Introduction

Signal processing applications span a vast array of disciplines, including entertainment, communications, space exploration, medicine, and more. Algorithms and hardware are prevalent in a wide range of systems, from highly specialized military systems and industrial applications to consumer electronics [1]. In the long history of development in the signal processing area, signals also have deep roots in time series analysis, which is related to data analysis in economics and statistics [2].

Fourier transform was first established by Joseph Fourier in the early 19th. Using the Fourier transform, we can transform complex signals into simpler forms, or we can also spawn complex signals from simple modes. It can be used not only to analyze natural signals in nature but also has a very wide range of applications in the industry. It can be said that Fourier's theory forms the core of signal processing theory today [3].

In the principles of signal processes, the Fourier transform plays a main role, and most algorithms are extensions of the Fourier transform. An example of this is the Hilbert Transformation based signal analysis.

The Hilbert transform can be traced back to 1905 when Hilbert worked on what became known as the "Riemann Hilbert" problem. The Hilbert transform has been applied to signal processing and the creation of analytic expressions for signals [4]. The signal analysis covered in this thesis, the analysis of data, relies heavily on the Hilbert transform and Euler's formula, which is a typical global signal analysis method. And to introduce the concept of the Hilbert transform, we need to start with the Fourier transform.

Overall, in the following sections, we will introduce the Hilbert algorithm and complex signal theories, with a brief explanation of Fourier Transformation at first, which contributes to the theory parts. In numerical parts, how to generate the instantaneous Amplitude (also called the envelope) of the complex signal will be shown by using data of the Nordic

electricity market from 1999 to 2007. The main aim is to observe the occurrence of extreme values in the Amplitude of signal. Therefore, we will make observations and do some simple statistical analysis of the envelope in the last section.

2.0 Fourier Transformation and Analytic signal

2.1 Fourier Transform

The significance of the Fourier transform in signal processing can first be understood simply as a transformation from the time domain to the frequency domain. Its symmetry and convolution characteristics will give a great deal of help in our understanding of Hilbert transformation.

According to the definition of the Fourier transform, its transform expression, and inverse transform expression is [5]:

$$X(\omega) = F(x(t)) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t} dt \quad (1)$$

$$x(t) = F^{-1}(X(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{i\omega t} d\omega \quad (2)$$

Here we construct a real function $x(t)$ and its Fourier transformation is $X(\omega)$. Know that $x(t) = x^*(t)$, two sides are simultaneously Fourier transformed to give $X(\omega) = X^*(-\omega)$.

The Fourier transform of the real signal contains negative frequencies; if the negative frequencies are removed, this makes:

$$Z(\omega) = 2X(\omega)u(\omega) = X(\omega) + \text{sgn}(\omega)X(\omega) \quad (3)$$

Where $u(\omega)$ is a step function, $\text{sgn}(\omega)$ is a signum function. Then by doing inverse Fourier transformation to $Z(\omega)$:

$$\begin{aligned}
z(t) &= F^{-1}(Z(\omega)) \\
&= F^{-1}(X(\omega) + \text{sgn}(\omega)X(\omega)) \\
&= x(t) + F^{-1}(\text{sgn}(\omega)) * F^{-1}(X(\omega)) \\
&= x(t) + i \left(\frac{1}{\pi t} * x(t) \right) \\
&= x(t) + i \tilde{x}(t)
\end{aligned} \tag{4}$$

This leads us to the analytic signal and the Hilbert transformation. $\tilde{x}(t)$ is the *Hilbert transformation* of $x(t)$, and the *Analytic signal* of $x(t)$ is $z(t)$.

2.2 Hilbert Transformation

Based on the extensive use of Euler's formula (in 1946 by Gabor), we can find a prototype to express an analytic signal. A complex signal (also called analytic signal) contains two parts: the real parts and the imaginary parts. The imaginary part is the real part's Hilbert transformation. The specific format is as follows:

$$Y(t) = u(t) + i v(t) \tag{5}$$

Where $v(t)$ is the Hilbert Transformation of $u(t)$, and $u(t)$ is orthogonal with $v(t)$. Such an operation actually achieves a change in the signal from a bilateral to a unilateral spectrum.

By the definition of the Hilbert transform, for a function $u(t)$, its Hilbert transform is the convolution of the function itself and $1/\pi t$ [6].

$$\tilde{u}(t) = H(u(t)) = \int_{-\infty}^{+\infty} \frac{u(\tau)}{\pi(t - \tau)} d\tau \tag{6}$$

Known from the definition of convolution, function (6) can be expressed

as:

$$\tilde{u}(t) = u(t) * (\pi t)^{-1} \quad (7)$$

To better understand the mathematical significance of the Hilbert transform, the properties of the Fourier transformation and convolution are presented below.

$$F\left(H(u(t))\right) = F\left(\frac{1}{\pi t} * u(t)\right) = \int_{-\infty}^{\infty} \frac{1}{\pi t} e^{i\omega t} dt \cdot F(u(t)) \quad (8)$$

Derived from function (8), we have the frequency response here:

$$H(\omega) = \int_{-\infty}^{\infty} \frac{1}{\pi t} e^{i\omega t} dt = -i \operatorname{sgn}(\omega) \quad (9)$$

Where $\operatorname{sgn}(f)$ is a signum function that:

$$\operatorname{sgn}(f) = \begin{cases} 1 & f > 0 \\ -1 & f < 0 \end{cases} \quad (10)$$

From the above expression, we conclude that the Hilbert transform shifts all positive frequency components by $-\pi/2$, and all negative frequency components by $\pi/2$, which is equivalent to a quadrature filter, with the amplitude keeping unchanged.

Similarly, from the above properties we can say that the Hilbert transformation of a cosine, is minus sine.

$$H(\cos(x)) = -\sin(x) \quad (11)$$

2.3 Instantaneous Amplitude

In fact, analytic signals can be better understood in the three-dimensional space, which is consisted by the complex plane and time, with x-axis representing real parts, y-axis representing time and z-axis for imaginary parts.

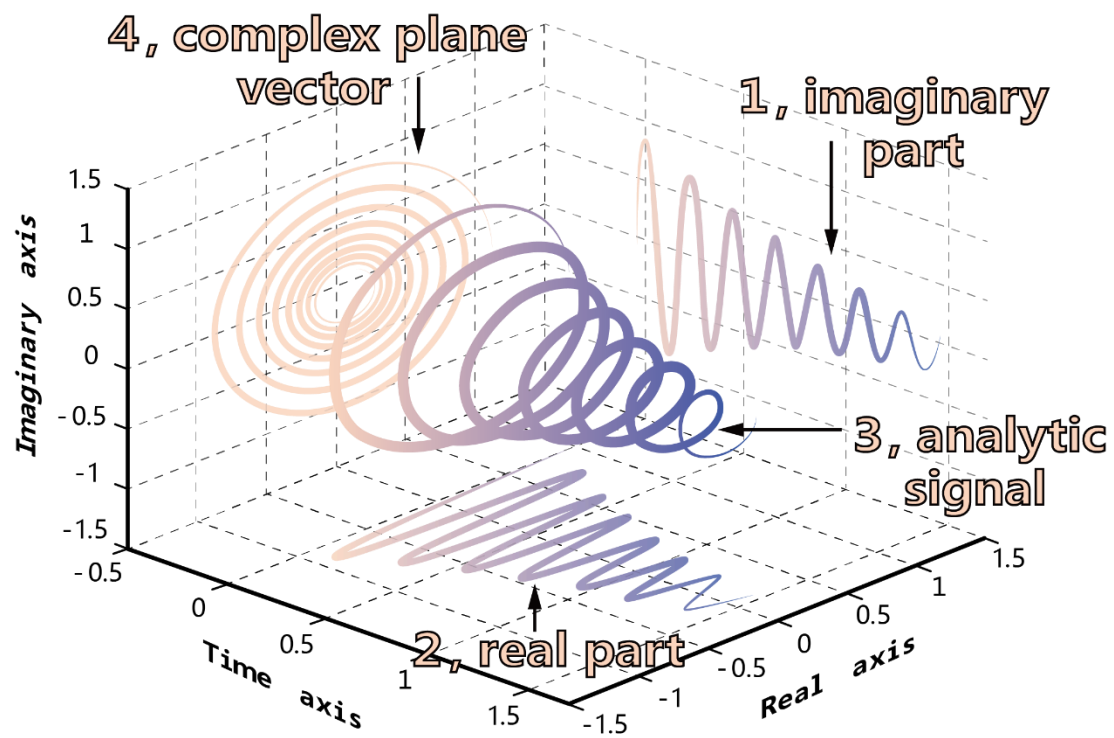


Figure 1: There-dimensional space

Going to the complex plane alone here, the rotation vector in the figure is the envelope of the resolved signal. In other words, instead of studying the signal in the frequency domain, one can look at the rotation vector in the time domain with instantaneous phase and instantaneous amplitude [7].

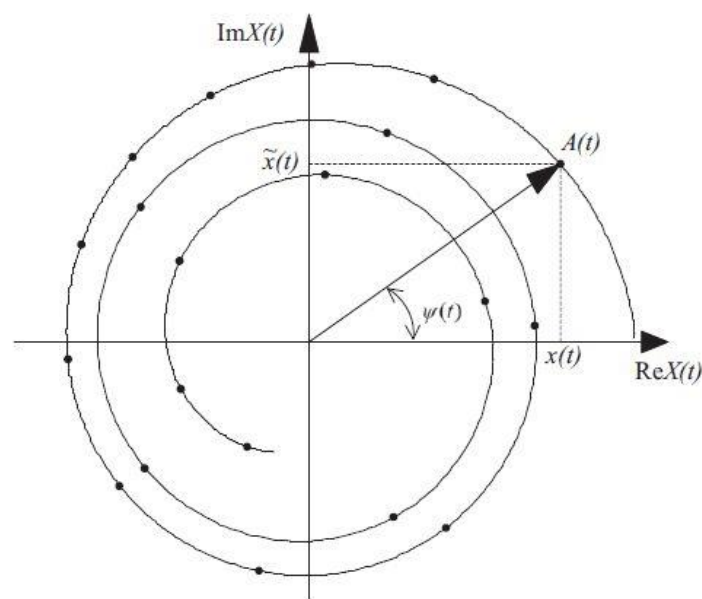


Figure 2: Complex Plane

It is easy to obtain the function of $A(t)$ and $\varphi(t)$ from Figure 2. To do this, we treat $A(t)$, $x(t)$ and $\tilde{x}(t)$ as three segments: $\varphi(t)$ is an angle of the closed right triangle, in which $x(t)$ and $\tilde{x}(t)$ are two right-angled sides. By a simple calculation of Trigonometric operations, we can obtain the hypotenuse $A(t)$ and angle $\varphi(t)$ of this right triangle. Hence, we have:

$$A(t) = \pm|X(t)| = \pm\sqrt{x^2(t) + \tilde{x}^2(t)} \quad (12)$$

$$\varphi(t) = \arctan \frac{\tilde{x}(t)}{x(t)} \quad (13)$$

We could then interpret these more mathematically by using a simple harmonic example:

$$x(t) = a \cos(\omega t) \quad (14)$$

Where $a = A(t)$, represents the amplitude; $\varphi(t) = \omega t$ represents the phase.

At first, we do HT to (14), and we have created an analytic signal defined before (we only consider positive frequency there):

$$X(t) = x(t) + i \tilde{x}(t) = A(t) \cos(\omega t) + i A(t) \sin(\omega t) \quad (15)$$

Using Euler's Formula, we have:

$$e^{ix} = \cos(x) + i \sin(x) \quad (16)$$

$$X(t) = A(t)e^{i\omega t} \quad (17)$$

Combining equation 15 with equation 17, it is easy to obtain the instantaneous amplitude (also called the envelope):

$$A(t) = \pm|X(t)| = \pm\sqrt{x^2(t) + \tilde{x}^2(t)} \quad (18)$$

Same as the results we interpreted from the complex plane.

3.0 Example of Analytic Signal

3.1 Numerical Example 1

In practice, the Hilbert command works by taking the Fourier transform of the original signal. Then we take the positive frequency part, and finally perform the inverse Fourier transformation again to construct an analytic signal (as mentioned in section 2.2).

As a testbed, to see how the algorithm goes on in an ideal case, given an artificial time series than before:

$$x(t) = a \cos(\omega_1 t) + b \cos(\omega_2 t) \quad (20)$$

Say with, $a = 1, b = 1.1, \omega_1 = 1, \omega_2 = 1.1$, and we set the range of t as $[0,100]$.

To analysis the real signal more intuitively, we transformed it into a complex signal using Hilbert's algorithm by Matlab and extracted its components.

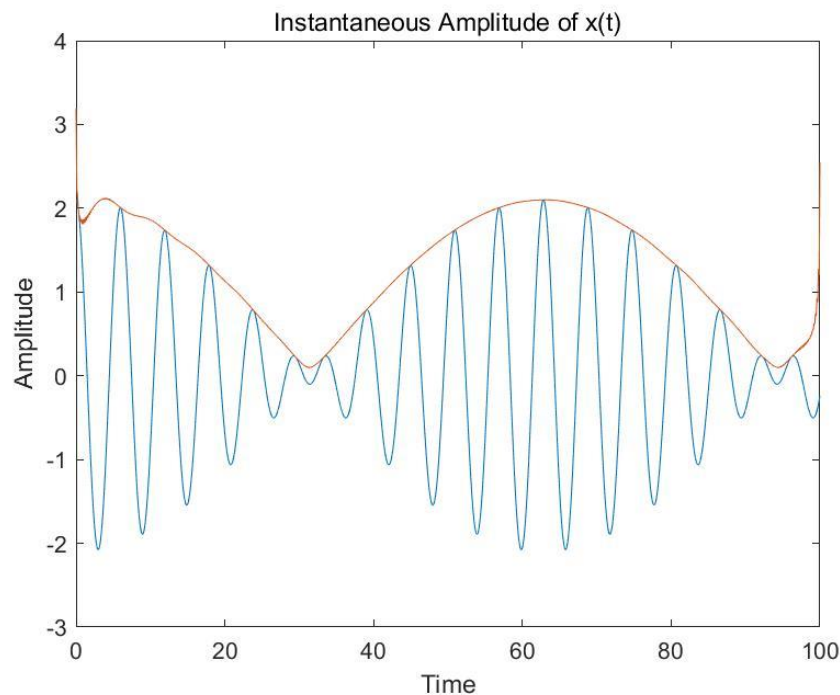


Figure 3: Amplitude of real signal $x(t)$

As shown on Figure 3, the result given by the algorithm is consistent with the one we derived in Equation 18. A slight disturbance can be observed at the ends of the curve, which is triggered by end effect. We would not discuss the specific theory in this thesis, but we will analyze its impact in the following sections. In addition, we can conclude that from this two-cosine model, the instantaneous amplitude is a smooth curve that fits the original signal well.

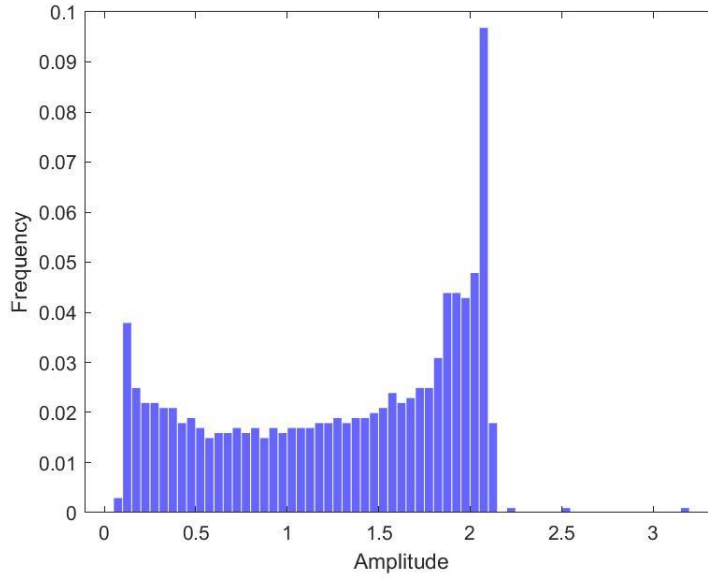


Figure 4: Histogram of Amplitude

To facilitate the analysis, we construct histogram plots of the instantaneous amplitude, and it can be seen from the plots that the peaks are partially located around 2.2, 0.1 for the lowest points, similar to what we observe in the envelope plots (also ignore the end effect here). Thus, the range of envelope values from about 0.2 to 2 encompasses most values, which are the daily fluctuations of this artificial signal.

3.2 Numerical Example 2

The next step is to build a new model based on the formula in Example 1 (the two-cosine example) with a slight variation and see what happens.

$$x(t) = 3.3 + 1.3(1.4t + 1.2) + 1.2 \sin(1.8t + 2.2) + \cos(0.8t - 1.6) \quad (21)$$

This time, we begin with plotting the diagram of complex signal itself:

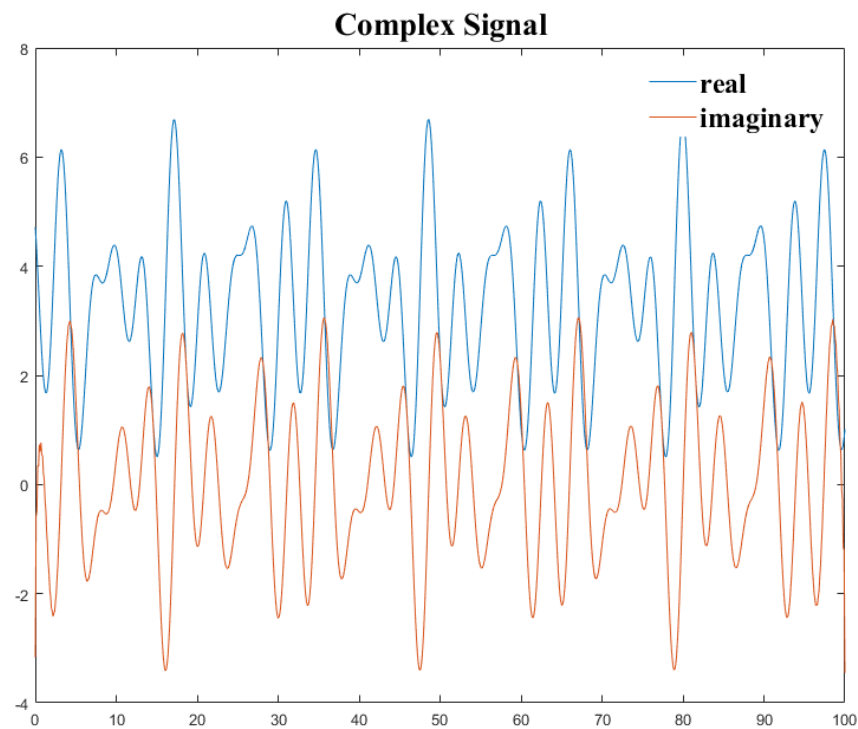


Figure 5: Real part and imaginary part of Hilbert function

From two results in Figure 5, it can be observed: the regularity of the complex signals remains relatively strong, and the changes in the curves of the real and imaginary parts are to some extent correlated.

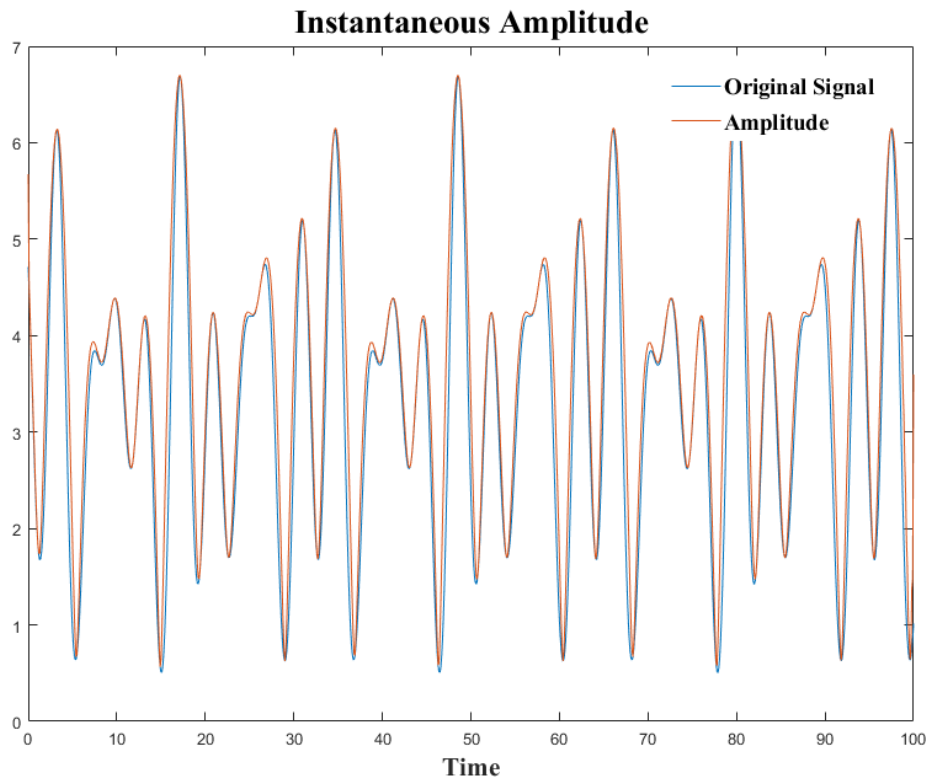


Figure 6: Instantaneous Amplitude of complex signal

We could say that the way the envelope curve fits the original signal in Figure 6 differs from Figure 3. This difference is that the newly constructed signal, although still highly periodic, is closer to the natural fluctuations of the signal, rather than the simpler signal in example 1. In reality, signals are often far more complex than artificial signals, due to internal and external factors.

4.0 Nord Pool Data Analysis

4.1 Background of Nord Pool Market

The Norwegian electricity market was initially established in 2003, when the leaders realized the uneconomical and low price of electricity caused by overcapacity. In order to ensure the coordination of regional electricity and the efficient use of electricity, the position of the transmission operator becomes very important. Hence harmonization of regional differences became a key to the success of the Nordic market. Over the years, it has

grown from a Norwegian electricity market to the Nordic market in 2000. After this, it not only became a service provider for other European markets, but also expanded its operations to other countries [8].

The Nordic Electricity Market has two main functional modules. The physical market, also called Elspot, organizes the physical delivery of electricity, where the market price and amount are determined by the previous day's supply/demand equilibrium. Therefore, although Elspot actually acts as a spot market, its deal processing could be seen as a one-day *futures contract*. As the opposite of Elspot, Elption and Eltermin are financial markets. They offer real financial contracts like futures and forwards to consumers aiming for managing risks [9]. Due to the fact that over 80% of the system was operated by ELspot, in this thesis, our data set was abstracted from the spot market.

As we mentioned before, the price of electricity can occasionally take on extreme values due to uncertainty in demand, accidents at transmission facilities, and one of the underlying triggers are the properties of non-storability of electricity. Extremes and disturbances can occur for a variety of reasons, and there are many possible reasons from a supply and demand perspective—for example, seasonal variations in electricity demand, with higher demand in winter. There could also be price fluctuations on the supply side, such as the cost of emissions from coal generation [10]. Unfortunately, this article will not discuss the reasons why extreme price fluctuations occur. Analyzing these 'accidents' or 'oscillations' will be the main melody of this project.

4.2 Analysis of data

4.2.1 Dealing with dataset: Logarithmic return

At first, we need to attach our dataset, which will be used in the project. The data set contains the electricity price (in EUR/MWh) from Jan 1st 1999 to Jan 26th 2007 at a sampling rate per hour. This process was done in R, with an output figure.

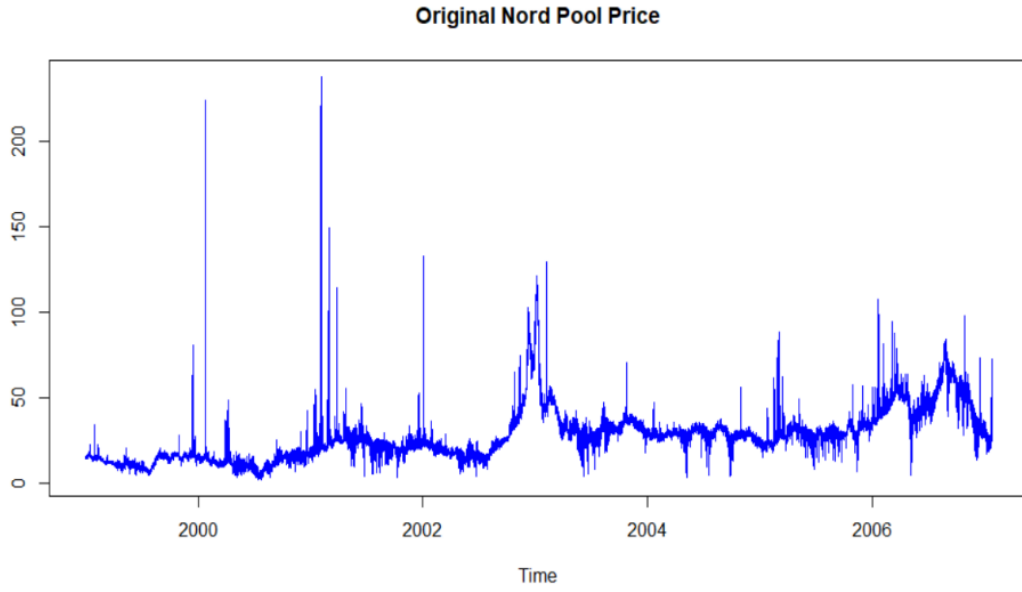


Figure 7: Nord Pool market price

Data plot was shown in Figure 7. The original data can be observed as a time series with an upward trend by time and shows non-stationary characteristics. Therefore, statistical approaches will fail with ill-defined mean values. Aiming to make this time series statistically meaningful for further analysis, we apply Logarithm return here to deal with the original time series. To compute daily logarithm returns, use the daily average price as original data.

The log return is deduced from the equation of simple return. See simple return's function as below:

$$R_s = \frac{P_t}{P_{t-1}} - 1 \quad (22)$$

And if logarithm is applied here, we could get the definition of log return:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (23)$$

Where P_t refers to the value of price at a specific time spot 't'.

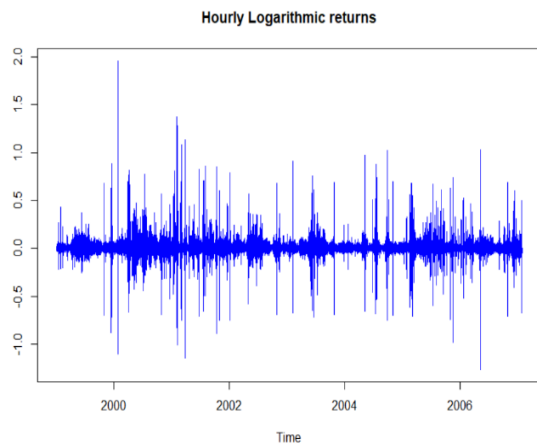


Figure 8: Hourly Log returns

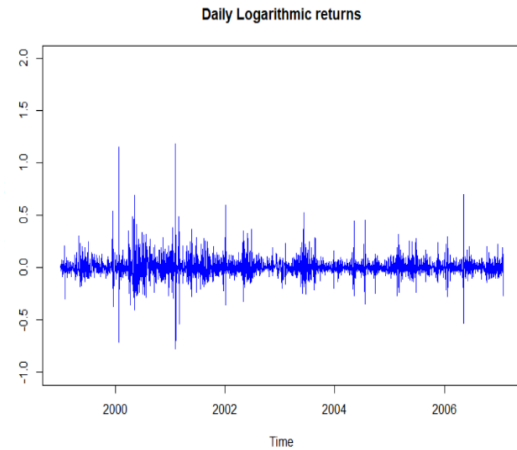


Figure 9: Daily Log returns

Results applied with logarithm return are shown in Figure 8 and 9. Obviously, data looks more stationary here. We see a kind of daily oscillation in Figure 9, and these are going to be analyzed in the following part of this project. Then we compute the mean value and subtract the mean value from the time series (so that the result has a mean zero).

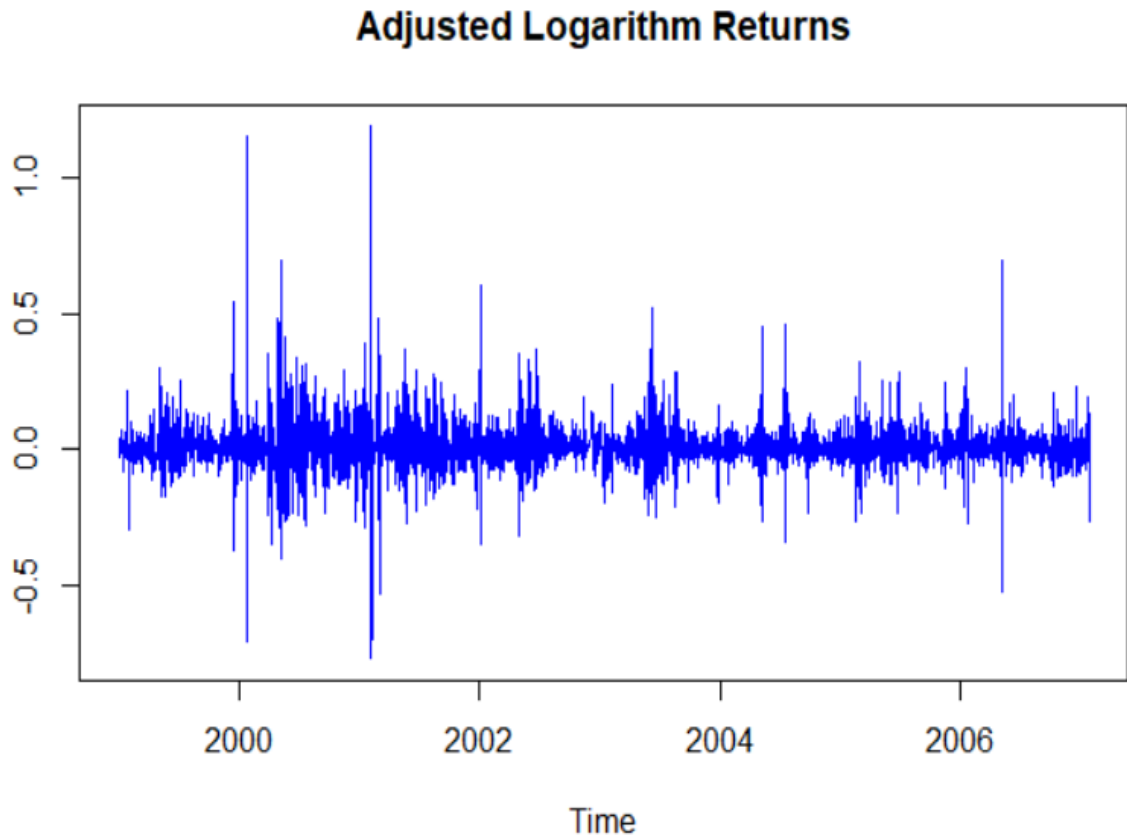


Figure 10: Adjusted Daily Logarithm returns

This way, the periodicity of the metadata has been adjusted, and the mean and variance are more sensible. As a result, we made raw data more suitable for use with statistical algorithms and analysis methods. The adjusted dataset (Figure 10, computed by R) will be the original signal for our subsequent analysis process.

4.2.2 Amplitude analysis

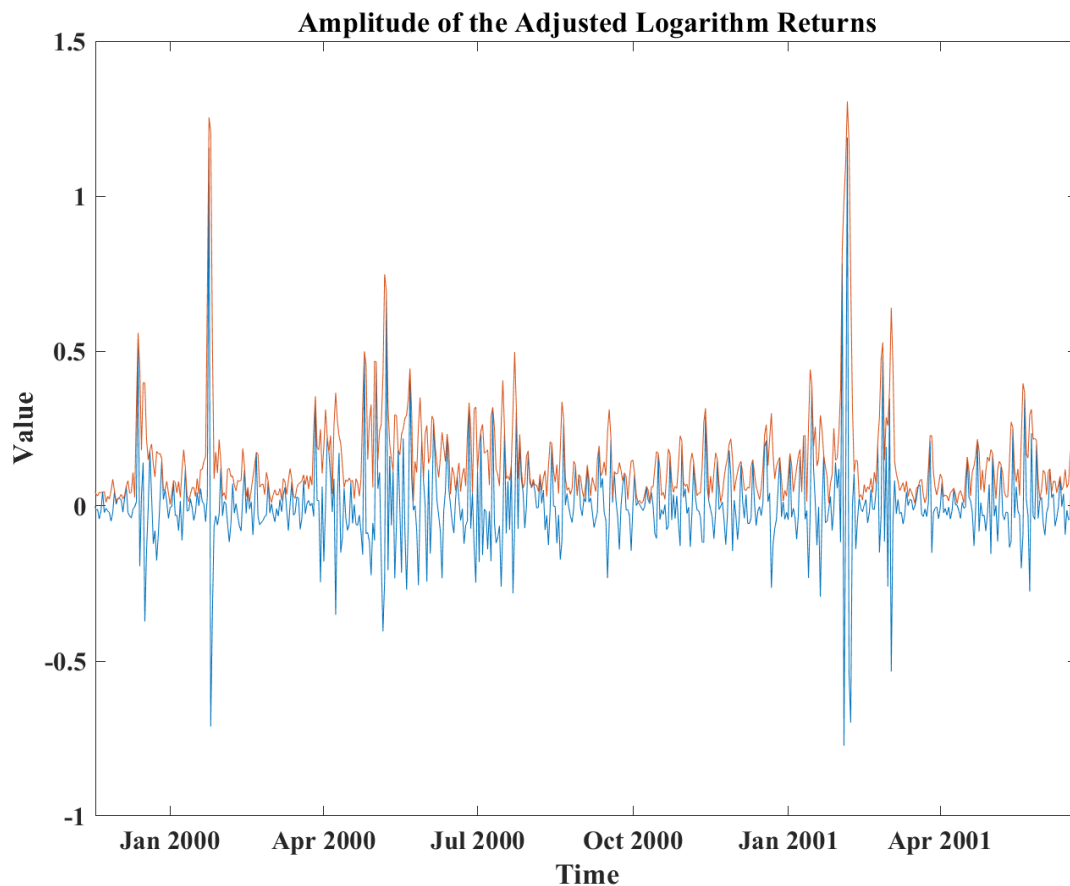


Figure 11: Instantaneous Amplitude of signal

By using Matlab command, Figure 11 was generated through combining the original signal and computed instantaneous amplitude curve. As can be seen by looking at some of the intervals, the envelope overlaps perfectly with the original signal and is able to match the trend of the original signal very well (Here, we only focus on the positive envelope). To observe changes in the envelope more precisely, we need to abstract a minor scale interval from the whole timeline.

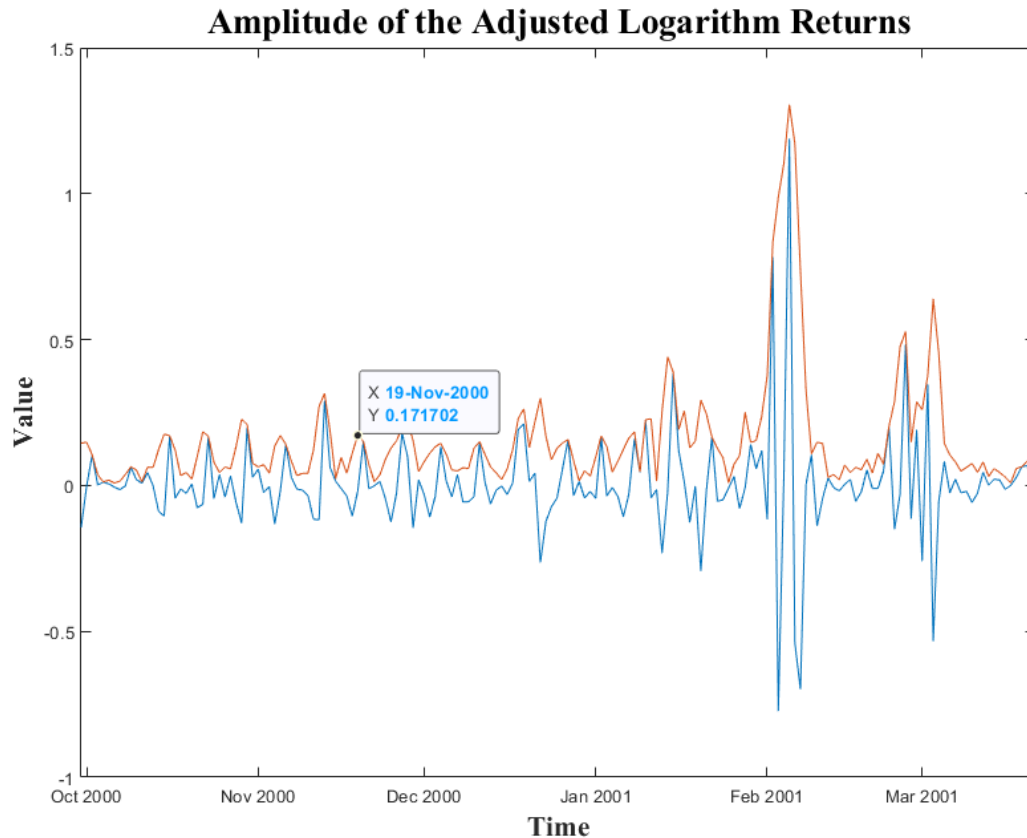


Figure 12: Instantaneous Amplitude in a year

From Figure 12, we can observe the fluctuations around the value on Nov 9th 2000. It is easy to conclude that the peak of the envelope seems to occur earlier than the original signal. And the curve in the chart up to December confirms our suspicions, with the peaks in October and November showing a similar nature. And this pattern is sometimes broken, for example, when the first extremes appear in early February when the envelope reaches its peak at almost the same time as the original signal. To test our guesses and find more patterns, more extreme values need to be observed.

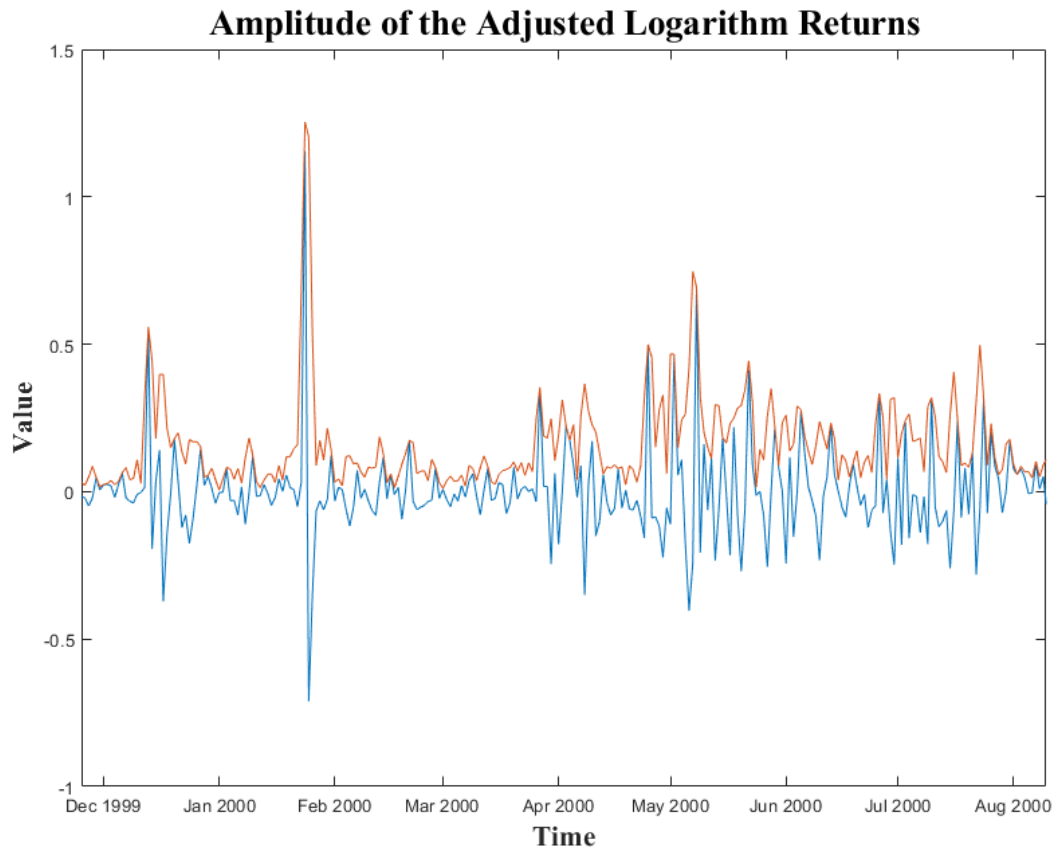


Figure 13: Instantaneous Amplitude from Dec 1999 to Aug 2000

Taking the interval from May to August 2000, a period of frequent oscillations on the timeline, the same conclusion can be drawn as before: most of the time, the envelope gives information ahead of the original curve. However, the two extreme values in December 1999 and late January 2000 confirm the failure of the envelope to predict this information in advance when extreme values (deviate significantly from the daily data) occur. Next, we can apply the histogram used in section 3 to try to find more regulations.

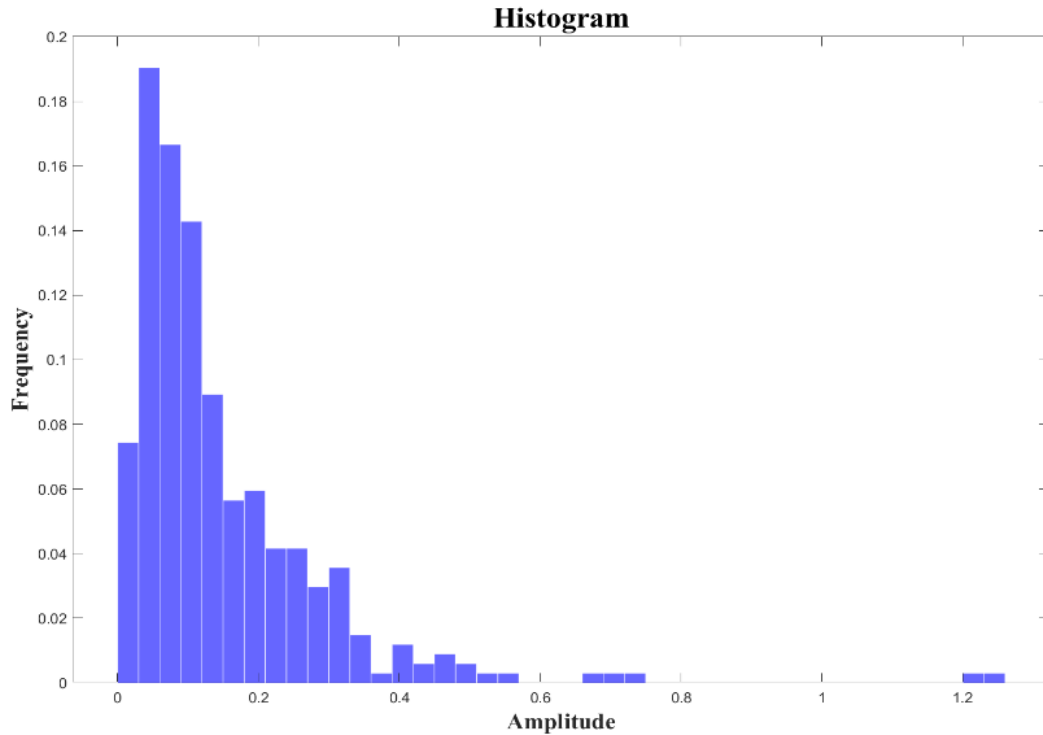


Figure 14: Histogram of Amplitude (Nov 15th 1999 to Nov 15th 2000)

The frequency histogram shows that for nearly half of the days of the year, the value of amplitude is below 0.1, proving that values below this are the market's daily. In contrast, values around 0.15 to 0.35 occur much less frequently and represent more minor fluctuations in the market. We can find lots of corresponding data from April to August 2000 in Figure 11. In addition, the 'shortest' squares in the histogram, which are the values that occur least frequently, such as those with horizontal coordinate values around 1.2 and 0.7, represent extreme values. They can also correspond to the two times of late January and early May 2000 respectively. Again, we need to make observations of the later data, choosing 2005-2006 for the envelope analysis.

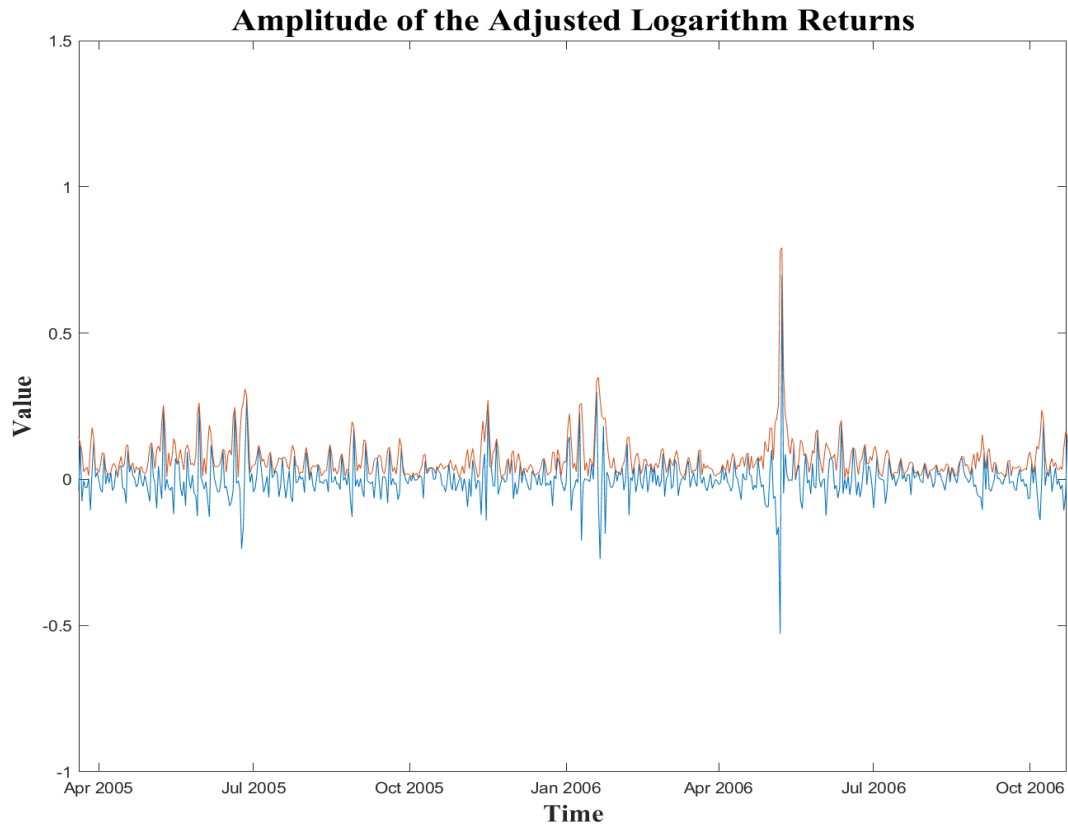


Figure 15: Instantaneous Amplitude (Apr 2005 to Oct 2006)

It can be intuitively perceived that the data for the year 2005 to 2006 is more stable and less volatile than the earlier data. This can also be seen from the raw data curves in Figure 7. As we mentioned in 4.1, there could be a number of reasons for this. And we can find that the extreme data appearing at the end of April 2006, although the envelope curve rises somewhat earlier, its peak still appears simultaneously with that of the original data, which is the same phenomenon as before.

But this year, taking smaller time periods, one can find curves that do not fit the pattern of instantaneous amplitudes predicting fluctuations in advance.

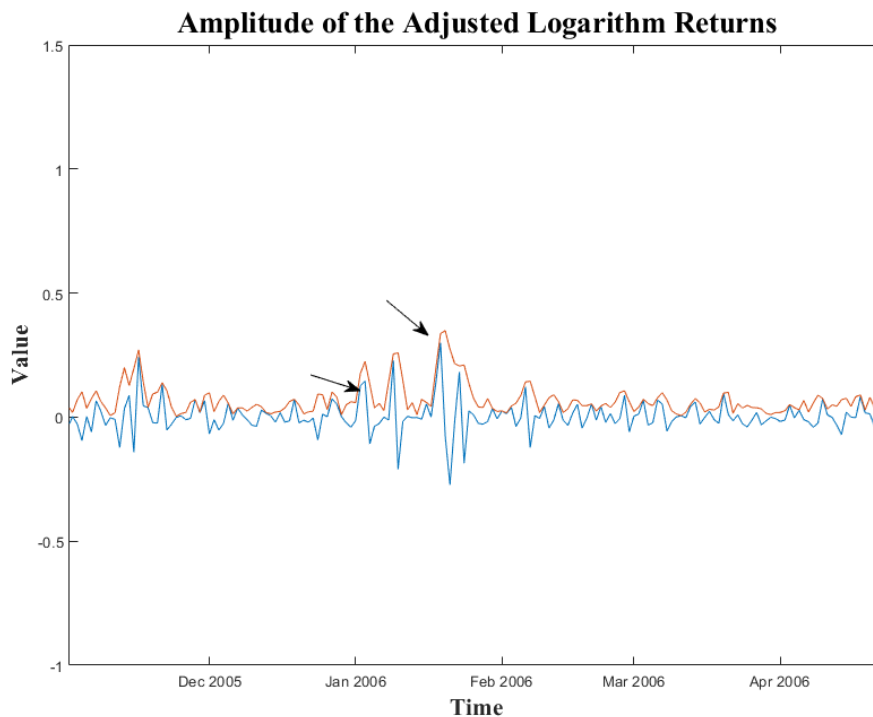


Figure 16: Instantaneous Amplitude (Nov 2005 to May 2006)

This is reflected in the section labeled in Fig. 16. This may indicate that when the signal is very smooth, relative fluctuations can still invalidate the envelope's prediction mechanism, even though the absolute magnitude of some fluctuations is not significant.

Then we continue to plot the histogram to see how Amplitude in Figure 15 performs.

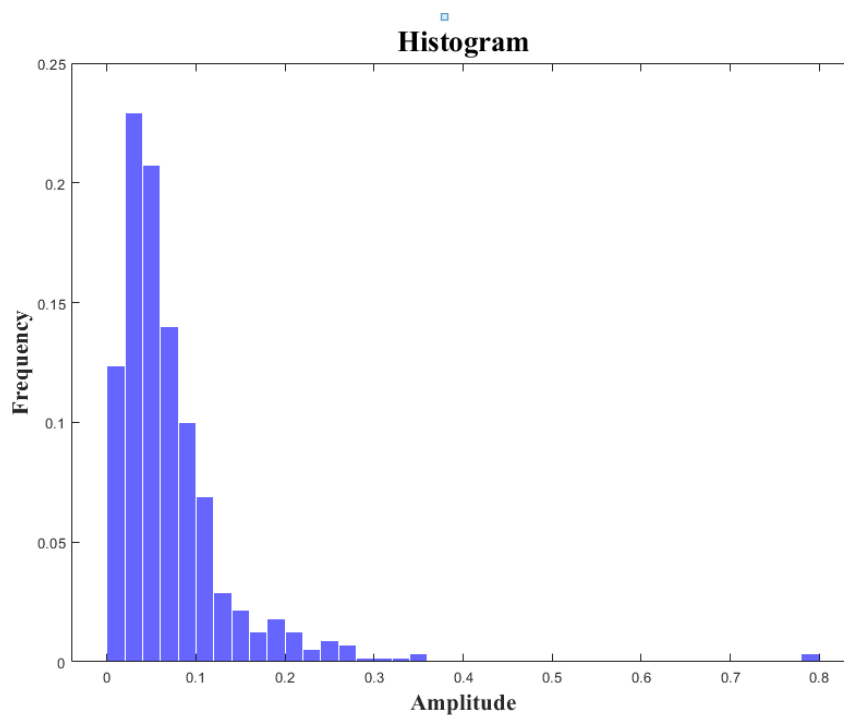


Figure 17: Histogram of Amplitude (Apr 1st 2005 to Oct 1st 2000)

The histogram shows us very clearly that values greater than 0.12 do not even sum to more than 0.1. Compared with Figure 14, this indicates that prices were much flatter in 2005-2006 compared to 2000. Extreme values around 0.8 can be found corresponding to the time point of May 8th 2006.

Next, we take two edges of the image to see if the breakpoint effect of the Hilbert transform affects our judgement of the extreme values.

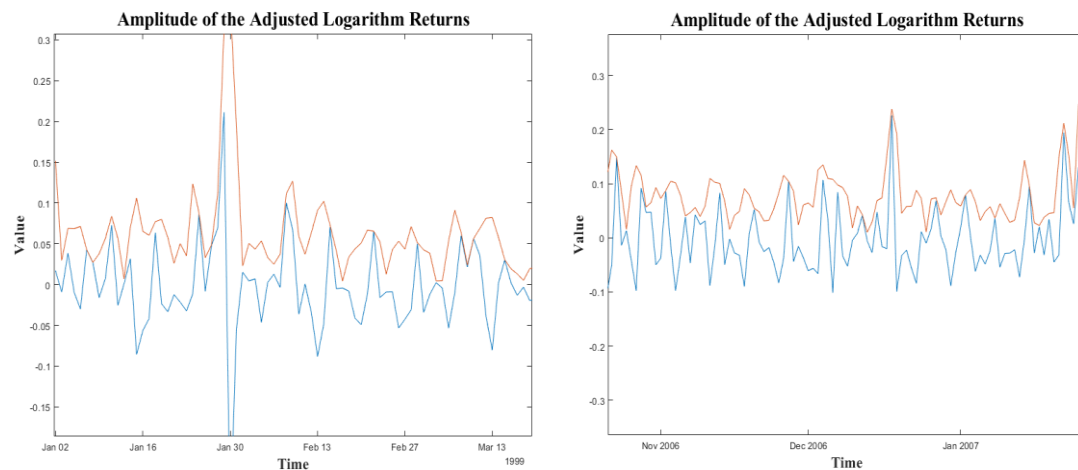


Figure 18: The left and right sides of Amplitude

It is not difficult to find that the end effects on both sides are not significant, and the envelope still reflects the trend of the original signal. This makes a direct global analysis possible. Therefore, we could go back from the miner scale to see the whole signal. Notice that data points which instantaneous amplitude values at 0.4 and above are the extreme points that we will be looking at. The parts marked by red circles are not easily distinguishable on the overall map, so we use a smaller scale.

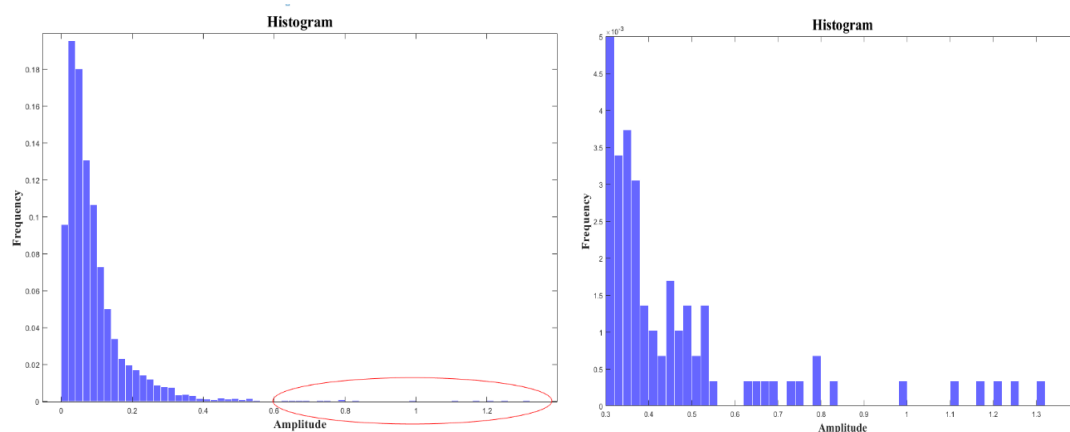


Figure 19: Histogram of whole signal

It can be observed that the histogram in Figure 19 has a similar distribution to histograms plotted before (Figure 14 and Figure 17). And it also well indicates where extreme values are.

5.0 Conclusion

This thesis is intended to explore whether signal processing is a suitable method for predicting and observing the occurrence of extreme values. So we introduced two artificial signals in section 3 as examples to see how the instantaneous amplitudes behave. The envelopes were found to fit the original signals relatively well, and for the first time, a histogram was applied.

Using the principles described in section 1, we applied the algorithm to data from the Nord Pool market using `r` and Matlab. First, we use log return to deal with original increasing data, in order to construct a stationary signal. Then the analytic signal was derived by Hilbert transform, and the image of instantaneous amplitude was calculated comparing with the original signal, supplemented by histogram analysis.

In the process, some patterns were discovered. Most of the time, the envelope is an excellent 'omen', since the envelope curve usually produces a change in trend earlier than the original signal. However, this mechanism can sometimes fail. For example, as we mentioned in the explanation (of Figures 13 and 16), when very extreme values appear, or when the curve is very smooth and regular, and fluctuations arise suddenly, its reaction may not be timely enough.

In addition, the ends effect we found in the artificial signal example did not have a serious impact on the overall data analysis. Still, it could potentially bring inaccurate predictions when the amount of data is relatively small.

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