Financial Mathematics MSci Project MTH717U 2018/19

Analysis of daily returns for spot price market data

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Abstract

The purpose of this thesis is to analyse the daily returns of spot price market data. In particular, we will explore the area of financial risk associated with the data. This thesis will discuss a mixture of economic and mathematical stances on the subject. The data that we will look at is from the Nord Pool market. The time series of this data shows the spot price taken every hour from 00:00 on January 1st 1999 to 23:00 on January 26th 2007. We will first take a brief look at the time series of prices itself. Later discuss some comparisons between our hourly and daily returns. We then discuss the daily returns themselves in greater detail and apply a few financial risk measures to these results in order to evaluate the worthiness of Nord Pool electricity as an investment.

Acknowledgements

I would like to thank Dr Wolfram Just, he has been my supervisor during the process of writing this thesis and has motivated and has guided me throughout.

I would also like to thank the School of Mathematical Sciences for helping me along the way and giving me the mathematical and financial background necessary for me to write and research this thesis.

Finally, I would like to thank my year 9 maths teacher Miss Copus for changing my view on maths and teaching me to love it.

Declaration of original work

Declaration made: 03/04/2019
Student Declaration:
I, **Kristen Lee Sharp**, hereby declare that the work in this thesis is my original work. I have not copied from any other students' work, work of mine submitted elsewhere, or from any other sources except where due reference or acknowledgement is made explicitly in the text, nor has any part been written for me by another person.

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Chapter 1

Introduction

1.1 What is risk?

The Oxford English dictionary defines risk as "a situation involving exposure to danger" [17]. Every day we are subject to some kind of risk and as humans we have a natural instinct to avoid it. In general, our brains are quite good at assessing the environment we are in and warning us of any incoming risks so that we can escape them. For example, we see a car coming and step out of the road or we see some pink chicken and decide not to eat it. These simple examples are every day things. They are things that, as we age and learn, become natural to avoid. In this context, risk is seen as an immediate danger to us, but then we may ask how risk is defined across different situations.

In general risk is "a possibility of loss or injury" [14]. According to Hasbro, risk is "is a strategy board game of diplomacy, conflict and conquest for two to six players" [22]. In a workplace risk is defined mainly by the health and safety precautions put in place by a company. In this thesis we will examine financial risk.

"Financial risk is the possibility that shareholders or other financial stakeholders will lose money when they invest in a company that has debt if the company's cash flow proves inadequate to meet its financial obligations." [3]. In this thesis, we will look at the risk associated with the Nord Pool data. In particular, we will look at different risk measures used within the financial sector and will apply these measures to the data. The main object of this thesis is to look at the risk related to the daily returns of the spot prices, however we will also explore the differences between daily returns compared with hourly returns.

1.2 What are the different measures of risk?

In finance there are several methods used to measure risk. In general these methods are used on a portfolio of assets however in this thesis we will be applying them to Nord Pool and analysing the results.

The most basic measures of risk are the mean and variance of the price of an asset. The mean is an average of the stock price. This is calculated as $\frac{1}{n}\sum_{i=1}^{n} x_i$ where n is the number of data points and x_i is the ith data point.The variance is a measure of the spread of data. It is calculated as $\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ where the meanings of n and x_i are as above and $\bar{x}$ refers to the mean.This is a suitable calculation for the population variance, however, we do not have the entire population of Nord Pool spot prices. Although our value of n is especially high it still only represents a sample of the data available. This population variance is then "biased by a factor of $\frac{n-1}{n}$." [24]. Thus, it would be better to use the formula for the sample variance. The formula for sample variance is similar to that of the population variance except it is divided by a factor of n-1 rather than n itself. Hence, the formula is given by $\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$. "The use of the term n-1 is called Bessel's correction" [24]. In an ideal world and investor would wish for the mean (or expected value) of an asset's price to be high and it's variance to be low. The use of mean and variance as a risk measure comes together under The Mean-Variance Portfolio Theory or Modern Portfolio Theory which we will discuss later.

Another highly used measure of risk is Value at Risk (or VaR).VaR is "a statistic that measures and quantifies the level of financial risk in a firm, portfolio or position over a specific time." [7]. Mathematically, VaR is defined as "the smallest number y such that the probability that Y:=-X does not exceed y is at least $1 - \alpha$." [23]

$$VaR_\alpha(X) = F_Y^{-1}(1-\alpha)$$

[[23]. Here, α is the confidence level, which lies between 0 and 1, and F_Y is the cumulative distribution function of the random variable Y. For this thesis we will take two values for the VaR. Firstly, we will look at the VaR where we assume that our returns are distributed according to a normal distribution. Mathematically, we will take the mean and standard deviation of our returns and will use these to calculate the inverse of the CDF of the Normal distribution. Mathematically, we will be looking for r such that

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{r} e^{\frac{-(x-\mu)^2}{2\sigma^2}} = (1-\alpha)$$

here μ and σ are the mean and standard deviation of our returns, respectively. α is our confidence level and r will be our VaR. Secondly, we will look at the historical VaR for our data. For historical VaR we count the number, N, of data points we have, we work out $(1-\alpha)\%$ of N, we then find the point which corresponds to this % and this will be our historical VaR.

Another measure of risk used within finance is the expected shortfall. Expected shortfall is also known as the conditional VaR. At the $\alpha\%$ level "expected shortfall is the expected return on the portfolio in the worst $\alpha\%$ of cases" [19]. Mathematically, we can define it as

$$\mathbb{E}(S_\alpha) = -\frac{1}{\alpha}\int_0^\alpha VaR_\gamma(x)d\gamma$$

where VaR_γ is defined as above and α is the confidence level.

Chapter 2

Background of the Data

2.1 The Nord Pool Market

2.1.1 History of Nord Pool

Nord Pool runs the largest electricity market in Europe."More than 80% of the total consumption of electrical energy in the nordic market is traded through Nord Pool." [21]. Nord Pool provide day-ahead and intraday markets, which we discuss is sections 2.1.3 and 2.1.4, respectively. Both of these markets are important because they allow Nord Pool to stabilise the delivery of power and ensure that it is always available. Nord pool is the first ever multinational exchange for trading in electricity. Nord Pool trades across the Nordic region (Norway, Denmark, Finland and Sweden), the Baltic states (Estonia, Lithuania and Lativa), Germany and the UK.

2.1.2 The Electricity Market

A financial market is a market which is used to trade financial products such as stocks, bonds, commodities and derivatives. In general, when people think about "the market" the first thing we think of is the stock market. However, when looking at the Nord Pool data we are looking at trades made on the electricity market. The electricity market is a particularly interesting one. When a trader buys or sells a share on the stock market the thing that is actually being bought or sold is a share certificate. This is similar for a bond, and for a derivative it is a contract that is being bought or sold. Realistically none of these things are really considered tangible. The difficulty in understanding electricity trading is that it it neither traded as an equity nor a commodity, it is somewhere in between. Although electricity itself is not a tangible commodity like wheat or livestock, it is still delivered and made use of unlike a share certificate. The main difference between energy and other commodities is that it is used as it is produced. When trading wheat, tonnes and tonnes of the wheat itself can be stored, similarly with corn, livestock, oil and any other commodity. Electricity, on the other hand, cannot be stored at a wholesale level. The nature of electricity can therefore make it a very volatile thing to trade. We will see this later when we look at the Nord Pool data in 2.2.

2.1.3 The Day-Ahead Market

The day-ahead market requires a company (the buyer) to assess the amount of power it believes it will need the next day. Once the buyer has figured this out they then decide how much they are willing to pay for this power hour-by-hour. At the same time the company that produces the power- for example, a nuclear power plant- will assess how much energy they believe that they can produce for the next day and how much they are willing to sell it for hour-by-hour. Both the buyers and sellers make bids on the day-ahead market for how much they are willing to buy or sell the power for, respectively. The deadline for bids is 12:00 CET. A computer then calculates the exact price of the power and then the trades are settled. Power delivery begins at 00:00 CET the next day and is delivered hour by hour for the agreed price. The agreed price is set where the sell price and buy price meet.[16] This can be seen in figure 2.1.

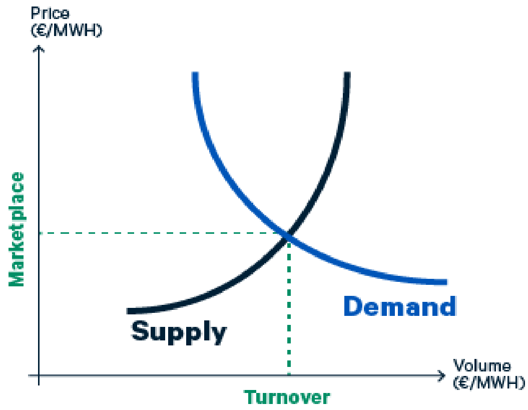


Figure 2.1: This graph shows where the curves of supply and demand meet. This is where the price of the power is set.[16]

2.1.4 The Intraday Market

The intraday market is a continuous market, meaning that trades can be made around the clock and buyers and sellers can trade volumes close to real time. Trades here are made up to an hour before delivery and the best prices are given to those who are first in line to trade. The intraday market is important because power is unreliable. If a power plant cannot deliver the amount of power promised, or, if it manages to produce more than expected, then the prices need to be adjusted accordingly.This is particularly important between 12:00, when the bids on the day-ahead market close, and 00:00, when the power is actually delivered.[15]

2.2 The Nord Pool Data

The data that we will use in this thesis is a set of spot prices. These spot prices are taken every hour from the 1st January 1999 to the 26th January 2007. Using Microsoft Excel I have sorted the data and created a time series of the spot prices. This is shown below in figure 2.2. In section 2.1.2 we found that the nature of electricity makes it a volatile thing to trade. This is seen quite obviously in figure 2.2. We see clear spikes where the price has quite drastically risen and then quickly fallen again. The presence of these spikes suggests that the variance in the prices may be quite high. We will explore what this later in 3.

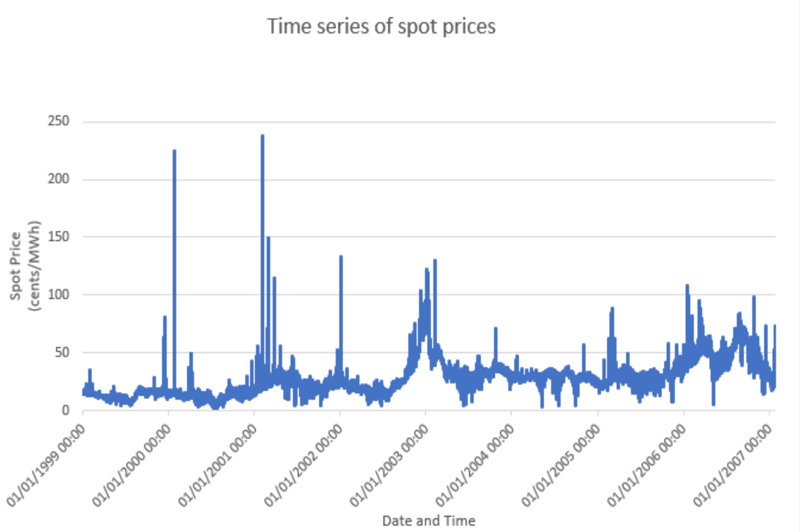


Figure 2.2: Time series of Nord Pool spot prices

2.2.1 The Hourly Returns

The Nord Pool data is given in raw spot prices, in order to compare the current price with the price from the previous hour we can calculate the hourly returns for the data. Hourly returns are given by

$$r_h = ln(\frac{x_n}{x_{n-1}})$$

here r_h is the hourly return, x_n and x_{n-1} are the nth and (n-1)th spot prices, respectively. I have used Microsoft Excel to calculate and graph the hourly returns, the graph is shown below.

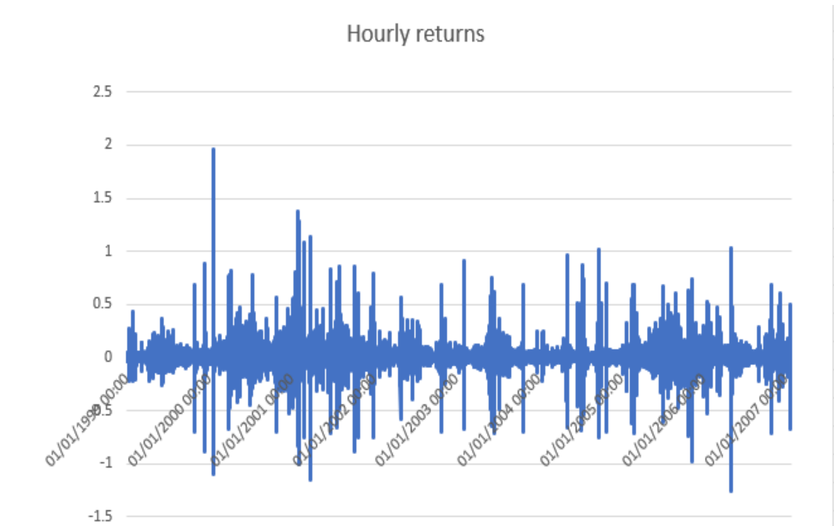


Figure 2.3: Hourly returns for the Nord Pool spot price data

2.2.2 The Daily Returns

In a similar fashion to the hourly returns, we can calculate daily returns using

$$r_d = ln(\frac{x_n}{x_{n-24}})$$

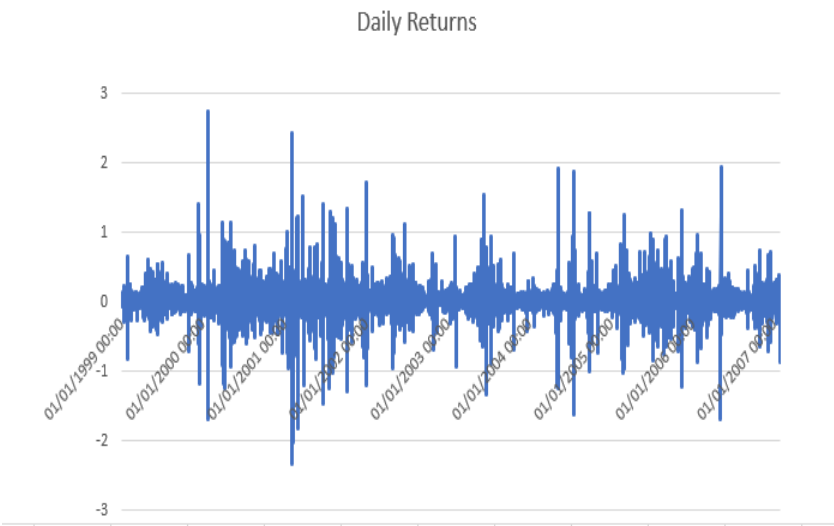


Figure 2.4: Daily returns for the Nord Pool spot price data

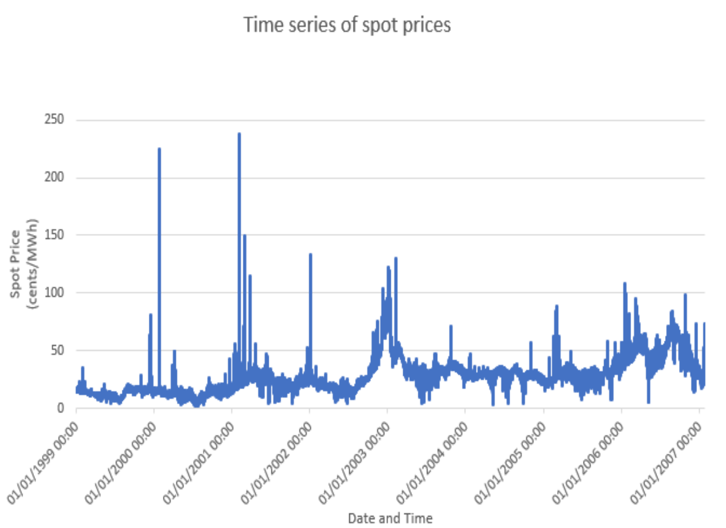
here r_d is the daily return, x_n and x_{n-24} are the nth and (n-24)th spot prices, respectively. Again I have used Microsoft Excel to calculate and graph the daily returns. I have included the graph below.

Chapter 3

The Nord Pool Data and Risk

3.1 Analysing the Spot Price Data

In section 2.2 we saw the time series of the Nord Pool data. In this graph we see a lot of volatility in



the spot prices. There is no apparent pattern and we see large spikes and drops throughout. In 1.2 we discussed the use of mean and variance as risk measures. Using Microsoft Excel I have worked out the mean and variance of the spot prices to be **27.47552196** and **216.4893081**, respectively. We can see from these numbers that the spread of the data is fairly large while the average price is relatively low. But what does this mean in terms of financial risk? This can be explained using Modern Portfolio Theory.

3.1.1 Modern Portfolio Theory

Modern Portfolio Theory is a theory that was proposed by Harry Markowitz in his paper "Portfolio Selection" [9]. The theory gives an idea on how to construct a risk-averse portfolio of assets based on their expected returns and market risk. The theory states that you can construct an **efficient frontier** of portfolios which give optimal returns based on a fixed level of risk. Below we see an illustration of the basic concept of the theory and how the efficient frontier is constructed and used.

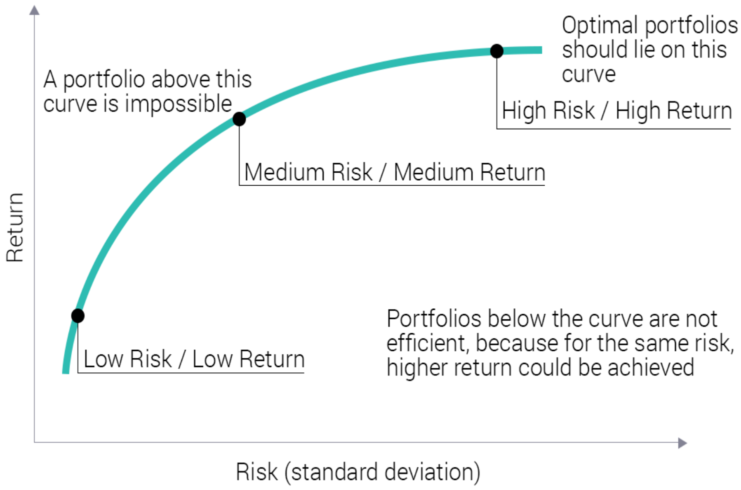


Figure 3.1: The Markowitz efficient frontier [10]

3.1.2 Modern Portfolio Theory and the Nord Pool Data

Clearly this theory requires an entire portfolio of assets and in this thesis we are only analysing the price of one single asset. Nevertheless, we can look at the image above and get an idea of where the Nord Pool data would be placed with regards to the Markowitz efficient frontier. Taking the square root of our above variance we get the standard deviation as **14.71357564**. In finance, the standard deviation of past spot prices is known as **Historic Volatility**. It is quite difficult to evaluate the significance of these numbers without any context. As we discussed in 2.1.2 electricity is infamous for it's high level of volatility. The easiest way to see this is to compare the volatility of the Nord Pool spot prices with the historic volatility of various equities and commodities. Below we see a table of different companies and the historic volatility of their stock prices between the 1st January 1999 and the 26th January 2007.

Company	Mean	Historic Volatility
Apple (AAPL)	3.67992	3.212612
Faroe Petroleum plc (FPM.L)	87.89744	30.79916
Google (GOOG)	158.2688	55.56904
Randgold Resources Limited (RRS.L)	491.2352	299.2769
British Petroleum (BP.L)	541.2705	78.68269
Nord Pool	27.47552196	14.71357564

Figure 3.2: Table of means and historic volatility for 6 companies including Nord Pool. All data was obtained from Yahoo Finance, 2019. [1]

Using this data and Microsoft Excel we can plot these means and standard deviations along with the Nord Pool data in order to compare. The plot of this data is shown below.

We can see clearly that in comparison to the other companies Nord Pool is relatively low risk, low price. In general a person investing in a portfolio of shares would be looking toward the low risk, high price side of this graph, which is not where Nord Pool falls. Statistically, however, this graph is not the best measure of comparison between these prices. We have discussed the mean and standard deviation of the prices themselves which is some sense is fine, but, "most business and economic time series are

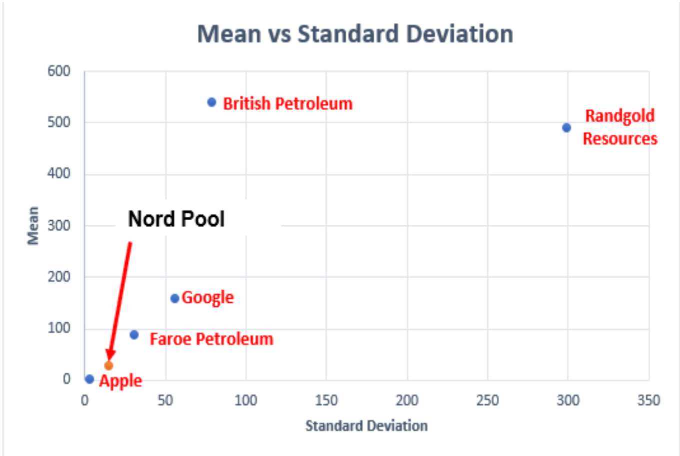


Figure 3.3: Graph of mean against standard deviation for the above companies

far from stationary when expressed in their original units of measurement"[13]. This means that in a mathematical sense the mean and standard deviation of the prices themselves are actually not well defined. In fact, as we saw in 3.1.1 Modern Portfolio Theory uses the expected returns rather than the mean of the prices themselves and therefore I have taken this analysis even further and I have used Excel to create an optimal portfolio for these data points. In order to create this optimal portfolio we first calculate the returns of our data for each of the companies including the Nord Pool data. The returns are calculated as $\frac{x_i}{x_{i-1}}$, we then use the average returns of the entire portfolio of assets to calculate the optimal portfolio. A table of our optimal portfolio results is shown below. The optimal portfolio is the

Optimal Portfolio					
Average Return	489.4072	500	510	520.0005	530
Standard Deviation	25.14117	25.17392	25.35825	25.82644	27.4913
Slope	19.46637	19.86182	20.1118	20.13443	19.27882
RRS	0%	0%	0%	0%	0%
GOOG	2%	0%	0%	0%	0%
BP	5%	4%	2%	0%	0%
AAPL	50%	50%	50%	43%	23%
FPM	44%	46%	48%	57%	77%
Nord Pool	0%	0%	0%	0%	0%

Figure 3.4: Optimal Portfolio of shares

turning point of the slope, in this graph the slope continues to increase up until the 4th portfolio, after which it drops. Hence for this combination of shares and commodities the optimal portfolio would consist of 43% in Apple, 57% in Faroe Petroleum and 0% in the rest of the companies including Nord Pool. This further supports the idea that Nord Pool electricity is a volatile and risky investment. Using this table of portfolios I was able to plot the Markowitz efficient frontier for our own data. This is shown below. This graph shows clearly what we discussed from the table above. After our chosen optimal portfolio we see the obvious drop in the slope of the efficient frontier. We also see that the Nord Pool data does not fall on this line. The optimal portfolio for this data was created using Microsoft excel. The method used on our data was a method found online[11]. Firstly, we work out the returns for each set of prices using the formula $\frac{x_i}{x_{i-1}}$. We then use the returns data to build a covariance table, we work out the covariance

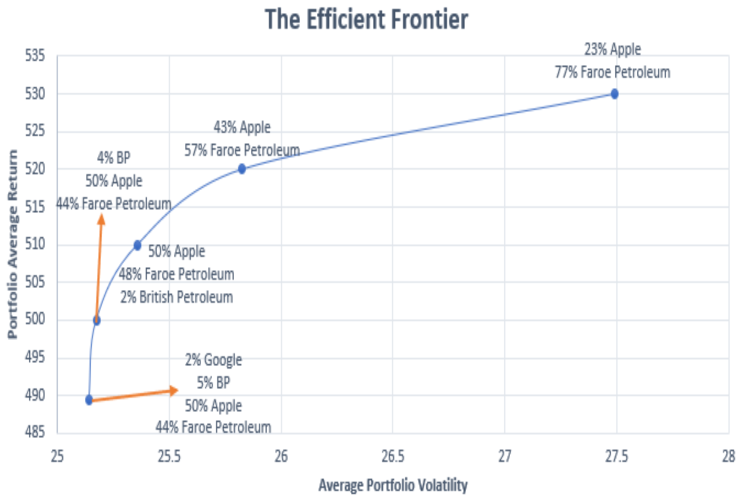


Figure 3.5: Markowitz efficient frontier for our optimal portfolio

of each of the pairs of assets using the excel formula COVARIANCE.S. we then arrange these numbers into a table, the table also includes the weighting of each of the assets in the portfolio. We then build a table of expected returns for each of the assets by averaging over their returns using the excel AVERAGE function. After this point we use the SUM PRODUCT excel function on the weights and covariances of each asset. We then have a cell with our target return for the portfolio and the standard deviation of the portfolio. We then create the actual optimal portfolio using Excel's "Solver" add-in several times until the slope reaches it's peak and then begins to decrease again. The exact method is given in more detail on quora.com[11]. Finally, we take the expected returns of each portfolio graphed against the standard deviation of the portfolio to give the graph of our efficient frontier. While this analysis gives us a good comparative standpoint there are a few things that should be considered. Firstly, this is a relatively small selection of assets. In practice a portfolio would usually be much larger than this, in particular for a bank or hedge fund a portfolio of assets may consist of hundreds or even thousands of assets as opposed to just six, and it is probable that several of these assets may be less favourable than Nord Pool. Secondly, electricity is very different from stocks and shares and almost falls into it's own category when it comes to trading because it is used as it is produced. Lastly, this model uses the returns rather than the logarithmic returns. In the next section 3.2 we discuss returns but in actual fact what we are referring to is the logarithmic hourly and daily returns. Using logarithmic returns essentially means that if we consider our prices to have a log-normal distribution then or log-returns will be normally distributed. This we will discuss in 3.2.1.

3.2 The Hourly and Daily Returns

In sections 2.2.1 and 2.2.2 we saw the hourly and daily returns of the spot price data. For this thesis we will focus predominantly on the daily returns but will refer to the hourly returns for some comparison. We will first compare some basic statistical measures for the hourly and daily returns. We will then look at the daily returns compared at 6pm and 2am and will discuss what this comparison means in terms of the risk of this asset. Finally, we will use the daily returns in the calculations of VaR and expected shortfall which we mentioned in 1.2 and we will use these to further evaluate the risk involved when buying electricity on the Nord Pool intraday market.

3.2.1 What do the Hourly and Daily returns tell us statistically?

We have already looked at the mean and variance of the spot prices themselves, as well as what these mean in 3.1.2. We can do the same for the hourly and daily returns. Below we have a table of the mean, variance, skewness and kurtosis of both the daily and hourly returns.

	Hourly	**Daily**
Mean	0.000007	0.000198
Variance	0.003491829	0.01589618
Skewness	1.930508913	1.188244702
Kurtosis	86.4285468	50.70323281

We see from this table that the mean and variance of the daily returns are seemingly both higher than the hourly returns while the skewness and kurtosis are lower. Linearity of expectations states that $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$. Under this notation, the mean of our hourly returns is $\bar{r}_h = \mathbb{E}[\frac{x_i}{xi-1}]$. Our daily returns are then given by $ln(\frac{x_i}{x_{i-24}}) = ln(\frac{x_i}{x_{i-1}}) + ln(\frac{x_{i-1}}{x_{i-2}}) + ... + ln(\frac{x_{i-23}}{x_{i-24}})$. Taking the expectation of this gives $\bar{r}_d = \mathbb{E}[ln(\frac{x_i}{x_{i-1}})] = \mathbb{E}[ln(\frac{x_i}{x_{i-1}})] + \mathbb{E}[ln(\frac{x_{i-1}}{x_{i-2}})] + ... + \mathbb{E}[ln(\frac{x_{i-23}}{x_{i-24}})] = 24\bar{r}_h$. This is reflected in our table since the mean of our daily returns is roughly 24 times the mean of our hourly returns. If the returns were independent then the variance would have the same additive property as the variance. This is not reflected in the table and thus we have correlations between returns.These correlations may be indications of a mean-reversion property of our time series and possibly some volatility clustering. We do see that our variance is higher for our daily returns which suggests that the data is more largely spread and experiences more spikes for the daily returns. We see that our skewness and kurtosis are lower for the daily returns than for hourly returns. This is somewhat intuitive. If the returns have a weak dependence then the Central Limit Theorem still holds and the returns should tend towards a standard normal distribution for longer periods.[18]. Hence as we go from 1 hour to 24 hours the skewness and kurtosis decrease. These are likely to tend closer to 0 if we were to increase the time periods further. "Skewness is asymmetry in a statistical distribution, in which the curve appears distorted or skewed either to the left or to the right. Skewness can be quantified to describe the extent to which a distribution differs from a normal distribution."[12].

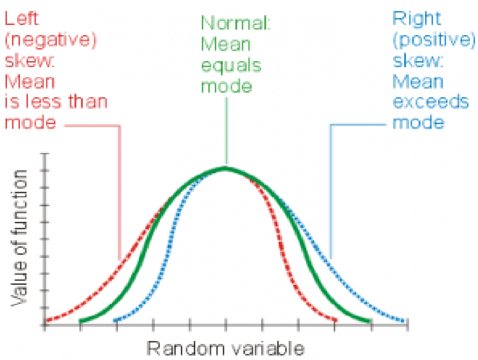


Figure 3.6: Diagram of skewness[12]

In our case the skewness for both the hourly and daily returns is positive and therefore the distribution of the returns is skewed to the right in both cases, however since the skewness of the hourly returns is slightly larger than the daily returns this suggests that the hourly returns deviate further from a normal distribution than the daily returns. A positive skew suggests that the distribution of the returns has larger right tails and therefore the occurrence of extreme events within the right tail are more probable. In our case this suggests that spikes in electricity prices are more probable than drops. Kurtosis is defined as the fourth moment of the standard normal distribution, similarly to skewness it is used as "a measure of the "tailedness" of a probability distribution" [20]. Given a random variable X we can calculate it's kurtosis using $kurt(X) = \mathbb{E}[(\frac{X-\mu}{\sigma})^4]$ where μ and σ are the mean and standard deviation of the distribution. The distributions of our hourly and daily returns are leptokurtic meaning that they have positive excess kurtosis. A distribution is considered leptokurtic if it's kurtosis has a value higher than 3, our hourly and daily returns both have kurtosis which exceeds 50. This, again, implies that the distribution has much fatter tails relative to the standard normal distribution. Mathematically this implies that the distribution decays at a slower rate. "In finance, fat tails often occur but are considered undesirable because of the additional risk they imply." [20]. The high skewness and kurtosis that occurs in both sets of returns further implies that the financial risk of the Nord Pool data is high.

3.2.2 Comparison of 2am versus 6pm

An interesting comparison to look at is the daily returns at specific hours of the day. I have chosen to consider the returns at 2am versus the returns at 6pm.

Below I have shown the graph of the daily returns at 2am. I find this graph particularly interesting

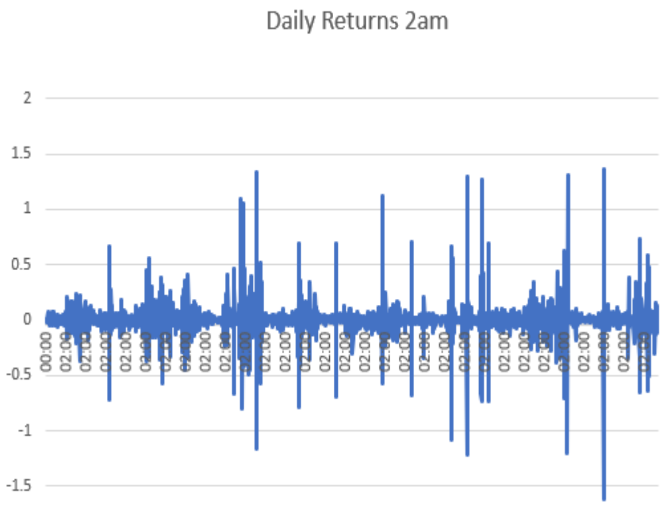


Figure 3.7: Daily returns for 2am

because these returns are clearly quite volatile. We may assume that in general returns on electricity prices at this time are more likely to be steady and relatively low seeing that we would probably anticipate lower electricity usage at 2am. For this data we can calculate the expected return to be **0.00027015** and the variance of the returns to be **0.015866828**. When we look at these numbers compared with the expectation and variance of the daily returns across the entire time scale we can see that in reality our data says quite the opposite of what we may expect. Our expected returns at 2am are actually higher than our overall expected return whilst the variance of the 2am returns remains roughly the same.

We can now look at the graph for the returns at 6pm. This graph is quite different from the 2am graph, large spikes are less frequent and we see far more volatility clustering. We can view volatility

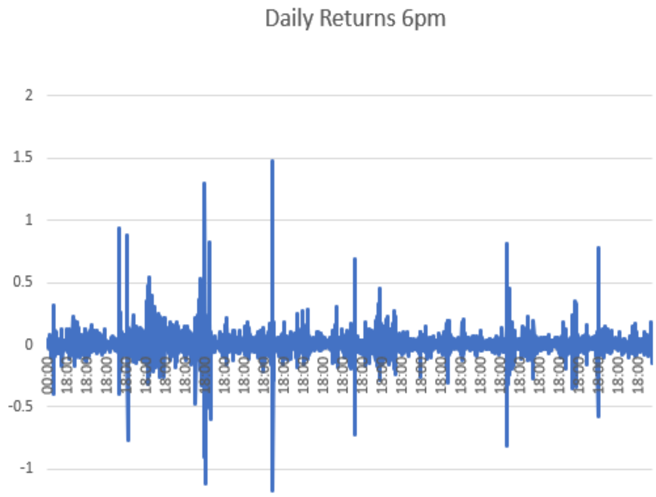


Figure 3.8: Daily returns for 6pm

clustering as "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes." [8].In other words, although asset prices and their returns are mathematically random, it is logical that if the returns are low they are unlikely to become drastically high and vice versa. Be that as it may, it is not sufficient to simply compare the graphs of the two sets of returns. The lack of large spikes in the 6pm data compared with the 2am data may lead us to make assumptions which are simply untrue. When we look at the actual numbers, we see that the variance of the 6pm returns is **0.015900568** which is actually higher than the variance of the 2am returns. Surprisingly, the expected return at 6pm is **0.000198402**. this means that if you invest in Nord Pool your return, on average, is likely to be higher at 2am than it is at 6pm, however your risk is likely to be slightly higher at 6pm in comparison to at 2am. We may choose to ponder why this could be, but realistically there are several factors that may contribute.

Looking directly at the two graphs it is interesting to see that some of the large spikes correspond to each other. Furthermore, taking a closer look at all of the graphs for daily and hourly returns we see a large spike about mid way through 2006. Focusing more closely at the data itself I have discovered that this spike corresponds to a volatile period on the 7th May 2006. I have then graphed the short period between May 6th at 00:00 and May 8th at 23:00. This graph is shown below. We can now see how, when packed together in a larger set of data, this series of events will become what seems to be a large spike. Examining the graph within a smaller range we see that the range in returns happens over a few hours. There is a large fall in daily returns after 00:00 on 07/05/2006, this corresponds in a drop in the price from 28.33 at 01.00 on 06/05/2006 compared with just 7.58 at the same time on 07/05/2006. We then see that later on our returns revert back to 0 and after 00:00 on May 8th the returns see a sharp increase corresponding to a lift in the price from 7.58 at 01.00 on 07/05/2006 to 25.58 at the same time on 08/05/2006. The exact cause of these fluctuations is very difficult to pinpoint, especially when looking at data that is over 10 years old. However, we can speculate that this fall and rise of prices is most likely due to variations in supply and demand. As discussed in section 2.1.2 electricity is not a commodity which is stored but, in general, is used as it is produced.This means that if too much electricity is being produced at a time when it is not necessarily in use the price is likely to drop as we saw on the 7th of May 2006. Similarly, if the demand for electricity is higher than it's supply, the price

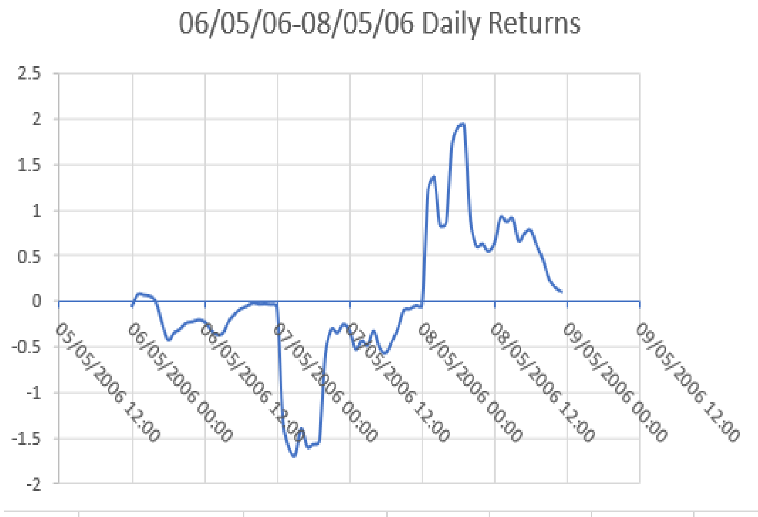


Figure 3.9: Graph of the daily returns from 06/05/2006 - 08/05/2006

tends to increase. An interesting mathematical implication of the link between supply and demand is the mean reversion property. In finance "mean reversion refers to the phenomenon that prices or returns will fluctuate around a long-term average level." [4]. Given that our average daily return is 0.000198, it makes sense that we see our returns often coming close to 0. This mean reversion property of assets is largely the basis of the "buy low, sell high" strategy of investors. The theory that the price of an asset will usually revert back to a long-term mean gives the impression that if the price is far from it's mean it is generally set to drop and vice versa. This, of course, has a big implication for financial market risk since investors are typically looking to make as much money as possible. Thus, if an asset has a larger average rate of return, then even when the return reverts back to it's mean your portfolio will be of larger value. Thus, we see again that with an average daily return of 0.000198, our Nord Pool data is not going to be the best investment.

3.2.3 Value at Risk and Expected Shortfall

In section 1.1 we briefly discussed the meanings of VaR and Expected Shortfall in financial terms and we saw a mathematical formulae for how they are calculated explicitly. In this section we will discuss the financial meaning of these two measures of risk in more detail and we will apply these measures to our Nord Pool data using Microsoft Excel, we will then discuss these numbers and what they actually mean in layman's terms. "Value at risk measures the largest loss likely (in the future) to be suffered on a portfolio position over a holding period with a given probability (confidence level)." [6]. In practice we take a given confidence level (say 95%) and calculate the largest loss that we are likely to incur over a short period (in practice this is typically 10 days) based on historic returns of the portfolio. Of course in our scenario we do not account for an entire portfolio, instead we are looking at just one asset. For the Nord Pool case I have calculated the 10 day VaR using a confidence interval of 95%. As discussed in section 2 I have used two different methods for calculating the VaR. The first method we will discuss is the calculation of VaR using the CDF of the Normal Distribution. The result that I got for the daily Normal Distribution VaR of the Nord Pool data was **-0.207382109** or approximately **-20.74%**, multiplying this result with $\sqrt{10}$ gives the 10 day Normal Distribution VaR as **-0.65579981** or roughly **-65.58%**. In layman's terms this means that in a 10 day period, we can say with 95% confidence that our loss will not exceed 65.58% and over 1 day our loss will not exceed 20.74%. For comparison we may wish to take into account the VaR of some other companies. I have looked at the VaR data for 5 other companies from a range of sectors. A table of these is below. As we can see from the table these comparative figures are far lower than

Company	1-Day VaR (%)	10-Day Holding VaR (%)
Apple	4.12	13.02858396
Facebook	3.14	9.929551853
Bioplast Pharma	7.44	23.52734579
Imperial Oil Limited	2.69	8.506526906
Electricite de France	1.74	5.502363129

Figure 3.10: Table of 1 and 10-Day VaR for various companies [2]

the values we received for our Nord Pool data. Even the highest 10-Day Holding VaR sits at roughly 42% lower than that of Nord Pool. In other words, Bioplast Pharma, who has the highest VaR of these companies, can say with 95% confidence that in a 10 day period you will not incur a loss of more than 23.53% if you were to invest in their shares. In perspective this is only 3.53% higher than the loss you may incur within only one day of trading Nord Pool electricity. However, there are several things to note. Firstly, our VaR is calculated from data that ranges over a large period of 8 years whereas it is difficult to say how much data is used by ycharts.com to calculate the VaR percentages in this table. Also, we must consider that Normal Distribution VaR is predicated on an assumption that the Nord Pool prices are normally distributed. We saw in section 3.2.1 that our skewness and kurtosis differ largely from what we would expect of a normal distribution and thus making this assumption leads to numbers which are essentially incorrect. A more accurate indication of the VaR for our data would be the Historical VaR. For our data we have N = 70727 data points. Taking α equal to 0.95, we get $(1-\alpha)$ as 0.05 (i.e. 5%). 5% of 70727 is 3536.35. We thus look at the 3537th smallest data point which is **-0.135341348** and the 3536th smallest data point which is **-0.135345726**. We then interpolate between the two to give a 1-day historical VaR of **-0.135344194** or approximately **-13.53%**. Again, we multiply with $\sqrt{10}$ to give a 10-day holding VaR of **-0.427995921** or approximately **-42.80%**. Simply put, according to our historical VaR, we can say with 95% confidence that in a 10 day period we will not incur a loss of more than 42.8%.Taking this smaller number in comparison with the companies in the table we see that the Nord Pool data still have a relatively large VaR, and is still nearly double the 10-day VaR of Bioplast Pharma. We can also see that this number is clearly much smaller than the number we calculated using the Normal Distribution VaR. Once more, we see a big indication that our returns are not distributed according to a normal distribution.

I mentioned earlier that we do not know how much data is used by ycharts.com, in general a good estimate of the VaR is based on roughly one years worth of data. Hence, if we pick a specific year we might get a slightly more accurate representation. In this case we can look at 1999 as the year for comparison. When taking the daily returns from 1999 alone we get a historical daily VaR of **-0.112800892** or approximately **-11.28%**. Our historical 10-day holding VaR using the 1999 data then comes out at **-0.356707741** or roughly **-35.67%**. This number tells us that on January 1st 2000, we could say with 95% certainty that we would not suffer a loss of more than 35.67% within a 10 day period. Again, this is a far smaller loss than our value of -65.58% according to the normal distribution mean and is also lower than our historical VaR calculated using all pf the data. We see that as we implement steps to make our VaR closer to a standard industry VaR it is gradually becoming far smaller.However, we can clearly see that the VaR for our Nord Pool data is still respectively much larger than for our 5 companies above.

We can now take a look at the Expected Shortfall or Conditional Value at Risk (CVAR).While VaR takes into account the maximum loss for a given α% of the returns, expected shortfall accounts for what is most likely to happen if the $(1-\alpha)$% of cases were to occur. We took our α as 95%, so our expected shortfall is the average loss that we would incur if the bottom 5% of cases were to actually occur. For our Nord Pool data this average amounts to **-0.271842045** or **-27.18%**. This means that if the bottom 5% of cases were to occur, the average loss that we would have is 27.18% over the period of a day. For a period of 10 days this average loss is **-0.859640026** or approximately -85.96%. In simpler terms this means that there is a 5% chance that we could incur a loss of 27.18% over the next day or a loss of 85.96% in 10 days time. This would obviously be a massive loss for any investor and despite it's low probability of occurrence is still something that we should definitely consider when evaluating Nord Pool as an investment.

3.3 A Note on Distributions

Thus far we have discussed our data in a combination of mathematical and financial ways. This section will take a more mathematical path where we discuss in further detail some conclusions that we have made regarding the distribution of our returns. Throughout this thesis we have sometimes assumed normality of our returns and on a few occasions the data analysis has proved us completely wrong. Firstly, we saw large numbers for skewness and kurtosis in section 3.2.1, we later saw a big difference between our historical and normal distribution VaRs in section 3.2.3. A good way to see what our distribution looks like is to plot it in the form of a histogram. Below is a graph where I have plotted the cumulative percentages of the data against the frequency of which they occur. This gives us the basic shape of the distribution of our returns. After graphing this distribution I used excel to simulate 70727

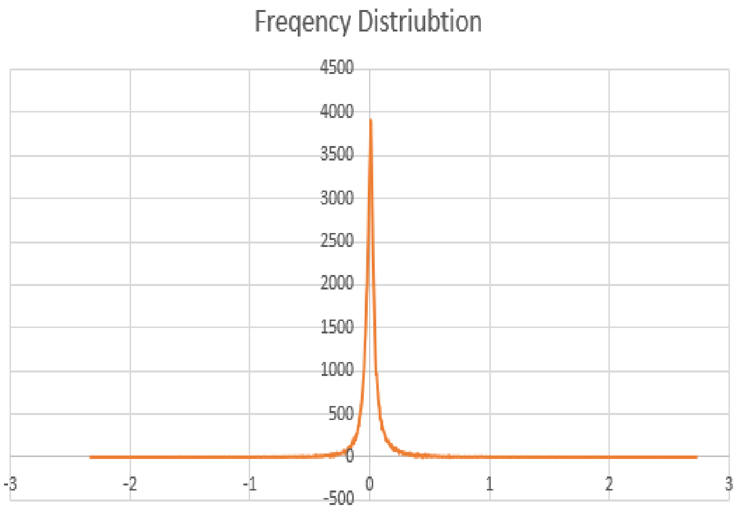


Figure 3.11: Distribution of daily returns

normally distributed random numbers with mean 0.000198 and variance 0.01589618. Hence I created a normal distribution with the same mean, variance and number of samples as our returns. I then graphed these numbers as a frequency distribution. I have then put together a combination of the two graphs for an easier comparison. Both graphs are shown below. Looking closely, this comparison graph shows

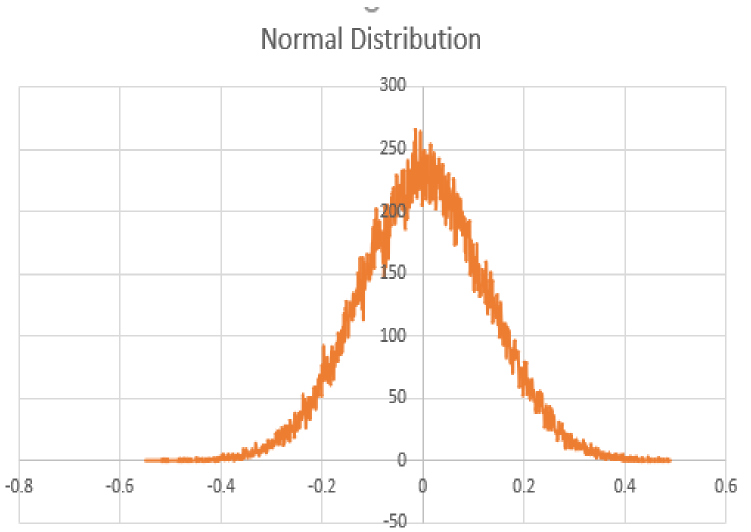


Figure 3.12: A normal distribution with mean 0.000198 and variance 0.01589618

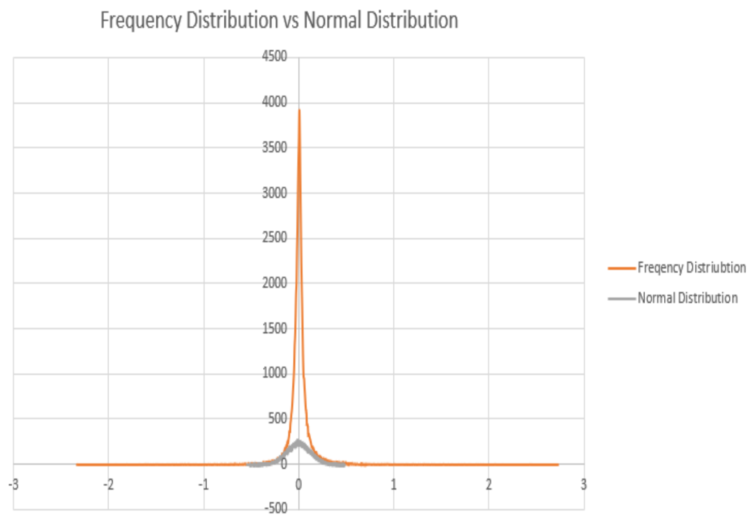


Figure 3.13: The distribution of our returns versus a normal distribution with the same expectation and variance

the positive skew of the data with respect to the normal distribution as we see our spike is to the left of the normal distribution curve. The most obvious difference is large spike which shows the indication of the excess kurtosis. This spike looks far closer to a Laplace distribution than a Gaussian distribution, however even the Laplace distribution has an excess kurtosis of only 3 in comparison with our daily returns which has a massive excess kurtosis of approximately 50.7, which we can see is obviously much larger. This large deviation from a normal distribution can make a large difference to a number of results within financial risk analysis, in particular we encountered the large difference in VaR calculations for the normal distribution VaR and the historical VaR. Modern Portfolio theory in section 3.1.1 is also predicated on the assumption that asset prices are subject to a normal distribution, and it is obvious that in our case the Nord Pool data does not follow this trend. An extension to Modern Portfolio Theory is the Capital Asset Pricing Model (CAPM) which recognises that a portfolio is subject to two different types of risk, the first is systematic (or market) risk which is non-diversifiable and the second is unsystematic risk which, in theory, can be reduced by having a portfolio of diversified assets. The systematic risk is quantified by the portfolio's beta. This is a widely used model for quantifying and reducing risk, however it is also a model which assumes that asset returns follow a normal distribution. This assumption of normally distributed returns is one that arises in several areas of finance and is something that we will consider further in our conclusions.

Chapter 4

Conclusions

The aim of this thesis was to look at the daily returns of our spot prices and do a financial risk evaluation taking the returns as a financial asset. Of course this is slightly abstract. Since we established that electricity is not a commodity which can be stored it is logical that businesses invest in electricity in order to use it rather than to hold it as part of a portfolio. Nonetheless, our time series of prices as well as their hourly and daily returns can be treated as a financial instrument and thus are subject to financial risk. It must still be considered whether all of the models we used are entirely suitable for the electricity market. Although portfolio theory is highly respected and often used on various financial instruments, its suitability in the context of an electricity market may be called into question. In practice Modern Portfolio Theory is used within the energy sector, but there are many papers and studies "aimed at improving the capacity of the model and adjusting it to the reality of the electricity market." [5] Which therefore suggests that there are some adjustments that need to be made to this model in order to consider it viable for evaluating risk within electricity markets.

We had two perspectives on our analysis, a mathematical stand point and a financial standpoint. Both of these perspectives suggested that, as an investment, the Nord Pool electricity market was not a great one in terms of the risk involved. In general the returns were low yield yet highly volatile with all of our risk measures coming up high against other assets.It has to be considered though that our risk measures as well as many other risk measures and financial models make several assumptions which simply are not true with regards to the Nord Pool data. A largely inaccurate assumption was the assumption of normality of our returns. Although, as we discussed, this distribution would most likely tend towards a normal distribution for a longer length of time.

In order to take this thesis and analysis further it would be interesting to explore the distribution of our returns even further. In particular, how do our daily returns fit with assumptions from other financial models used in industry? Do our returns follow a Geometric Brownian Motion as assumed under the Black-Scholes model? Or do they follow closer to a jump diffusion model like the one suggested by Robert Merton in 1998? We could also take the analysis from modern portfolio theory further and explore some of the methods which have been suggested in research for fitting this model more closely to our Nord Pool data. These are questions which would, of course, require further research and analysis of our data, but would be imperative to a complete and in depth financial analysis of this data.

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