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Analytic Signal Processing for Financial Time Series

A study on extreme events in spot market data using the amplitude and phase of time series

Thuy Trang Nguyen, ID 160592643

Supervisor: Dr Wolfram Just

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School of Mathematical Sciences
Queen Mary University of London

Declaration of original work

This declaration is made on September 11, 2020.

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Abstract

This thesis covers the concept of signals and signal processing, and how they are used to extract information about the data, specifically for electricity market prices. In particular, we will be analysing if the amplitude and phase has any relation with the appearance of extreme spikes in the time series. In this paper, we will be using the Nord Pool market prices as a financial time series, which contains houtly electricity prices from the 1st January 1999 until the 26th January 2007.

Some mathematical techniques used in signal processing will be covered, where we will study concepts of analytic signal, Hilbert transform and Fourier transform, and how they associate with deriving amplitude and phase from the signal. Those notions will be then applied on the Nord Pool financial time series using R commands.

We will discover how the amplitude varies for the wave signal, and if the phase provides any information regarding extreme events appearing in Nord Pool electricity prices.

Contents

Introduction

1.1 Signal and Signal Processing

Before we consider analysing extreme events of Nord Pool market data, we should first go through some fundamental terms and notions. Specifically, what does signal actually mean for us? *"Ages ago, signal referred to some physical manifestation of information that changed with time and/or space."* [2]. Nevertheless, as time goes on, the signal has a wider meaning and the definition depends on the field we are taking into account. The list of examples of signal processing can be endless, including audio, video, communication, musical data, chemical, medical and so on. In our discussion, however, we are considering the definition of signal in signal processing arena. And here, comes another question of what is signal processing?

Although it might not be that evident, signal processing appears everywhere in our daily life, like in CD players, cameras, and even computers. Signal processing is an area of electrical engineering that deals with analysing, modifying and synthesising of signals like sounds and images. It appears in many diverse application fields like audio signal processing, image processing, control systems, and even financial signal processing, which we will cover to analyse the financial data of Nord Pool in order to estimate the movement of electricity prices.

Coming back to our first question, in signal processing, a signal is *"a function that coveys information about the behaviour of a system or attributes of some phenomenon."* [5]. For our discussion we are referring to a signal as a function of some independent variable, which is in our case the time. And signal processing is, in other words, an action that performs on an input signal to produce an output signal.

Signal processing is the procedure of the features of some signal, in order to get a signal with more helpful for us properties. For that purpose, characteristics of a signal like amplitude and phase may be modified. Signal processing is, therefore, applied for signal generation and modification, as well as extracting information from input signals. The field of signal processing advantages form development of areas such as electrical engineering, machine learning, applied mathematics and statistics.

Signals in signal processing are generally studied with respect to time, frequency, or both simultaneously, and the transform called Fourier transform, studied further later on, can be applied on signals to change the time-domain signals to frequency-domain, or the other way around depending on the requirements of our analysis. Signals can be classified as analog signals and digital signals. For analog signals the variables interpreting the signal are continuous in terms or the time and amplitude. For digital signals, on the other hand, the signals are discrete in time and amplitude. Typically, signals are in analog form, which then becomes digital by the sampling process.

1.2 Financial Signal Processing

Like mentioned, the area we will be focusing on here is financial signal processing, where we will be taking time varying signals from Nord Pool market, which has data of electricity prices throughout 7 years, to extract the information the changes in prices of the market.

Formerly, the finance section had very little overlap with the information-processing and signal section. But due to a graduate raise in the interaction between the fields, as finance started to cover more computing, signal-processing technologies together with mathematics applications, the signal-processing together with computer science, mathematics and statistics play now a meaningful role in applications in financial risk and portfolio management. With the emergence and growth of signal processing and its mathematical approaches, it became therefore a highly significant use in the financial industry, where independent time variables are collected as data. Those variables are mainly described as discrete-time signals.

Having sufficient historical data collected, predictions of the price movements can be made over some time using financial signal processing and its algorithms. Although predictions may not be certain, the outcome might guide the investor in their financial strategies and investments in aim to try to beat the market.

Understanding the Electricity Market

The use of electricity has extremely changed our life and has become our daily essential necessity. We probably cannot imagine doing our job and tasks without it. But how exactly do we get the electricity to our home or businesses? In other words, how do electricity market works? Electricity is rather a different type of commodity than other tangible commodities. The key features that make electricity distinct from natural gas, crude oil or other commodities are that it is interchangeable, 1MWh electricity from coal or natural gas produce the same amount of energy, the energy has to be produced and used concurrently as the storage for electricity are hardly achievable at the present time, and also the power grid has to be balanced to avoid black-out. The price of the electricity in the market is set by supply and demand of the commodity. Those are dependent of the features such as seasonal maintenance, wind speed, price of fuel, temperature or time of a day. Electricity is generally not storable, therefore it is available only on demand.

The electricity industry involves four main sectors: generation, transmission, distribution and end user. Electricity is generated at power stations by using natural gas, coal, wind, hydro or other energy sources. There are number of types of power stations depending on each company, which then make judgements depending on government policy, environment and market signals. The energy then travels across country over high voltage transition networks and distribution networks carries the electricity into our homes, offices or factories from the transmission lines at lower voltages. At the end user sector retailers fix retail rates for consumers.

In electricity market the electricity is enabled to be purchased, sold or traded through the bids and offers. The prices are determined using the principles of supply and demand. In the wholesale market the electricity is purchased or sold before it reaches the end users. It occurs while generators present their commodity to retailers, who then introduce the re-priced electricity to the market. Participants of the wholesale market does not necessarily own the sources and they might not necessarily serve the end consumer directly. The power market in general consists of day-ahead and real-time market pricing. The day-ahead market offers participants commitment to purchase or sell electricity a day in advance operating day. In the real-time market, on the other hand, participants buy or sell the electricity on the day of the operation day.

In the market, the electricity prices are more volatile than the prices of different commodities as they are not storable and have limited transmission. As a result of substantial price and volume risks that can be spotted in the market, participants perceive the importance of risk management. The participants can meet trading risks in the electricity market due to extreme price volatility at the period of peak supply and demand shortage. Price risk appear on the account of hard to predict "price spikes". Many risk management techniques are used in the financial industry to prevent participants from facing the risks in the market. Those include hedging agreements, portfolio optimisation and asset valuation.

Nord Pool

Nord Pool is the Europe's leading power market, which is owned by Euronext and Nordic Transmission System Operators and Litgrid. The organisation provides to its clients day-ahead and intraday market services, as well as consultancy services. They have currently 360 companies across Europe that are working with them.

Nord Pool has built and shaped their market for over 25 years and still continues to grow. The power exchange was formed in 1991 as the Norwegian parliament determined to deregulate the trade for electrical power. In 1993 Statnett Market AS has become an independent company and in 1996 was replaced by Nord Pool ASA and Norwegian-Swedish electricity market was set up. In 1998 Finland decides to enter Nord Pool ASA and the company launch their office in Denmark. The Nordic market became completely integrated in 2000 due to Denmark's join. In 2002 Nord Pool's spot market appeared as an independent company under name of Nord Pool Spot. Later on, in 2005, the power market then sets the Kontek bidding area in Germany. In 2008 the superior turnover and share in the market was noted in the firm's history. In the early 2016, the company changed the corporate image to from Nord Pool Spot to Nord Pool.

Nord Pool's product is a clear and reliable price of the power that is processed every hour daily in the market. They propose productive and secure day-ahead and intraday markets. They offer a liquid trade in the electrical market and function as the information to the market. The markets of Nord Pool are currently performed from offices in 6 countries - Norway, Sweden, Finland, Estonia, UK and Germany.

In Nord Pool, for the day-ahead trade, customers may offer for sale or purchase energy for the following 24 hours in a closed auction. Over 300 clients make at least 2000 orders daily, and about 500 TWh of electrical energy is traded annually across 14 countries. In general, the day-ahead market in considered a starting point for preparation of the following 24 hour duration, which is then amended or intraday market with further possibilities are offered. The intraday market, enabling clients to trade nearer upon the physical delivery, can therefore operate with the day-ahead market in order to meet the supply and demand law. The attractiveness of intraday market rises as it lets participants to take into account any sudden and unplanned modifications in consumption. The company offers a variety of orders for their customers, both markets, to optimise the balance between supply and demand they provide.

Although the Nordic electricity market can be stated as the creator of modern energy exchange, the Nord Pool still seeks to perfect the power market in the future. The company focuses on 3 crucial factors to prevent any close to delivery time concerns. They aim for the market model of the organisation to be dependent on the nature of the business leaded by participants and the requirement for the protection of supply for system operators. They also hold the view, that the market should rely on producers and consumers, to be the main solvers for stability of power system. The last main factor is for the market to be constructed to give an easy access and systematise principles and products over broad areas, aiming to avoid barrier limitations and optimise market liquidity.

Mathematics for Signal Processing

4.1 Sinusoidal Wave Signals

Before we go deep into what is sine wave signal, let us recap the term of signals. A signal is a function that gives information of how some quantity differs over some time or space. Signals pass through systems, where they are processed to produce other signals.

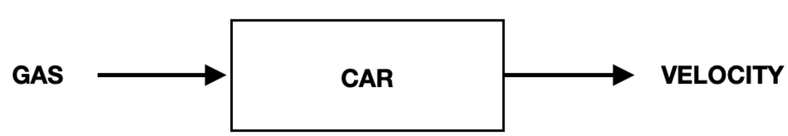

Figure 4.1: Signal passing through a system

The key distinction of signals in this thesis, is continuous time (analog) versus discrete time (digital). Continuous time signal, x(t), occurs whenever the independent variable t is continuous.

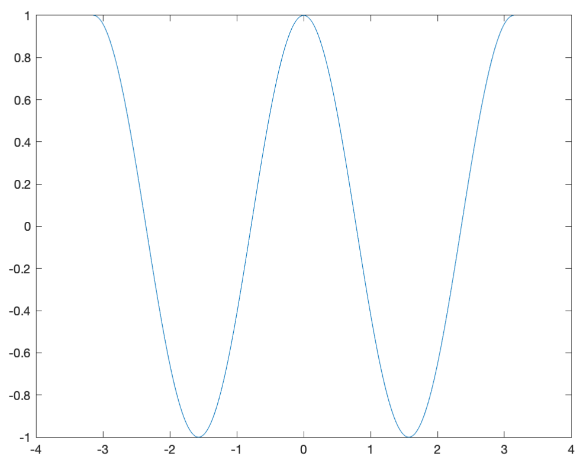

Figure 4.2: Continuous Signal

The signal is a discrete time signal, denoted as x[t] if the independent variable t consists of discrete values only.

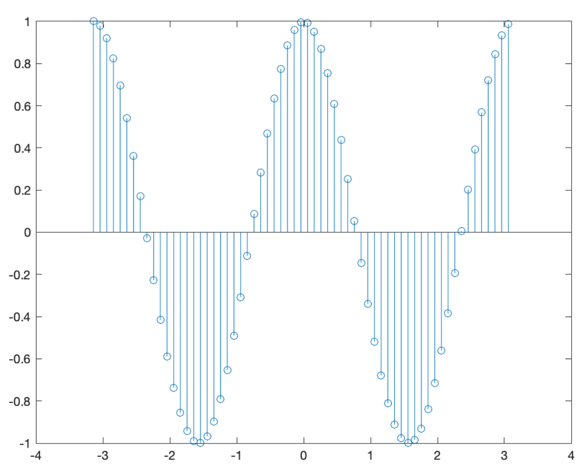

Figure 4.3: Discrete Signal

Sinusoidal signals are special signals, which are presented as a smooth periodic oscillation. The most basic function of the cure that describes it, is given by:

$$y(t) = Asin(2\pi ft + \theta) = Asin(\omega t + \theta), \tag{4.1}$$

where A is an amplitude of the wave, f is ordinary frequency, ω is an angular frequency, and θ is a phase.

Amplitude is the height from the centre line to the peak of the wave. We can also define it, as the height from the lowest and highest point in the wave and divide it by 2. Frequency f is the number of oscillations that occur each unit time and angular frequency ω measures the angular displacement per unit time. The relationship between frequency and angular frequency is defined by the equation:

$$\omega = \frac{2\pi}{T} = 2\pi f, \tag{4.2}$$

where T is the period, which is the length of time of one cycle in the reoccurring event, and it is the reciprocal of the frequency f. The phase defines how far the wave is shifted from the usual position, in other words the position of a point in time on a cycle. Those characteristics define the nature of a signal.

The following graph presents an example of a periodic signal wave expressed in terms of sine function and the parameters of the wave:

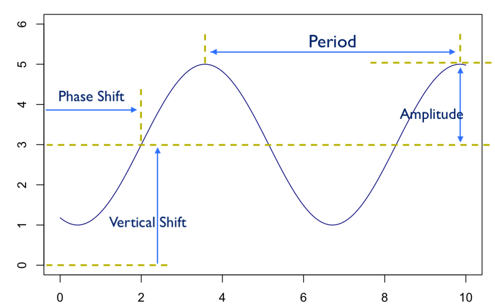

Figure 4.4: Sinusoidal Wave

The function of sinusoidal signal is a sine like function, simply put, the function can be constructed by shifting, stretching or compressing the sine function.

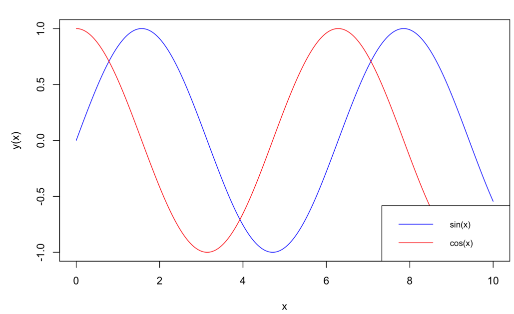

Figure 4.5: Graph of sin(x) and cos(x)

Figure 4.5 show that the sine and cosine functions are sinusoids of different phases as a cosine wave can be represented as a sine wave with $\pi/2$ shift:

$$cox(x) = sin(x + \pi/2), \tag{4.3}$$

and therefore, it is sinusoid.

The complex sinusoid is presented as:

$$Acos(\omega t + \theta) + jAsin(\omega t + \theta) \tag{4.4}$$

for a continuous time t, and as:

$$Acos(\omega nT + \theta) + jAsin(\omega nT + \theta) \tag{4.5}$$

for a discrete time T.

As we can see from the formula, the real part is a cosine signal and imaginary part a sine signal.

$$Re = Acos(\omega nT + \theta) \tag{4.6}$$

$$Im = Asin(\omega nT + \theta) \tag{4.7}$$

For the computations, one might find it easier to deal with complex exponential formula:

$$x(t) = Ae^{j(\omega nT + \theta)}, \tag{4.8}$$

which is produced by the relationship between trigonometric functions and exponential functions, namely, Euler's formula which states that for any real number x:

$$e^{jx} = cos(x) + jsin(x). \tag{4.9}$$

Sine and cosine functions can, therefore, be interpreted in terms of the exponential functions:

$$cos(x) = \frac{e^{jx} + e^{-jx}}{2}, \tag{4.10}$$

and

$$sin(x) = \frac{e^{jx} - e^{-jx}}{2j}. \tag{4.11}$$

4.2 Basic operations on signals

Having introduced the notion of signals, we can now present some operations that affect the signals, namely, signal processing. Those operations are performed over the independent variable of the signal, which is mostly the time parameter.

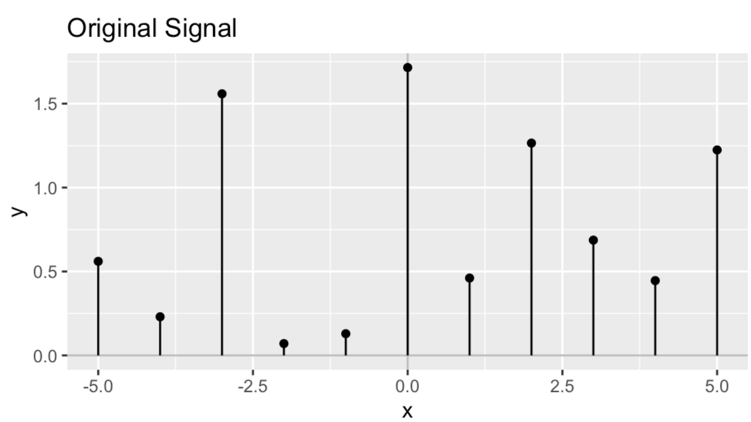

Figure 4.6: Original Signal

Flipping: This operation is basically flipping the sample around y axis, which is also called time reversal. The process can be simply done by multiplying the time variable by -1.

Example 4.2.1.

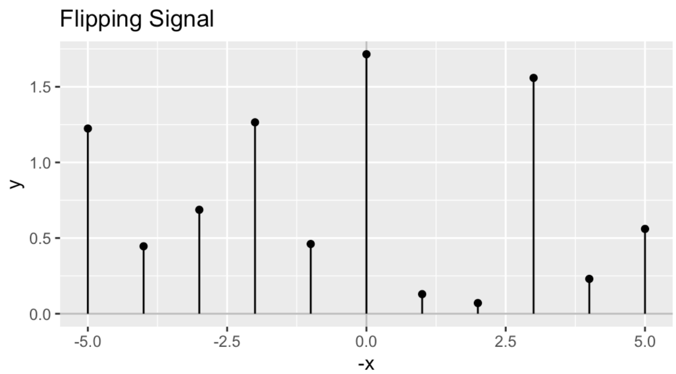

Figure 4.7: Flipping Signal

Scaling: The signal process compresses or dilates a signal by multiplying the independent variable by some quantity a. If $a > 1$, the signal is narrower, which is compression, and if $a < 1$, the signal is wider and named dilation. However, in case of a discrete signal, the operation works slightly differently.

Example 4.2.2. Let us scale a discrete signal x[t] with some quantity a, which is greater than 1, say 2:

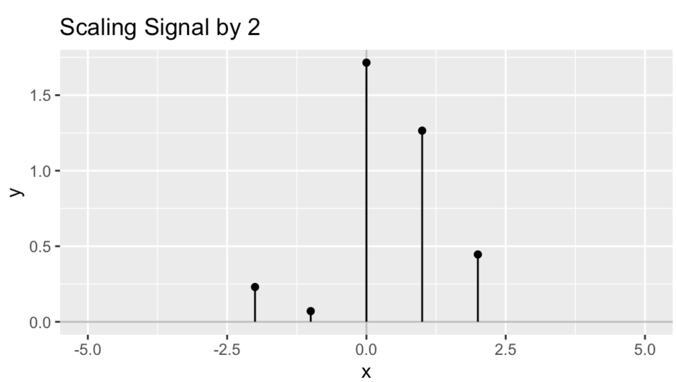

Figure 4.8: Scaling Signal by 2

Now, for a less than 1, say 1/2:

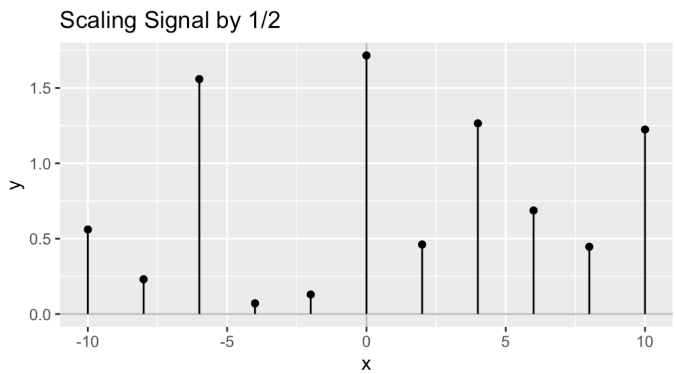

Figure 4.9: Scaling Signal by 1/2

We notice for discrete signals when $a > 1$ the samples are lost, in other words we are losing some information. On the other hand for $a < 1$, the signal is slowing down by some factor.

Shifting: This is performed by subtracting or adding by some quantity b to the independent time variable. Subtraction shifts a signal to the right, which causes the signal to be delayed. Addition shifts a signal to the left, which advances the signal by some added quantity.

Example 4.2.3.

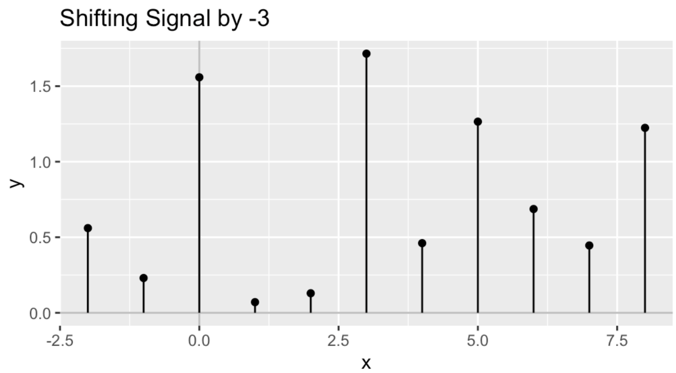

Figure 4.10: Shifting by -3

4.3 Properties of systems

Periodicity: Occurs whenever signal behaviour repeats after a fixed length of time T, and is mathematically defined as:

$$x(t) = x(t + nT),\ n = 1, 2, 3\dots, \tag{4.12}$$

where T is the fundamental period.

Causality: A system is causal if the output at time t, only depends on the input up to time t.

Example 4.3.1. $y[t] = x[t] - 3x[t-1]$ is casual.

Stability: The system is BIBO stable (the bounded input bounded output) if:

$$||x[t]||_\infty < \infty \tag{4.13}$$

implies that

$$||y[t]||_\infty < \infty. \tag{4.14}$$

That is, if bounded input leads to bounded output.

Even and Odd: Every signal has even and odd parts to make some conversions easy. The following equation decomposes a signal as a sum of even and odd parts:

$$x(t) = x_e(t) + x_o(t) = \frac{1}{2}(x(t) + x(-t)) + \frac{1}{2}(x(t) - x(-t)), \tag{4.15}$$

where

$$x_e(t) = \frac{1}{2}(x(t) + x(-t)), \tag{4.16}$$

$$x_o(t) = \frac{1}{2}(x(t) - x(-t)). \tag{4.17}$$

Linearity: A system is linear if a linear combination of inputs cause the same combination of outputs (Superposition Principle).

Example 4.3.2. Let $x1[t]$ and $x2[t]$ be inputs with the corresponding outputs $y1[t]$ and $y2[t]$ respectively. If for input $z[t] = ax1[t] + bx2[t]$ the output is $ay1[t] + by2[t]$, then the system is linear.

Time invariant: This means that the system behaves the same way, regardless of when input is applied. The system is time invariant whenever the shift in time domain results the same shift to the output.

Example 4.3.3. Let the system be defined as $y[t] = x[t] - 4x[t-1]$ and the shifted version of the input $z[t] = x[t - t_0]$. Then, putting z[t] into the system: $z[t] = z[t] - 4z[n-1] = x[t-t_0] - 4x[t-t_0-1] = y[t-t_0]$. Therefore, the system is time invariant.

LTI systems: In real-word, systems are typically modelled as linear time-invariant systems (LTI) as it is often a good approximation, since there are many system that are almost linear, and analyses are easy and powerful. The key concept is the superposition for LTI systems:
$a_0x[t] + a_1x[t-1] + a_2x[t-2] + \ldots = a_0y[t] + a_1y[t-1] + a_2y[t-2] + \ldots$.

4.4 Analytic Signal and Hilbert Transform

An analytic signal is a signal with non negative frequency, which is used in signal processing. Let us consider a sinusoidal wave, given by $cos(2\pi ft)$. The value is the same for both positive and negative frequencies. Therefore, if given the value of frequency, we know the number of oscillations per unit time, but not the direction (the frequency can be positive or negative). Analytic signal does not have this limitation.

The important concept here, is that the analytic signal is obtained by applying the Hilbert transform to the input signal. Hilbert transform extracts the analytic representation of real-valued signal s(t). It is simply a method that derives the complex components from a signal, for the analytic signal to be represented as:

$$s_a(t) = s(t) + jH(s(t)), \tag{4.18}$$

where $s_a(t)$ is the analytic representation of a signal $s(t)$, and $H(s(t))$ is the Hilbert transform of s(t). The analytic representation of a signal is, therefore, defined as a complex number with original signal as the real part, and its Hilbert transform as the imaginary part.

The Hilbert Transform for a given signal $s(t)$ is defined as:

$$H(s(t)) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{s(\tau)}{t - \tau} d\tau. \tag{4.19}$$

The Hilbert transform has the impact of transforming the signal as +/- 90 phase shift of its all components. As a result, performing the Hilbert transform on signal only changes its phase and the amplitude and frequency remains unchanged.

Example 4.4.1. For our cosine function, considered above:

$$H(cos(2\pi ft)) = cos(2\pi ft - \frac{\pi}{2}) = sin(2\pi ft). \tag{4.20}$$

And therefore:

$$s_a(t) = cos(2\pi ft) + isin(2\pi ft) = e^{i2\pi ft}, \tag{4.21}$$

where last equality is the complex exponential, used numerously in signal processing.

A crucial part of the signal processing, is that it is commonly written in terms of the amplitude and the phase, which takes the form:

$$x(t) = acos(\theta(t)), \tag{4.22}$$

where $a(t)$ is an instantaneous amplitude and $\theta(t)$ is an instantaneous phase.

Taking into account the previous formula, however, there can be numerous pairs of $a(t)$ and $\theta(t)$, which concludes that the above definition might be incoherent, as it does not relate a given real signal with a precise pair of the instantaneous amplitude and phase. The solution to this problem is the concept of a canonical pair of a(t) and $\theta(t)$. In order to get a pair of a(t) and $\theta(t)$ that has a one-to-one relationship with signal $s(t)$, we relate $s(t)$ with its analytic signal $s_a(t)$. From the formula (4.18) we know that $s(t) = Re(s_a(t))$, therefore there is a desired one-to-one relationship if the analytic signal is real. However, since $s_a(t)$ is a complex function, is can be defined as:

$$s_a(t) = a(t)e^{\theta(t)}, \tag{4.23}$$

where $\theta(t)$ is modulo 2π, and $a(t)$ is non-negative. Therefore, applying analytic signal and changing the formula (4.18) for polar coordinates, makes it possible to associate a unique pair of a(t) and $\theta(t)$, called canonical pair, with a real signal $s(t)$.

Example 4.4.2. Considering a very simple case with a signal $s(t) = 4cos(2)$, then:

$$H(s(t)) = 4sin(2)$$

and

$$s_a(t) = 4cos(2) + i4sin(2)$$

.

Now, changing $s_a(t)$ for polar coordinates:

$$a(t) = |r| = \sqrt{x^2 + y^2} = \sqrt{(4cos(2))^2 + (4sin(2))^2} = \sqrt{4^2(cos(2)^2 + sin(2)^2} = 4,$$

and

$$\theta(t) = tan^{-1}(\frac{y}{x}) = tan(\frac{4sin(2)}{4cos(2)}) = arctan(tan(2)) = 2.$$

Example 4.4.3. Let us, now, analyse a more complex example with the sum of 2 cosines: $s(t) = 0.4cos(3.5t) + 0.9cos(4t)$.
The equation can be evaluated in terms of exponentials as:

$$s(t) = 0.4(\frac{e^{i3.5t} + e^{-i3.5t}}{2}) + 0.9(\frac{e^{i4t} + e^{-i4t}}{2}).$$

Defining an analytic signal by keeping the positive frequency part we obtain:

$$s_a(t) = 0.4e^{i3.5t} + 0.9e^{i4t}.$$

We already know that $s(t) = Re(s_a(t))$, and it, therefore, can be defined as:

$$s(t) = Re(s_a(t)) = Re(A(t)e^{i\theta(t)}),$$

where $A(t) = |s_a(t)|$ is the instantaneous amplitude, and $\theta(t) = arg(s_a(t))$ is the phase of a signal.
Hence:

$$A(t) = |0.4e^{i3.5t} + 0.9e^{i4t}|$$
$$= |0.4e^{i3.5t}(1 + \frac{0.9}{0.4}e^{i(4-3.5)t})|$$
$$= |0.4e^{i3.5t}||1 + \frac{0.9}{0.4}e^{i(4-3.5)t}|$$
$$= 0.4\sqrt{(1 + \frac{0.9}{0.4}cos(0.5t))^2 + (\frac{0.9}{0.4}sin(0.5t))^2},$$

and

$$\theta(t) = arg(0.4e^{i3.5t} + 0.9e^{i4t})$$
$$= arg(0.4e^{i3.5t}(1 + \frac{0.9}{0.4}e^{i(4-3.5)t}))$$
$$= arg(0.4e^{i3.5t}) + arg((1 + \frac{0.9}{0.4}e^{i(4-3.5)t})$$
$$= 3.5t + arg(1 + \frac{0.9}{0.4}e^{i0.5t}).$$

4.5 Computing the Fourier Transform

From the previous subsection it is known, that if the function of the signal is defined as the sum of sinusoids, then Hilbert transform, as well as analytic signal, can be easily interpreted. However, signals are ordinarily expressed in time space rather than frequency space, and this is when the mathematical tool called Fourier transform is stepping in.

One of the Fundamental Secrets of the Universe: *"All waveforms, no matter what you scribble or observe in the universe, are actually just the sum of simple sinusoids of different frequencies."*[18]

Practically, everything can be represented by a waveform, such as images, sounds or stock prices. Fourier Transform enables to visualise these waveforms, by decomposing waveform into its sine and cosine components. A huge strength of LTI systems is, therefore, the capability to use some transformation methods, which takes the input into the system that outputs some new domain, which is the Fourier domain, and that makes solving LTI systems easier.

4.5.1 Fourier Series

An incentive of studying Fourier transform arose from first, introduction of Fourier series by a French mathematician, Joseph Fourier. Fourier Series states that every periodic, continuous-time signal can be mathematically defined as the sum of sinusoids.

Given the continuous, periodic signal defined as $x(t) = x(t + T)$, which goes to infinity, there is a part in the signal that repeats each T period.

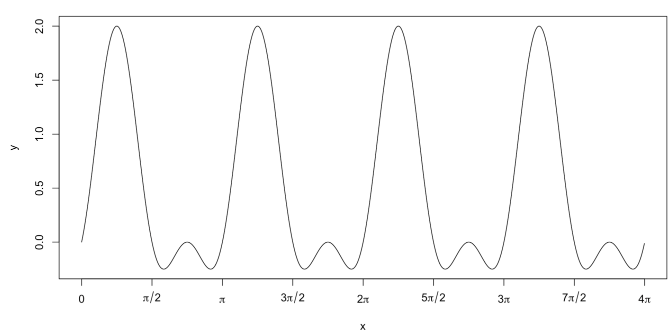

Figure 4.11: Example of a periodic wave signal

The most natural periodic signals that also have the same period T are sinusoids. And if each sinusoid is periodic with period T, then similarly does the complex signals or the summation of their combination. Therefore,

$$e^{jkw_0t} = cos(kw_0t) + jsin(kw_0t) \tag{4.24}$$

is periodic with period T, and so does

$$\sum_{k=-\infty}^{\infty} a_k e^{jkw_0t}. \tag{4.25}$$

This demonstrates, that a given periodic signal, like the one in Figure 4.11, can be constructed in such a way to look like the sum of sinusoids. The aim is, then, to find each a_k for a given x(t) and a fixed integer k. Those constants coefficients are worked out by the following equation:

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jkw_0t}dt \tag{4.26}$$

Hence, a signal can be expanded as the Fourier Series, which, in trigonometric form, is is defined as:

$$\hat{f} = a_0 + \sum_{m=1}^{\infty} a_m cos(\frac{2\pi mt}{T}) + \sum_{n=1}^{\infty} b_n sin(\frac{2\pi nt}{T}) = \sum_{m=0}^{\infty} a_m cos(\frac{2\pi mt}{T}) + \sum_{n=1}^{\infty} b_n sin(\frac{2\pi nt}{T}), \tag{4.27}$$

where the constants

$$a_0 = \frac{1}{T}\int_0^T f(t)dt \tag{4.28}$$

$$a_m = \frac{2}{T}\int_0^T f(t)cos(\frac{2\pi mt}{T})dt \tag{4.29}$$

$$b_n = \frac{2}{T}\int_0^T f(t)sin(\frac{2\pi nt}{T})dt \tag{4.30}$$

are Fourier coefficients, which define the weight of each sinusoid.

4.5.2 Fourier Transform

Fourier series applies only to continuous-time periodic functions. However, in real life we deal with aperiodic signals most of the time. Recall that expansion of Fourier series is $\sum_{k=-\infty}^{\infty} a_k e^{jkw_0t}$, and we can define a_k as $a_k = \frac{1}{T}\int_0^T x(t)e^{-jkw_0t}dt$, which we can expand as $\frac{1}{T}x(kw_0)$ Fourier transform is an extension of Fourier series, where signal x(t) becomes an aperiodic function, when the period of the signal approaches to infinity, T $\rightarrow \infty$. As a result:

1. $w_0 = \frac{2\pi}{T} \rightarrow 0$
2. $kw_0|_{w_0\rightarrow 0} \rightarrow w$
3. The summation of the expansion becomes an integral:
   $x(t) = \lim_{T\rightarrow\infty} x(t) = \lim_{T\rightarrow\infty} \frac{1}{T}\sum_{-\infty}^{\infty} TX(k)e^{jkw_0t}$
   $= \lim_{w_0\rightarrow 0} \frac{w_0}{2\pi}\sum_{-\infty}^{\infty} X(kw_0)e^{jkw_0t} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w)e^{jwt}dw,$
   which is called the inverse transform.
4. And finally, the Fourier transform becomes:
   $X(w) = \lim_{T\rightarrow\infty} X(kw_0) = \lim_{T\rightarrow\infty}\int_T x(t)e^{-jkw_0t}dt = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$

Summarising, for x(t) being a continuous function, if a signal is periodic, we get the Fourier series with the discrete set of a$_k s$, and for a non-periodic signal, we get the Fourier Transform with continuous $X(w)$.

4.5.3 Discrete-Time Fourier Transform

Now, to analyse the discrete-time signals, like our Nord Pool time series, which has samples at each fixed interval, we define the Discrete-Time Fourier Transform (DTFT), which converts a discrete-time signals at each n interval, x(n), into continuous frequency-domain signals.

Given the FT of a continuous-time signal:

$$X(w) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt.$$

If x(t) is sampled with sampling frequency 1/T, then we obtain:

$$X(w) = \sum_{-\infty}^{\infty} x(nT)e^{-jwnT} = \sum_{-\infty}^{\infty} x(n)e^{-jwn}, \tag{4.31}$$

where $x(n)$ represents $x(nT)$.
In other words, instead of integration, all the discrete values are summed. We, therefore, obtain a complex function DTFT which can be expanded as:

$$X(w) = \sum_{-\infty}^{\infty} x(n)cos(wn) + j\sum_{-\infty}^{\infty} x(n)sin(wn). \tag{4.32}$$

Therefore,

$$X(w) = Re(X(w)) + Im(X(w)). \tag{4.33}$$

DTFT can be, hence, defined as an amplitude and phase, with phase angle computed as:

$$\theta = tan^{-1}(\frac{Im(X(w))}{Re(X(w))}) \tag{4.34}$$

and amplitude:

$$|X(w)| = \sqrt{Re(X(w))^2 + Im(X(w))^2} \tag{4.35}$$

Note, that this is still a continuous function of w, the equation above does not make the frequency domain discrete. In discrete time, however, there is a fixed set of frequencies with the frequency range of $(-\pi, \pi)$. This is because the function of the phase is ambiguous, if we shift the phase by, for example, $2\pi k$, the result we get is $e^{j\theta+2\pi k} = e^{\theta}$. This shows that DTFT can be evaluated for any w, but it is 2π through value, therefore, the range normally has the restricted attention to the range of $-\pi$ to π.

4.5.4 Discrete Fourier Transform

As we noted, DTFT of a signal is continuous in terms of frequency, which requires incredible huge amount of storage and time to compute. It is impossible for DSP (digital signal processing) processor to process such a signal. We, therefore, need an alternative solution to this problem with a finite number of samples of $X(w)$ taken.

This is form of result is defined as Discrete Fourier Transform (DFT) which takes samples at each fixed interval with finite time duration and converts a discrete signal at each n interval, $x(n)$, into equally spaced DFT, $X(k)$:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N} = \sum_{n=0}^{N-1} x(n)[cos(\frac{2\pi}{N}kn) - jsin(\frac{2\pi}{N}kn)], \quad (4.36)$$

$k = 0, 1, 2, ..., N-1$,
where the inverse transform is defines as:

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}, \quad (4.37)$$

$n = 0, 1, 2, ..., N-1$.

As a result, we get a sequence of N frequencies, which are equally spaces by n interval. Note that, here, n represents a time index and k a frequency index. For simplicity we will represent $e^{-j\frac{2\pi}{N}}$ in the Equation 4.36 as W_N, resulting the above equations for DFT and IDFT to be expressed as:

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{kn} \quad \text{for } 0 \leq k \leq N-1, \quad (4.38)$$

and

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn} \quad \text{for } 0 \leq n \leq N-1, \tag{4.39}$$

respectively.

<u>Some properties of W_N^{kn}:</u>

1. For the sequence of length N:
   $W_N^{nN} = W_N^{kN} = e^{-j2\pi kN/N} = e^{-j2\pi k} = 1 = W_N^0$.

2. W_N^{kn} is periodic in terms of n:
   $W_N^{k(n+rN)} = W_N^{kn} W_N^{rkN} = W_N^{kn}$ for $-\infty < r < \infty$.

3. W_N^{kn} is periodic in terms of k:
   $W_N^{n(k+rN)} = W_N^{kn} W_N^{rnN} = W_N^{kn}$ for $-\infty < r < \infty$.

4. For even number of samples, $n = 2r$:
   $W_N^{kn} = W_N^{2rk} = e^{2(-j2\pi kr/N)} = e^{-j2\pi kr/N/2} = W_{N/2}^{kr}$.
   In other words, for W_N^{kn} with even samples of length N can be converted as $W_{N/2}^{kr}$ of length $N/2$.

Therefore, from above features of complex sequence W_N^{kn}, we can conclude that a sequence with N number of samples, has a periodic DFT: $X(k+N) = X(k)$

The DFT takes finite sequence of signals and transforms it to finite length of transform, which takes finite amount of memory and time. To our disadvantage, however, sometimes the sequence of signals for is too long, which takes a large amount of memory and time to derive. Those signals, in real-life, cannot be proceeded in DSP processors. Using the properties of DTF, a simplified version of transform was constructed called Fast Fourier Transform, giving a fast and efficient computation of the DFT. The implementation provides nearly the same results as the DFT, but with less computation time as it reduces the number of operations, by decomposing DFT into smaller DFTs.

Implementation with R

Having covered background of some essential mathematical techniques used in signal processing, namely for extracting instantaneous amplitude and phase from the wave signals, we can now explore how they can be used using R commands. And then, we will be using those R commands to analyse the financial time series of Nord Pool electricity prices.

Example 5.0.1. Let us consider first the case of signal with 2 cosines, which we covered in Example 4.4.3:

$$s(t) = 0.4 * cos(3.5 * t) + 0.9 * cos(4 * t),$$

represented in Figure 5.1 using the code:

```
#Defining signal s(t) and independent variable t
t=seq(0,100,0.01)
s=0.4*cos(3.5*t)+0.9*cos(4*t)

#Plotting signal s(t)
plot(t,s,type="l")
```

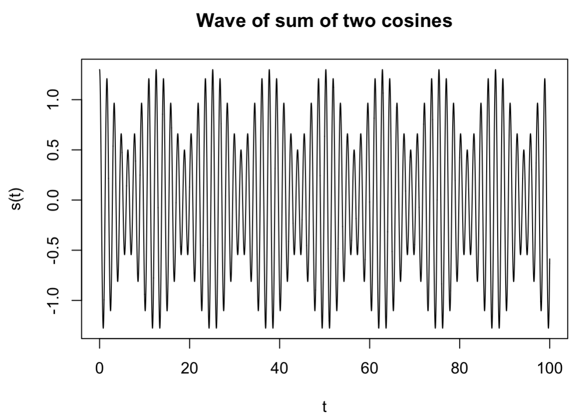

Figure 5.1: Signal of 2 cosines

First we apply Fast Fast Fourier Transform on signal using commands in R represented with the code below:

```
#Implementing Fast Fourier Transform
z=fft(s,inverse=FALSE)
z[1]=0
z[5002:10001]=seq(0,0,5000)
w=fft(z,inverse=TRUE)/length(t)
```

fft function returns z, which is the discrete Fourier Transform of the sequence, and fft when inverse=TRUE extracts a complex signal defined by w.

We then can calculate the instantaneous amplitude and phase of the signal using:

```
2*abs(w)
```

for amplitude, and

```
atan2(Im(w),Re(w))
```

for phase, where w is the transformed signal.

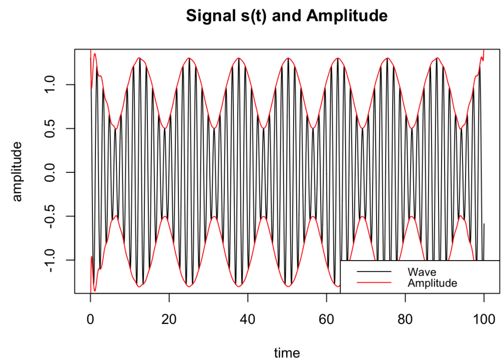

Figure 5.2: Amplitude

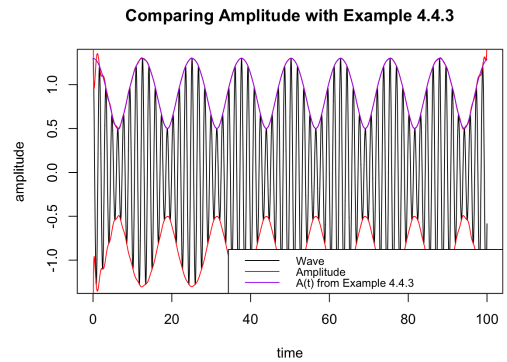

Figure 5.3: Comparing A(t)s

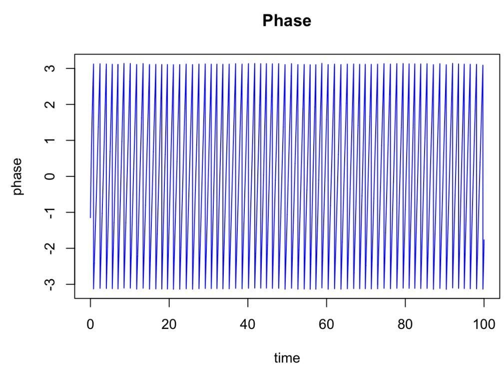

Figure 5.4: Phase

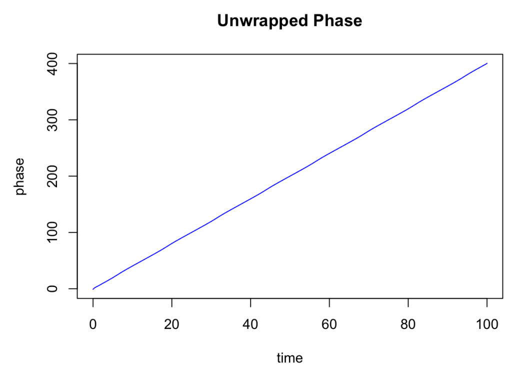

Figure 5.5: Unwrapped phase

Figure 5.2 shows the output of instantaneous amplitude. The amplitude seem to envelopes the original signal. The amplitude vary approximately between 1.3 and 0.5 for the upper envelope and between -0.5 to -1.3 for the lower envelope. The envelope on the instantaneous amplitude is, therefore, +/- A(t), where A(t) represents the instantaneous amplitude. Comparing with the amplitude calculated in Example 4.4.3, on the Figure 5.3, we observe that the lines for both amplitudes overlap.

Figure 5.4 shows the output of phase of the wave. The signal seems to vary with the same phase, in other words, there are no sudden extreme changes in a signal. Figure 5.5 shows phase in the unwrapped form of the Figure 5.4, which was plotted using the following code:

```
#Function for unwrapping a phase
up=atan2(Im(w),Re(w))
for (i in 2:length(t))
{ difference=up[i]-up[i-1]
  if (difference > pi)
  {up[i:length(t)]=up[i:length(t)]-2*pi}
  else if (difference < -pi)
  {up[i:length(t)]=up[i:length(t)]+2*pi}  }
#Figure 5.5
plot(t,up,type="l",main="Unwrapped Phase",
col="blue",ylab="phase",xlab = "time")
```

The difference between them is, that the former one is constrained to $(-\pi, \pi)$, and this type of phase is called wrapped phase, and the other one is called unwrapped phase. Figure 5.6 shows the comparison between the unwrapped phase in Figure 5.5, and the phase function calculated in Example 4.4.3.

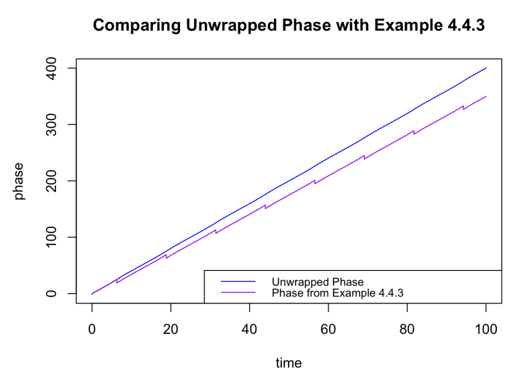

Figure 5.6: Comparing phases

Example 5.0.2. Now, we will analyse a signal of sum of cosine and sine:

$$s(t) = 0.7 * cos(2 * t) + 0.8 * sin(3 * t),$$

shown in Figure 5.7.

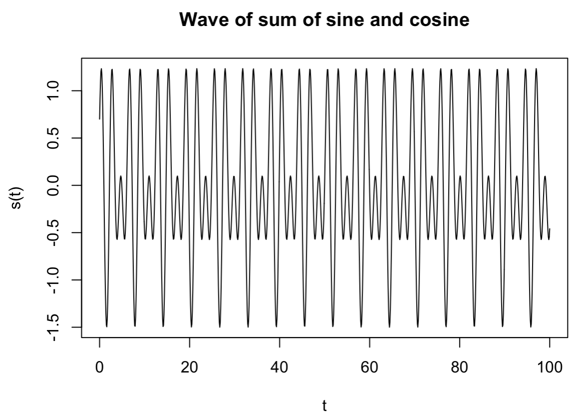

Figure 5.7: Signal of sum of sine and cosine

Applying the same R commands on the wave signal, as in previous example, we get the instantaneous amplitude represented in Figure 5.8, instantaneous phase in Figure 5.9 and unwrapped phase in Figure 5.10. The amplitude, here, is a proper envelope of the wave, and it varies roughly between -1.6 and 1.6. The phase of the signal seems to have a phase slip after each constant interval of about 6.5, which is more visible on from the bumps in the unwrapped phase in Figure 5.10. This is indicated by the sudden decreases and increases of the signal after each interval.

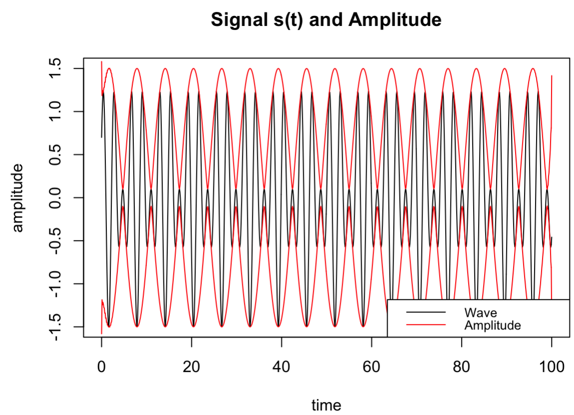

Figure 5.8: Amplitude

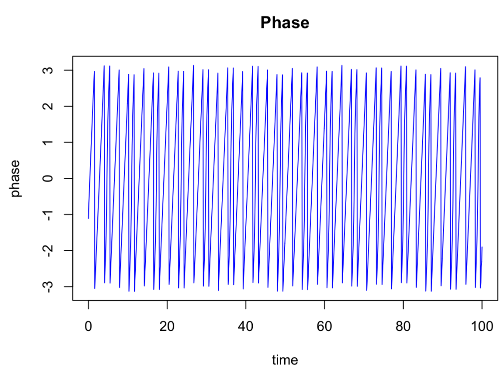

Figure 5.9: Phase

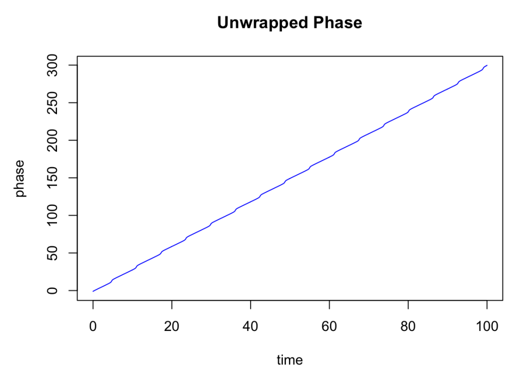

Figure 5.10: Unwrapped phase

Example 5.0.3. Let us now analyse more complex example:

$$s(t) = (1 + 0.7 * t) * cos(2 * t),$$

which is represented in Figure 5.11.

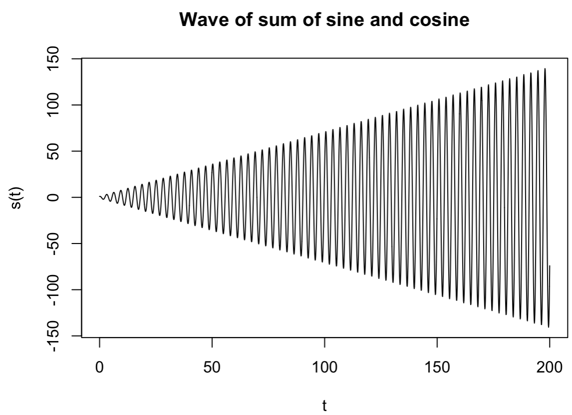

Figure 5.11: Signal with growing amplitude

As before, we get the output of instantaneous amplitude and phase, presented in Figure 5.12 and Figure 5.13 respectively. The amplitude is varying linearly, which seems to match with the time varying amplitude of the input signal. There are no large phase slips indicating any sudden extreme values in the signal, which can also be confirmed by a straight line of the unwrapped phase in Figure 5.14. This agrees with the original signal, as waves of the signal seems to enlarges gradually, and there are no extreme spikes appearing.

At this time, we can now use those mathematical techniques and R commands, studied above, on Nord Pool data, and analyse the instantaneous amplitude and phase of the electricity prices of the company and observe if there is any relation between the outputs of the signal and appearance of the extreme spike prices.

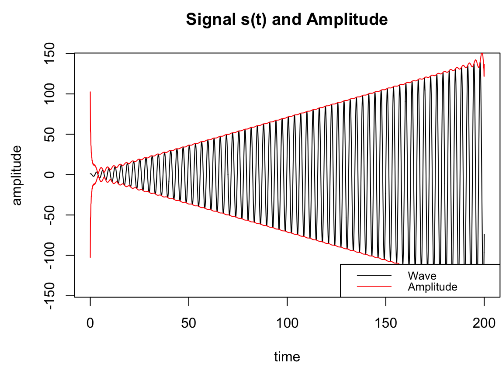

Figure 5.12: Amplitude

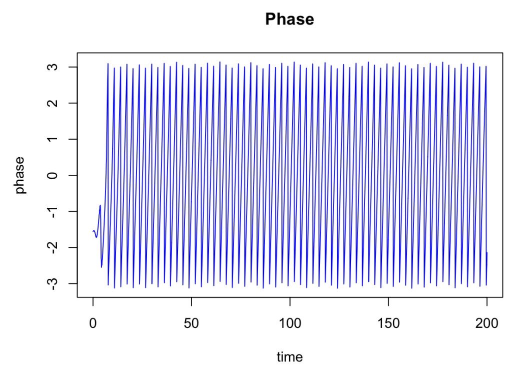

Figure 5.13: Phase

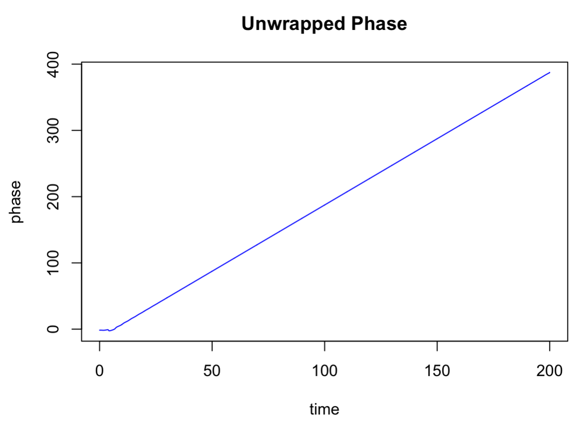

Figure 5.14: Unwrapped phase

Analysing Nord Pool Market Prices

6.1 Nord Pool Data

In this paper, we are going to study the market prices of Nord Pool market. The data is represented in the form of electricity prices at each hour interval, which have been noted for 8 years, from 1st January 1999 to the 26th January 2007. We can visualise the data as the time series graph in Figure 6.1, which has been imported and formatted as time series accordingly using R code:

```
#Importing Nord Pool Data
setwd("/Users/thuytrangnguyen/Desktop/Dissertation")
data=read.table("BEUR.txt",header=FALSE)

#Creating Time Series
library(xts)
dates<-seq(from=as.POSIXct("1999-01-01 00:00"),
to=as.POSIXct("2007-01-26 23:00"), by="hour")
time_series<-xts(data, order.by=dates)
```

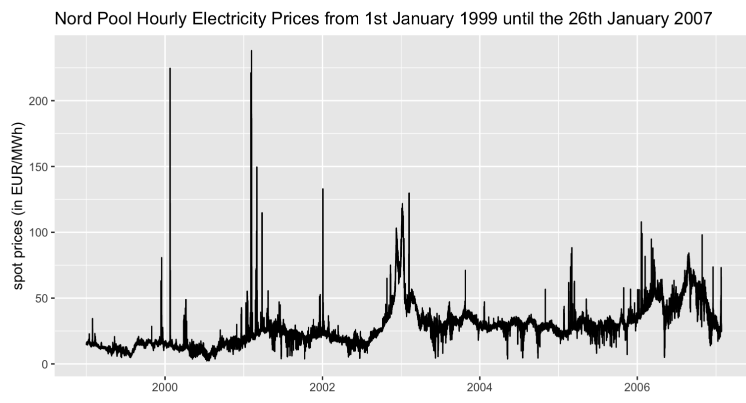

Figure 6.1: Nord Pool Hourly Electricity Prices

The graph shows 70,752 data points, which represent spot prices in Euro per MegaWatt Hour with sampling rate of 1 hour. The time series is evidently not stationary, which can be supported using tests for stationarity, such as Augmented Dickey-Fuller test, Ljung-Box test and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, where the null hypothesis for the first test is that the time series is non-stationary, and the null hypothesis of the last two tests shows the stationarity. It is worth noting, however, that the Ljung-Box test points out the absence of serial correlation, which is stronger than stationarity. Therefore, the time series can still be stationary for p-value less than 0.05 for Ljung-Box test.

The results for stationary tests of the Nord Pool data time series are represented in Figure 6.2. The Ljung-Box and KPSS tests indicate non-stationarity (p-value$<$0.05), however ADF test shows some stationarity (p-value$<$0.05). This might indicate that even though the data is non-stationary, as there is a slight increasing trend noticeable, the time-series perhaps shows some stationary behaviour.

```
        Augmented Dickey-Fuller Test

data:  na.omit(time_series)
Dickey-Fuller = -6.5468, Lag order = 41, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In adf.test(na.omit(time_series)) : p-value smaller than printed p-value
> Box.test(na.omit(time_series), type="Ljung-Box")

        Box-Ljung test

data:  na.omit(time_series)
X-squared = 69008, df = 1, p-value < 2.2e-16

> kpss.test(na.omit(time_series))

        KPSS Test for Level Stationarity

data:  na.omit(time_series)
KPSS Level = 150.31, Truncation lag parameter = 20, p-value = 0.01

Warning message:
In kpss.test(na.omit(time_series)) : p-value smaller than printed p-value
```

Figure 6.2: Stationarity Tests for time series

The graph also reveals are some extreme spikes, which appear in the market prices. The largest spikes seem to arise roughly each year, especially in 2000, 2001, 2002 and 2003. Those price spikes can cause price risks, which are not beneficial to the electricity market. The highest spike occurred in 5th February 2001 of 238.01 EUR/MWh.

The statistics of the time series are shows as follows:
Min: 2.33
1st Q: 16.75
Median: 25.88
Mean: 27.48
3rd Q: 32.79
Max: 238.01

There also seem to be many outliers occurring in the data, which are indicated by the box-plot and histogram shown in Figure 6.3.

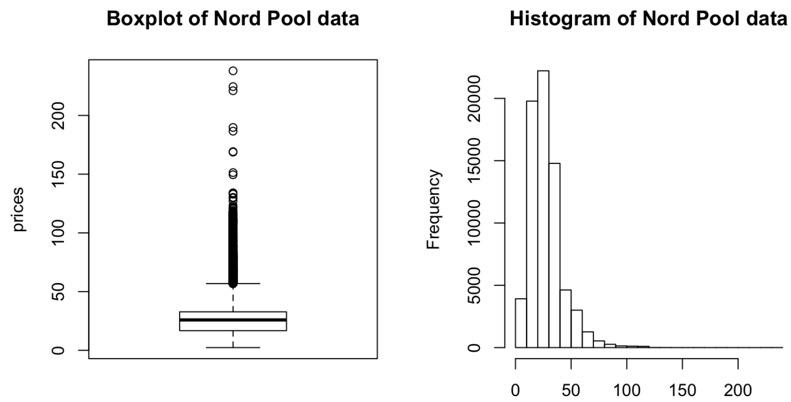

Figure 6.3: Boxplot and Histogram of time series

Box-plot reveals the appearance of high spikes in the financial time series. The histogram is right skewed as it can be seen by looking the right tail of the graph, meaning that most of the data fall into the low values category with few large values.

Having already an overall review of the data, let us now analyse the instantaneous amplitude and phase of the time series, which are derived using R commands for Fast Fourier Transform, explained in Chapter 4, and showed in practice in Chapter 5.

The Figure 6.4 represents the complex signal wave with instantaneous amplitude. The wave seems to vary between the lower and upper envelope of the amplitude. There is no periodicity noticed in the amplitude as the time series of the Nord Pool data does not any periodic pattern visible. The graph in Figure 6.5 shows the original time series and the instantaneous phase. The phase slip appears to be showed up during the spike prices. To analyse it further we will, later on, split the time series to analyse the data more closely.

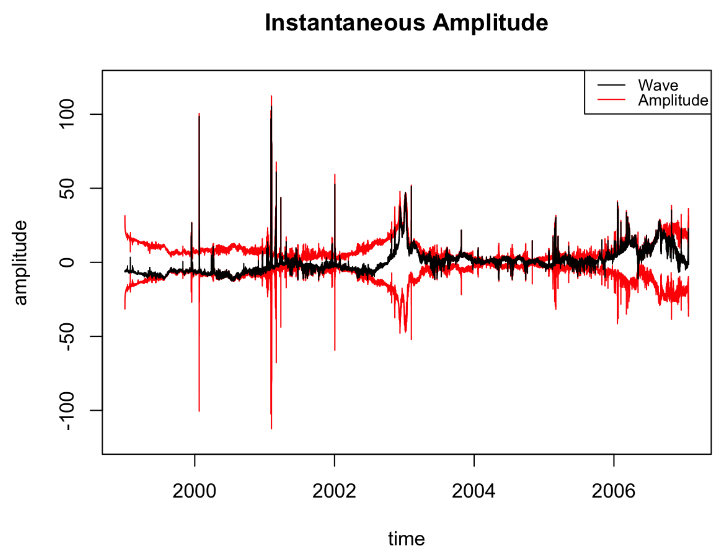

Figure 6.4: Amplitude of time series

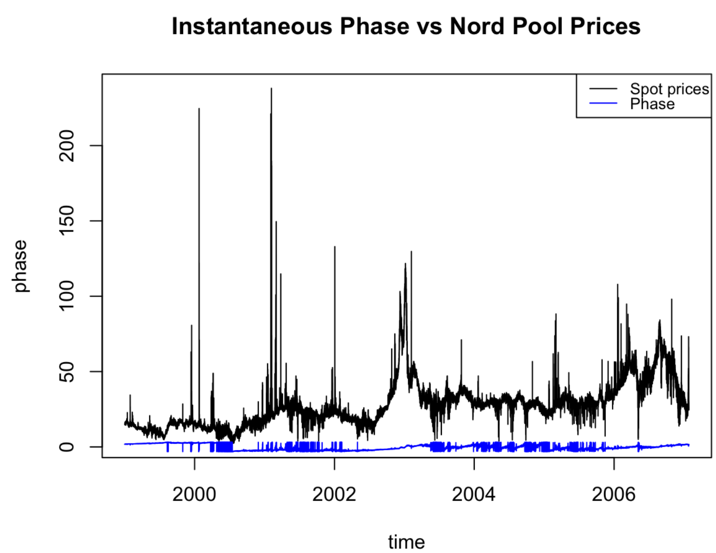

Figure 6.5: Phase of time series

As the data seems not to be stationary, let us now look at the logarithm returns of the data instead and check if the data becomes stationary. The new data is derived, from the original time series, using the function:

```
log_returns=diff(log(time_series))
```

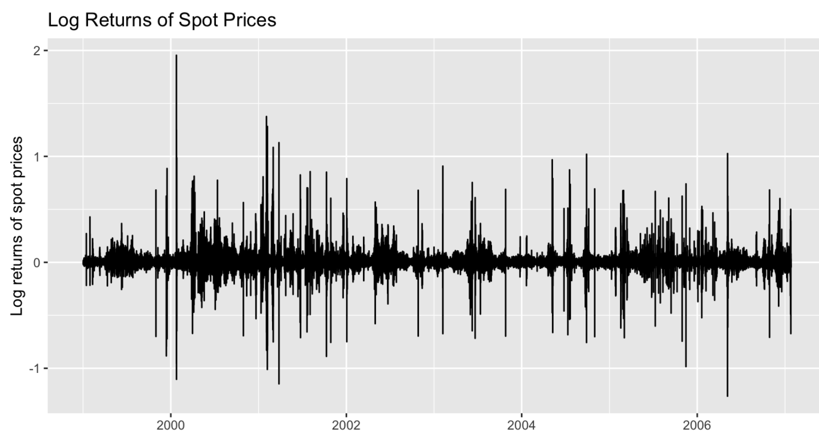

Figure 6.6: Log returns

The time series of the log returns, presented in Figure 6.6, appear to be stationary. For confirmation, three tests for stationarity are derived and the results are shown in Figure 6.7.

```
        Augmented Dickey-Fuller Test

data:  na.omit(log_returns)
Dickey-Fuller = -52.191, Lag order = 41, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In adf.test(na.omit(log_returns)) : p-value smaller than printed p-value
> Box.test(na.omit(log_returns), type="Ljung-Box")

        Box-Ljung test

data:  na.omit(log_returns)
X-squared = 10616, df = 1, p-value < 2.2e-16

> kpss.test(na.omit(log_returns))

        KPSS Test for Level Stationarity

data:  na.omit(log_returns)
KPSS Level = 0.0027265, Truncation lag parameter = 20, p-value = 0.1

Warning message:
In kpss.test(na.omit(log_returns)) : p-value greater than printed p-value
```

Figure 6.7: Stationarity Tests for log returns

The p-value of ADF test is below 0.05 , which rejects the hypothesis that the data is non-stationary. KPSS tests also confirms the stationarity of time series. However, the p-value of Ljung-Box is less than 0.05, but as mentioned before, the test of Ljung-Box is much stronger than stationarity. Therefore, it is safe to conclude that the log returns of the prices are stationary.

The graph in Figure 6.8 shows how the envelope of amplitude almost matches with the transformed signal. This is because, unlike the prices, the log-returns are stationary.

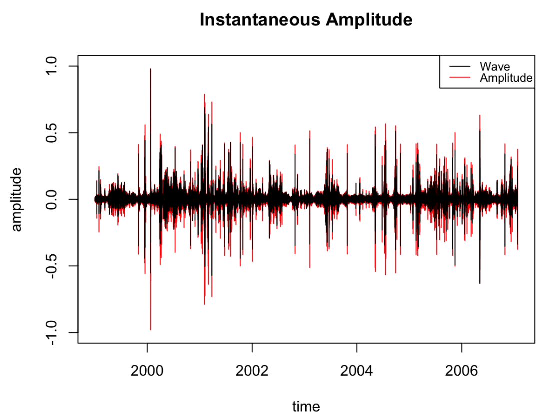

Figure 6.8: Amplitude of log returns

The instantaneous phase of the log-returns of time series does not show any specific pattern in relation with the signal, as the data is stationary. The phase varies between $-\pi$ and π.

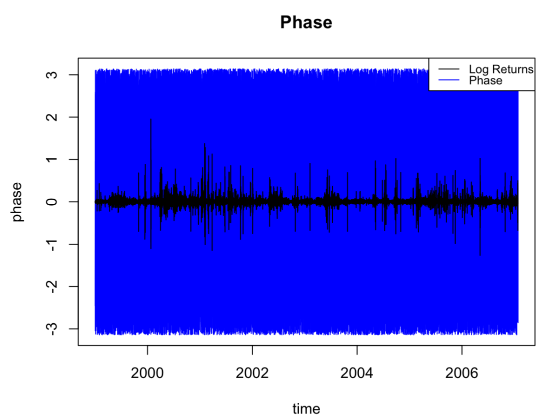

Figure 6.9: Phase of log returns

6.2 Average Nord Pool Data

The graph below in Figure 6.10 represents the time series of average Nord Pool prices, which is constructed by averaging prices for each hour of each day of a week, starting from Friday 00:00 to Thursday 23:00. The graph, therefore, represents 7*24=168 data point, where each average price is calculated by taking the mean of each 421-422 prices. Unlike, the original time Nord Pool time series, the average of Nord Pool data shows some seasonality pattern. We can therefore conclude that there is some daily seasonality pattern in the Nord Pool prices. The pattern seems to be influenced by the hours of the day. It is also noticeable from the graph, that there are two days where the average prices are quite lower than the other days of the week. The range of average prices vary between approximately 23 and 33 EUR/MWh.

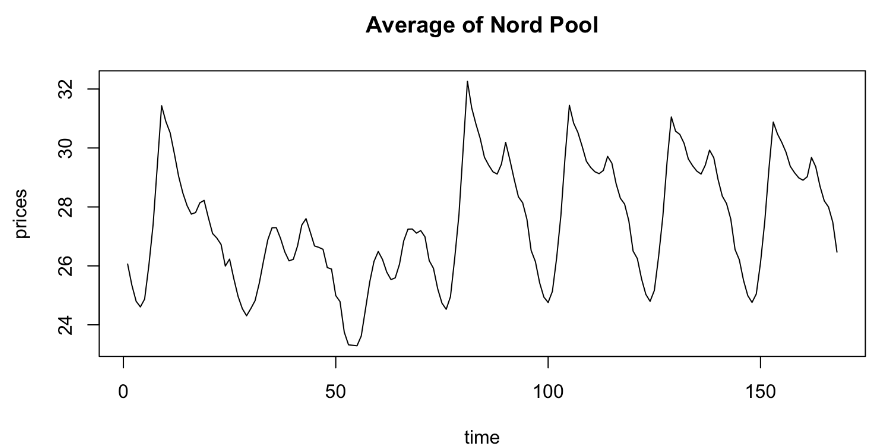

Figure 6.10: Average prices

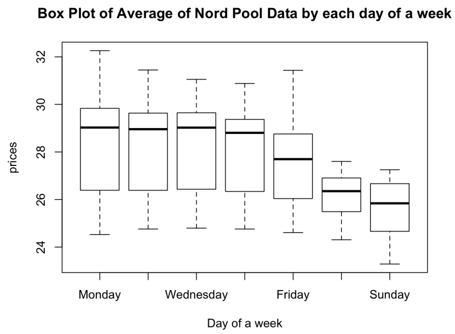

Figure 6.11: Box Plot by week day

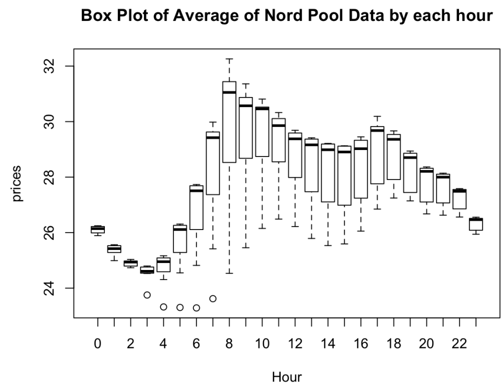

Figure 6.12: Box Plot by hour

The two box-plots in Figure 6.11 and Figure 6.12, of each seven-day week and of each hour, support in visualising in more deep, which days and hours effect the previous graph in Figure 6.10.

The first box-plot confirms our previous observation that there are 2 days with lower electricity prices, which are Saturday and Sunday. This might be due to the fact, that majority of the offices are closed during the weekends and people tend to have these days off, which lower the prices of the electricity as, like mentioned in the second chapter, the prices of electricity are based on the demand and supply. The second box-plot indicates that there is a rise of prices in the mornings, from 3am, with the peak at 8am. The prices, then, stay relatively high at between 29 and 30 EUR/MWh, and reaches the second peak at 5pm. The prices decline afterwards with the lowest price at 3am, and rises to 8am again.

The instantaneous amplitude and phase of the average data are calculated using, again, the Fast Fourier transform, and are represented on the graphs in Figure 6.13 and Figure 6.14. The illustration of the amplitude shows how the wave varies between upper and lowers amplitude envelope. The

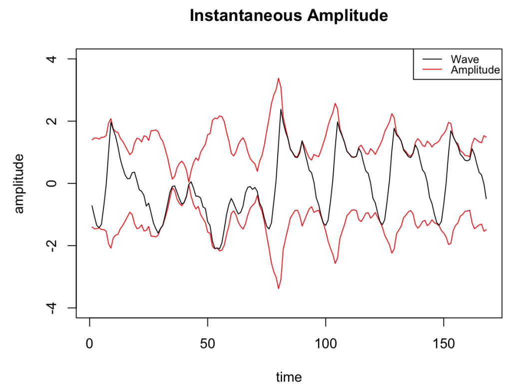

Figure 6.13: Amplitude

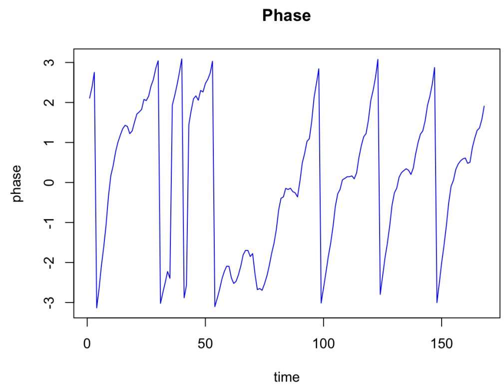

Figure 6.14: Phase

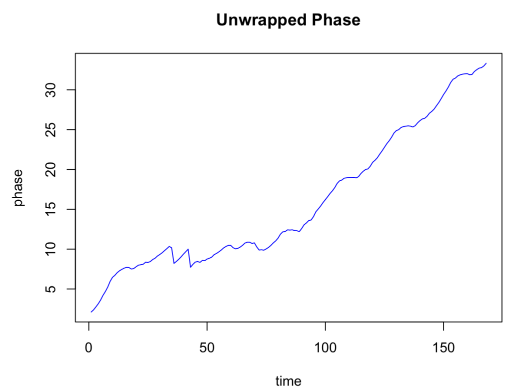

Figure 6.15: Unwrapped phase

instantaneous amplitude demonstrate some periodicity, which accounts to the pattern in the Figure 6.10. The fluctuations of the amplitude slightly changes during weekends, where the prices were the lowest. The graph of instantaneous phase shows the compatible way of action as amplitude, where the phase varies with similar pattern each weekday, with a large phase slips on weekends. From the unwrapped phase in Figure 6.15, the phase bumps also are most noticeable on Saturday and Sunday.

6.3 Price Spikes

As noted before, that there are some high spikes in Nord Pool prices, which we will now have closer look at. The most extreme spikes occurred in early 2000, 2001, 2002 and 2003. The results of those high spikes at the beginning of each year might be due to the fact that the temperature tends to be the lowest during January and February. The graphs of time series of each spike and their instantaneous amplitude and phase are represented below respectively.

The first graph in Figure 6.16 shows the time series of prices between 01/01/2000 to 29/02/2000, where the first extreme price spike appeared. The prices seem to be maintain in between 10 to 30 EUR/MWh for those 2 months, with extreme high spikes on 24th and 25th January, indicating that on those 2 days the weather was the coldest, which increased the demand of electricity, resulting in the increase in price. There is also daily seasonality pattern visible.

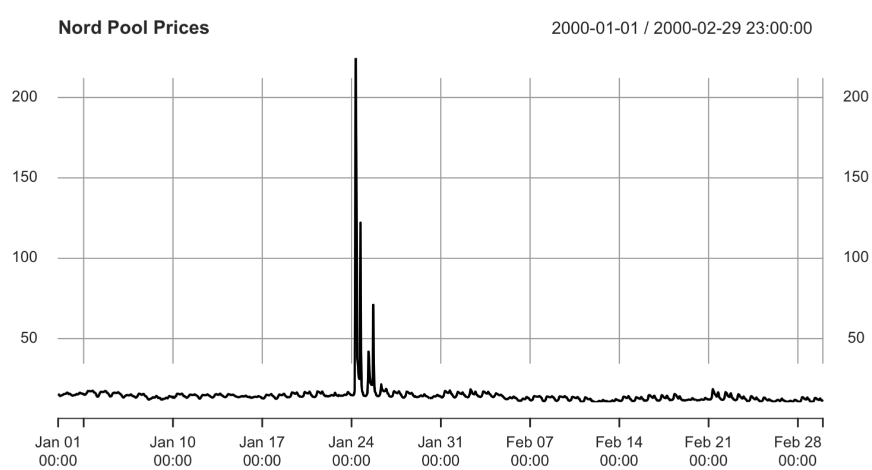

Figure 6.16: Time series of 01/01/2000 - 29/02/2000

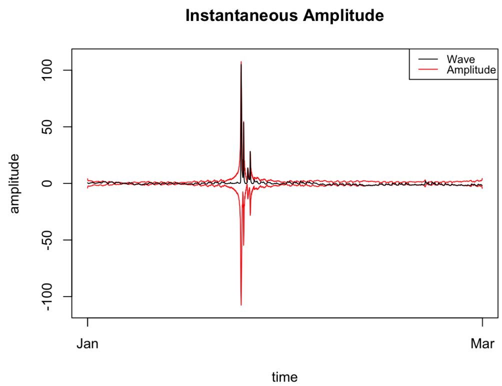

Figure 6.17: Amplitude

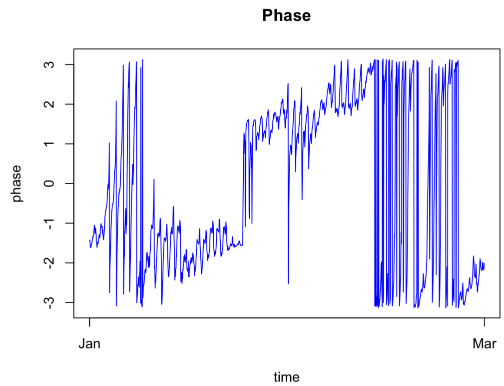

Figure 6.18: Phase

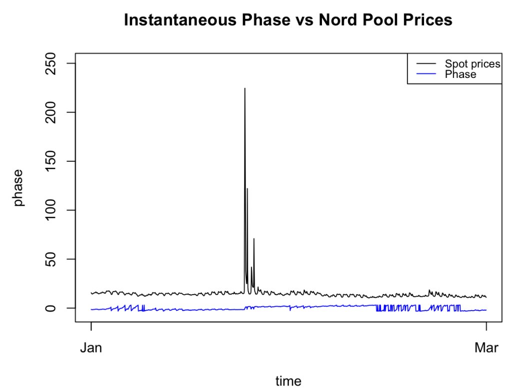

Figure 6.19: Phase and wave

Using the same method as before, the outputs of the instantaneous amplitude and phase of this time series are presented on graphs as follows in Figure 6.17 and Figure 6.18. The complex wave of signal varies between the lower and upper envelope of the amplitude with the spikes following the upper envelope. The extreme spikes do not seem to affect the prices after the spikes. The instantaneous phase shows the significant difference in phase. The Figure 6.19 compares the phase with the time series. The phase slips are noticeable during spikes.

The next time series represent the prices from 01/01/2001 to 01/07/2001. Through those 6 months, there were many price spikes recorded. The highest spikes occur during the first 3 months from January to March. After the end of April the prices do not exceed 50 EUR/MWh, and there are some downwards spikes observed. This is the effect of the change in season, and following with it the change in the weather.

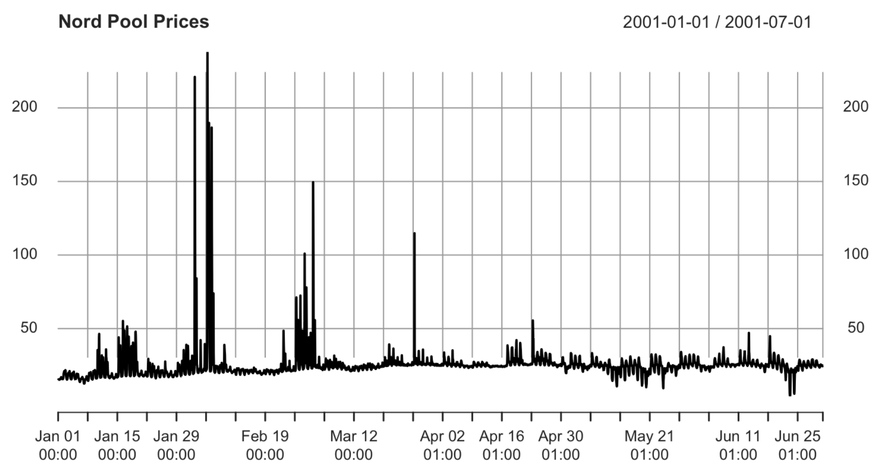

Figure 6.20: Time series of 01/01/2001 - 01/07/2001

As with the time series analysed in Figure 6.16, the prices lie within the lower and upper amplitude line with the spikes following the upper envelope whenever the price is dramatically increasing, and reaching lower envelope when there is a decrease in price. The instantaneous phase in Figure 6.22, as well, looks like it have the same behaviour as the previous time series. In other words, there are large phase slips visible, especially where there were 3 highest spikes occurring. This relationship between spikes and phase slips can also be observed in Figure 6.23.

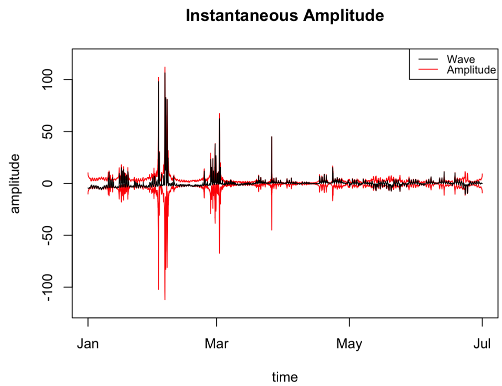

Figure 6.21: Amplitude

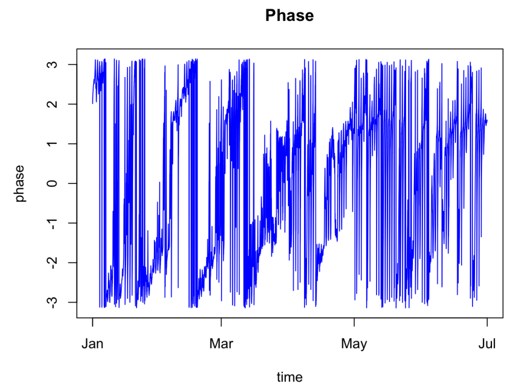

Figure 6.22: Phase

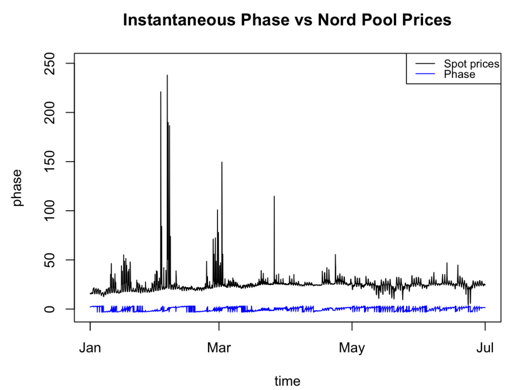

Figure 6.23: Phase and wave

To confirm the observations noted on behaviour of the amplitude and phase affecting the signals, we will analyse two more time series with spike prices, and see if the same conclusions were spotted.

Figure 6.24 shows the hourly prices from 01/01/2002 to 01/02/2002. The prices show a periodic pattern with extreme spikes at the beginning of the time series, and some small spikes at the end of time series. The outputs of analysis of instantaneous amplitude and phase are presented in Figure 6.25 and Figure 6.26.

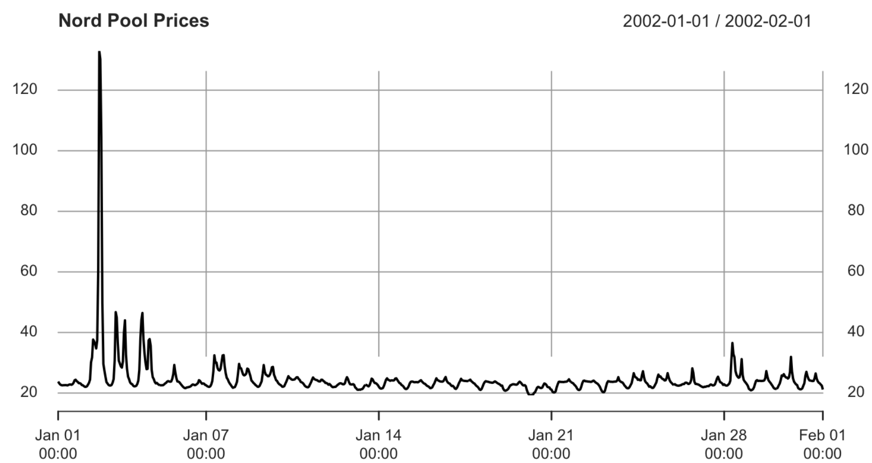

Figure 6.24: Time series of 01/01/2002 - 01/02/2002

Figure 6.25 agrees with our observation that there are some spikes at the beginning and end of Figure 6.24. Figure 6.26 shows how the phase looks like comparing to time series.

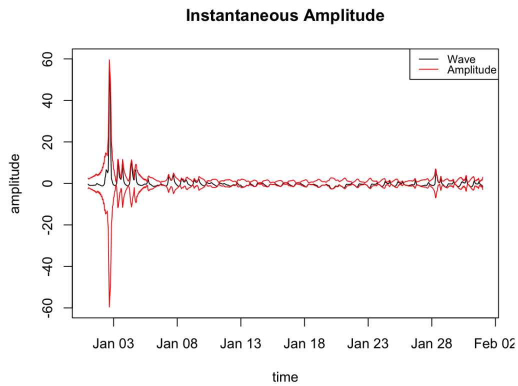

Figure 6.25: Amplitude

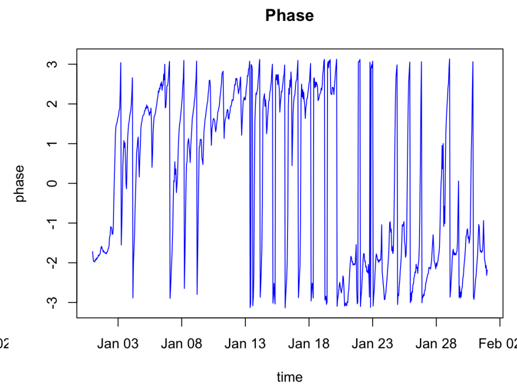

Figure 6.26: Phase

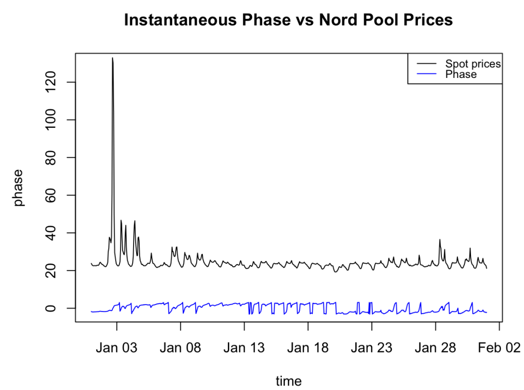

Figure 6.27: Phase and wave

The Figure 6.28 shows the how to prices vary between 01/01/2003 and 01/07/2003. The prices gradually increase until it reaches 121.75 EUR/MWh on 06/01/2003, and then gradually decrease. A month after, on 06/02/2003 there is a sudden spike of 129.75 EUR/MWh. As similarly observed in year 2001, the prices after end of March do not exceed 50 EUR/MWh due to change in the temperature. There are also bottom spikes noticeable in May and June. Figure 6.29 shows the instantaneous amplitude using the data in Figure 6.28. For first 2 months the time series follows the upper amplitude line, and then slowly following the lower amplitude line. Figure 6.30 shows

Figure 6.28: Time series of 01/01/2003 - 01/07/2003

the output of the phase of Figure 6.28 and Figure 6.31 shows their relation. The graphs show how there are sudden changes in price value during phase slips. There is especially noticeable phase slip around the beginning of February where the largest spike appeared. The extreme spike does not seem to affect the prices of the electricity after. Figure 6.31 shows the comparison of the time series and the phase slips. The graph shows how the price changes dramatically on February, resulted from the low temperature on that day. And there are also fluctuations in price in between May, where there were changes in the temperature.

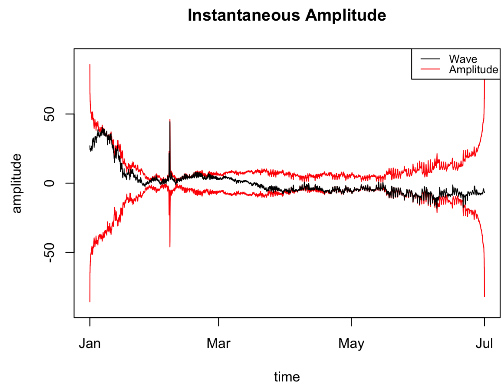

Figure 6.29: Amplitude

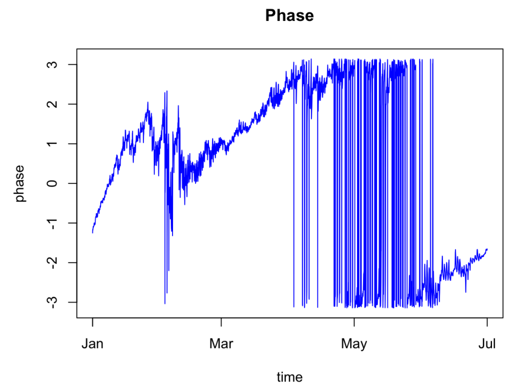

Figure 6.30: Phase

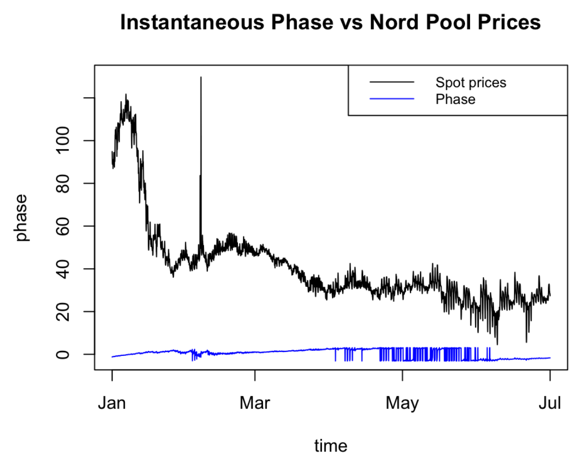

Figure 6.31: Phase and wave

Let us now analyse the time series between 15/04/2004 and 15/10/2004, where there were 3 extremely decreasing spikes prices, presented in Figure 6.32. The largest downwards spikes appeared in May, July and September with the lowest piece value of 3.84 EUR/MWh reached on 09/05/2004. The prices stayed below 40 EUR/MWh, which can be explained by the high temperature during those months

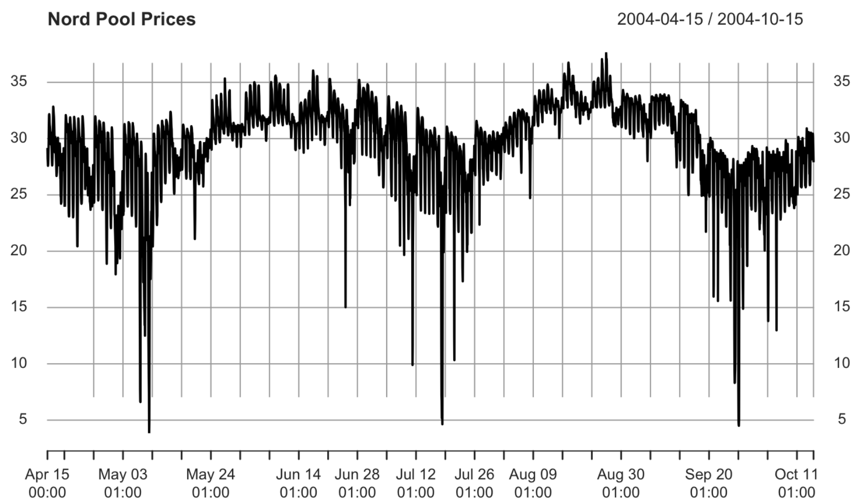

Figure 6.32: Time series of 15/04/2004 - 15/10/2004

The instantaneous amplitude in Figure 6.33 show downwards spikes which adapts with the sudden spike in the time series. The phase in Figure 6.34 and Figure 6.35 show the phase slips occurring around May, July and September, where the sudden decreases in prices were observed.

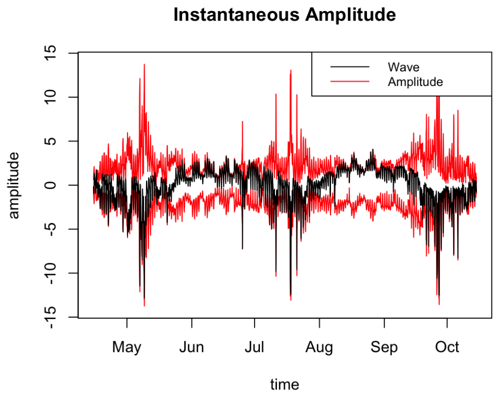

Figure 6.33: Amplitude

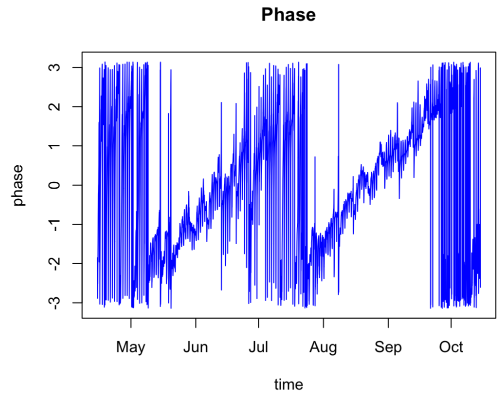

Figure 6.34: Phase

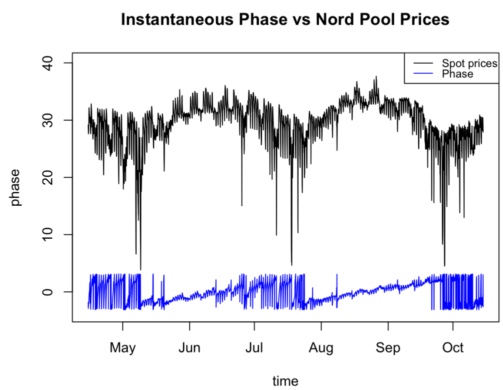

Figure 6.35: Phase and wave

From those observations of spikes we can, therefore, conduct that the instantaneous amplitude accommodates with the complex signal time series, and the phase slips provide us with the information about extreme price spikes occurring in the Nord Pool market prices.

Conclusions

In this paper, we explored the importance of signal processing in many application fields, and then we made a specified focus on financial signal processing to exact the information from the financial time series. Hourly Nord Pool market prices were used in this paper to analyse extreme events of the market. To understand the Nord Pool pricing, we also reviewed how does electricity market work, and observed how the electricity prices are affected by the demand of the commodity. We observed that extremely high spike prices appeared to occur mostly on the days with the low temperature, particularly at the beginning of the year.

Instantaneous amplitude and phase were extracted from analytic signal, which is derived by applying the Hilbert transform to a signal. Fourier Transform, which is an important application of decomposing signals, supports in obtaining analytic signal by removing unrequited negative frequencies. Since analytic signal is a complex function, we interpreted it in terms of instantaneous amplitude, and instantaneous phase.

After deriving instantaneous amplitude and phase of Nord Pool time series, we learned how amplitude adapts with a signal and discovered that there is a relation between the phase and price spikes. Analysing closely the high spikes of Nord Pool electricity prices, it is deduced that the phase slips seem to indicate the appearance of extreme events of prices.

Further studies could be considered on the discovering the space slips in advance of the extreme events to predict the movements of the electricity prices.

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