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IDENTIFICATION OF BUBBLES AND CRASHES IN FINANCIAL MARKETS: COMPARISON BETWEEN GEOMETRIC BROWNIAN MOTION AND REAL DATA SETS

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ABSTRACT

The aim of this dissertation is to analyse what happens in financial markets in order to turn a bubble into a crash. The core of this project is about a time-series analysis of stock crashes recently proposed by Tobias Preis and H Eugene Stanley regarding volatility close to switching points.

I have implemented numerically this model in C++ and I have tested it both with real data like indices prices and with a simulated process like Geometric Brownian Motion. Then, I have compared the conclusions of this model to the definitions of bubble and crash that are widely used in finance, trying to link the results of this study to the behaviour of investors in financial markets during a crisis.

CONTENTS

INTRODUCTION

This dissertation is about bubbles and crashes in financial markets and it can be thought as made by three main sections: a theoretical one in which these phenomena are shown according to the classical economic literature, another theoretical one that is about the contribute of physicists and mathematicians on these topics and in particular about the work made by Tobias Preis and H Eugene Stanley and a practical one in which I have tested the model given by Preis and Stanley.

In the first section (Chapter 1), bubbles and crashes are not described as two different and uncorrelated events, but they are presented as two parts of an entire phenomenon, that is a crisis. For this part I have used as source the paper “Bubbles, Financial Crises and Systemic Risk” written by Markus K. Brunnermeier and Martin Oehmke and I have explained these two phenomena according to the definition given by Hyman Minsky.

Then, in the second section (Chapter 2), this dissertation focuses on the contribute of “econophysics” in finance and economics and in particular on the study provided by Tobias Preis and H Eugene Stanley. These authors have written a paper regarding switching phenomena in financial market fluctuations, where they have found that volatility increases just after a switching point and then it returns to its previous values. The conclusion of this paper is that crashes in financial markets are not outliers, but they are a consequence of the formation of increasing and decreasing trends. This is a strong conclusion, therefore, my purpose in this dissertation is to analyse and test the model provided by Preis and Stanley and to understand if it holds and if its conclusion agrees with the one given by Brunnermeir and Oehmke, based on Minksky's work.

The third section is the largest of this dissertation and it comprehends Chapter 3, Chapter 4 and Chapter 5. I have tested the above mentioned model both with real data and with some simulations of the evolution of stock prices (I have used the stochastic process known as Geometric Brownian Motion), in order to understand whether it holds or it does not. I have written a program with C++ which generates paths of Geometric Brownian Motion. Then I have studied these paths and real data using another program written by me to analyse the volatility in financial markets as Tobias Preis and H Eugene Stanley have done in their paper. The outputs of this program are then plotted with Wolfram Mathematica and these graphs are described in the dissertation.

In the final chapter (Chapter 5) there are considerations and conclusions regarding Preis and Stanley's model.

CHAPTER 1

BUBBLES AND CRASHES

Crises have always been an important feature of financial markets through years, but, even though they have had and they still have a huge impact on trader's gains and losses, nobody has come out with an appropriate model for this kind of phenomena yet and they remain some of the most fascinating and dangerous topics in finance.

Everybody knows the global financial crisis of 2007-2008 and the American economic crisis of 1929, the two most famous crises in financial markets which have spread through all other economic sectors and have become global issues. I will keep them as examples through all this chapter, even though in financial markets one can find smaller bubbles and crashes with less devastating consequences very commonly.

We can think about crises as processes made by two phases: one in which bubbles grow and one in which we face crashes (bubbles burst). These two phases have been called respectively the run-up phase and the crisis phase by Markus K. Brunnermeier and Martin Oehmke in their paper "Bubbles, Financial Crises and Systemic Risk". From this definition therefore it is possible to notice that bubbles and crashes are not completely uncorrelated events, but they are actually two sides of the same coin [1]. Anyway I am going to discuss them in detail separately.

1.1 BUBBLES

A sustained mispricing of assets is called a bubble. Generally it starts with the introduction of some innovation that makes assets prices raise. Then, somehow, investors become overconfident about this innovation and they make assets prices increase even further. At this point a bubble has been created: in fact assets prices have grown more than they should have, creating a gap between their real price and their fair price.

During the whole run-up phase the bubble keeps growing, making the imbalance bigger and bigger; in fact more and more investors are influenced by markets and they buy assets with the feeling that their prices will keep growing in the future.

All this process that I have briefly described is split in 5 phases by Hyman Minsky [1]:

1. the displacement phase, in which expectations of greater profits and economic growth are results of an innovation;
2. the boom phase, during which investments increase, credit increases (investors buy on margin) and volatility is usually low;
3. the euphoria phase, in which overpriced assets are widely traded by investors in the markets; during this phase prices increase sharply and investors start thinking that there might be a bubble, but they are still confident that they can sell assets at a higher price in the very near future;
4. the profit taking phase, in which sophisticated investors start selling their assets and taking their profits;

5. the panic phase, in which most of the investors sell their assets and consequently their prices drop; this is the start of the crisis phase and therefore I have decided not to include it in the description of bubble, but in the one regarding crash.

Before the crash of 1929 in the USA, there had been a run-up phase in which bubbles had grown both in the stock and in the real estate market during the whole 20s (especially from 1927 to 1929). The bubble in the stock market was credit-financed: investors bought on margin [1], it means that they were so confident about market conditions that they borrowed money in order to invest in the market.

Even in the crisis of 2007-2008 bubbles had spread both in the stock and in the real estate market [1]. This time the innovation that let bubbles grow was the introduction of a new form of mortgage securitization and the diffusion of derivative products such as CDOs (Collateralised Debt Obligation).

1.2 CRASHES

At some point, for some reason, investors understand that they were not so right in their predictions and they start selling assets, generating panic in the markets and making assets prices fall drastically. That is a crash: the specific changing point in which the majority of investors turn from buyers and holders of assets to sellers is known within economic literature as Minsky moment [1] and it is the beginning of the crisis phase.

The gap between the real and the fair prices that has been created and has widely grown during the whole run-up phase suddenly materialises and makes victims among investors. Generally it is not easy to understand what the trigger of a crash is and sometimes it is not even important to understand what it is; in fact even small news can lead to a crash due to amplification. For instance, in the crisis of 2007-08 the triggering event has been identified in the subprime mortgage market, but even though the subprime market represented only about 4% of the overall mortgage market, this crisis has spread through all other sectors [1]. Investors influence each other with their behaviours and they create a domino effect by which everyone sells assets. As a consequence assets prices sharply decrease and the crisis spreads through other sectors. Furthermore if sophisticated traders are sure that the prices of these assets will fall in the near future, they can speculate short-selling. It means that they borrow assets from some owners, they immediately sell these assets in the market and they will buy them back later returning them to the person they borrowed them from. It is obvious that these traders will have a gain if the price at which they have sold assets is higher than the price at which they have bought them back, so they want the price to decrease during this period. They can therefore speculate during a crash because they are pretty sure that prices will fall. Of course, with their behaviour, these traders will make prices decrease even deeper.

Usually governments and central banks have to intervene with bailout plans and regulations in order to make a crisis end, to let economy rise once again and to avoid similar cases in the future.

CHAPTER 2

PREIS AND STANLEY'S MODEL

Tobias Preis and H Eugene Stanley are two physicists who have applied their concepts and their knowledge to economics, in particular to the current economic crisis. Doing this, they have joined a quite new academic discipline called "econophysics" [2], in which concepts of statistical physics are applied to economics in order to discover some new models to describe phenomena in financial markets.

Physicists are not the first to contribute to these topics; in fact mathematicians have worked on this field for a while as well. The first important result of a mathematician in finance came in 1900, when Louis Bachelier discovered that returns on share prices are Gaussian distributed, with few chances to see very large fluctuations in prices (to our purpose it means that the probability to see a crash is very small). This model is known as "random walk" (sometimes "drunkard's walk") because share prices are assumed to increase or decrease randomly by an amount that has a characteristic value (it has a normal distribution with parameter mean and variance) [2].

In more recent times, econophysicists can use a massive quantity of financial data for their analyses and, thanks to that, they have discovered that Bachelier's Model is not that good in describing the nature of returns in share prices. To be more precise, they have found that real data have fatter “tails” than the Gaussian distribution, so the probability to see very huge changes in share prices (and therefore crashes as well) is higher than in the “random walk” given by Bachelier [2].

Paramaswaram Gopikrishan and Vasiliki Plerou found in the late 1990s that returns on share prices do not follow a Gaussian distribution, but they are better described by an “inverse quartic power law”. This law is very consistent in describing the probability of very large movements that happen once every few decades; for instance, events corresponding to 100 standard deviations with a Gaussian model have a probability to occur of just about 10^{-350}, while with the inverse quartic power law they have a likelihood of 10^{-8} [2]. It means that large movements can happen slightly more frequently in an inverse quartic power law model rather than in a Gaussian model, even though this difference is not that huge.

The inverse quartic power law is a good model, but it has a big weakness: it does not take into account the time when transactions are made. Individual price changes were just put in the bins of the histogram and the time ordering of the data was not considered [2]. Preis and Stanley, in their model, have therefore developed this aspect of the analysis.

2.1 PREIS AND STANLEY’S MODEL: BRIEF EXPLANATION

Tobias Preis and H Eugene Stanley have analysed huge data sets consisting of three fluctuating quantities: the price of each transaction, the volume of each transaction (the number of exchanged shares in each transaction) and the time between each transaction and the next [2].

Their work is focussed on finding out whether it is possible or not to describe market behaviour near “switching points” in data by some general laws. A switching point is a point in which, given a defined delta of time (Δt), a certain trend ends and the opposite trend starts. These points are commonly known as local minima and local maxima. A trend that goes from a local minimum to a local maximum is called “uptrend”, while a trend that goes from a local maximum to a local minimum is a “downtrend” [2].

They have analysed a database containing 13,991,275 German DAX Future transactions recorded with a time resolution of 10 msec in order to describe “microtrends” and a database containing 2,592,531 daily closing prices of stocks in the S&P500 in order to describe “macrotrends” [3].

The first step was to rescale all these data, because the time between switching points can vary hugely; they have “renormalised” setting $\varepsilon = 0$ at the beginning of a trend (uptrend or downtrend) and $\varepsilon = 1$ when the trend ends [2]. Hence in an uptrend $\varepsilon = 0$ corresponds to a local minimum and $\varepsilon = 1$ to a local maximum, while in a downtrend $\varepsilon = 0$ is a local maximum and $\varepsilon = 1$ is a local minimum. They have then analysed all these trends from

$\varepsilon = 0$ to $\varepsilon = 2$ and it means that they have taken into account all the returns in the trend and all the ones just after it.

Their analysis revealed striking scale-free behaviour of volatility after each switching occurs [3]: the volume of each transaction sharply increases at the end of a certain trend, while the time interval between each transaction drops [2].

The result of this analysis is therefore that catastrophic bubbles occurring on large time scales are no outliers, but they are single representatives caused by the formation of increasing and decreasing trends on different timescales; in fact the formation of increasing and decreasing trends is scale-free in that the same law holds over nine orders of magnitude (from 10 ms to 10^2 days) [2-3].

2.2 DATA SETS

In their model, Tobias Preis and H Eugene Stanley have used two different database.

For the first analysis they have used a time series of the German DAX Future contract (FDAX) [3]. A future contract is an agreement to buy or sell an asset (called underlying) at a certain time in the future for a certain price [4]. In our case the underlying asset is the DAX index, which contains the performance of the 30 best German companies. This data set is made by a time series of 13,991,275 transactions of three disjoint three month periods and it contains the transaction prices, the volumes and the corresponding time stamps [3]. In this data set they have data for different transactions every 10 ms, so it is useful to analyse microtrends.

For the second analysis, focussed on macrotrends, they have used a time series of daily closing prices of all the stocks contained in the S&P500 index, that is an index containing the best 500 stocks traded in the USA. In order to avoid the effect of inflation over a long time period like that, they have used the logarithm of stock prices instead of the simple closing prices [3].

2.3 RESCALING PROCESS

Let p(t) be a discrete variable representing the transaction price of trade t. A transaction price p(t) is a local maximum $p_{max}(\Delta t)$ of order Δt if there is no higher transaction price in the interval $[t - \Delta t , t + \Delta t]$, while it is a local minimum $p_{min}(\Delta t)$ of order Δt if there is no lower transaction price in the interval $[t - \Delta t , t + \Delta t]$ [3].

Let t_{min} and t_{max} be respectively the time at which we face a local minimum and a local maximum. For an uptrend (from local minimum to local maximum), the renormalised time scale ε is given by [3]:

$$\varepsilon(t) \equiv \frac{t - t_{min}}{t_{max} - t_{min}} \qquad (1)$$

with $t \geq t_{min}$.

While for a downtrend (from local maximum to local minimum), ε is given by [3]:

$$\varepsilon(t) \equiv \frac{t - t_{max}}{t_{min} - t_{max}} \qquad (2)$$

with $t \geq t_{max}$.

At this point, in an uptrend $p(\varepsilon = 0) = p(t_{min})$ and $p(\varepsilon = 1) = p(t_{max})$, while in a downtrend $p(\varepsilon = 0) = p(t_{max})$ and $p(\varepsilon = 1) = p(t_{min})$.

Now Tobias Preis and H Eugene Stanley proceed to analyse every trend in the interval [0,2] of ε, so that they have trends both just before and after the critical value where $\varepsilon = 1$.

2.4 VOLATILITY ANALYSIS

Now the authors focus on fluctuations. They have used the local volatility $\sigma^2(t)$, which is given by [3]:

$$\sigma^2(t) = \big(p(t) - p(t-1)\big)^2 \qquad (3)$$

for t > 1.

In their analysis they have studied the mean volatility $\langle\sigma^2\rangle(\varepsilon, \Delta t)$ normalised by the average volatility $\bar{\sigma}$, where for uptrends:

$$\langle\sigma^2_{pos}\rangle(\varepsilon, \Delta t) = \frac{1}{N_{pos}(\Delta t)} \sum_{i=1}^{N_{pos}(\Delta t)} \sigma_i^2(\varepsilon) \qquad (4)$$

and

$$\bar{\sigma}_{pos} = \frac{\varepsilon_{bin}}{\varepsilon_{max}\Delta t_{max}} \sum_{\varepsilon=0}^{\varepsilon_{max}/\varepsilon_{bin}} \left(\sum_{\Delta t=0}^{\Delta t_{max}} \langle\sigma^2_{pos}\rangle(\varepsilon, \Delta t) \right) \qquad (5)$$

while for downtrends:

$$\langle\sigma^2_{neg}\rangle(\varepsilon, \Delta t) = \frac{1}{N_{neg}(\Delta t)} \sum_{i=1}^{N_{neg}(\Delta t)} \sigma_i^2(\varepsilon) \qquad (6)$$

and

$$\bar{\sigma}_{neg} = \frac{\varepsilon_{bin}}{\varepsilon_{max}\Delta t_{max}} \sum_{\varepsilon=0}^{\varepsilon_{max}/\varepsilon_{bin}} \left(\sum_{\Delta t=0}^{\Delta t_{max}} \langle\sigma^2_{neg}\rangle(\varepsilon, \Delta t) \right) \qquad (7)$$

where $N_{pos}(\Delta t)$ and $N_{neg}(\Delta t)$ are respectively the number of positive and negative trends of order Δt, and $\sigma_i^2(\varepsilon)$ is the local volatility at position ε in the i-th trend [3].

Preis and Stanley have found that both for the German DAX future contract time series and for the S&P 500 index time series there is a jump in the volatility when $\varepsilon = 1$, or, in other words, when a new local switching point is reached.

In particular the jump is bigger for downtrends than for uptrends [3]. For a downtrend, $\varepsilon = 1$ corresponds to a local minimum: just before the minimum is reached, investors who have a long position are facing a loss, so they are driven by panic and they sell their assets in order to stop their losses, while investors who are in a short position sell their assets as well in order to maximise their profit; just after the minimum, assets prices start to increase, in fact, other market participants, who were monitoring the situation, decide that this is the right moment to buy these assets. For an uptrend, when $\varepsilon = 1$ a local maximum is reached: at this point investors can be driven by expectations of even higher prices to buy more assets, but, on the other hand, they can be driven by fear to maximise their profit and therefore to sell their assets making their prices fall just after the local maximum.

The evolution of the volatility just before and after a switching point is consistent with a power law scaling behaviour [3]. The authors have used the volatility aggregation $\sigma^{2*}(\varepsilon)$, that is the mean volatility averaged for layers from Δt_{cut} to Δt_{max}, and they have found that

$$\sigma^{2*}(|\varepsilon - 1|) \sim |\varepsilon - 1|^{\beta_{\sigma^2}} \quad (8)$$

For the DAX future contract β_{σ^2} is 0.01 before a local maximum and -0.3 after it, while it is 0.04 before a local minimum and -0.54 after it. For the S&P 500 index β_{σ^2} is -0.05 before a maximum and -0.4 after it, while it is -0.09 before a minimum and -0.5 after it [3].

2.5 VOLUME ANALYSIS AND INTER-TRADE TIME ANALYSIS

Tobias Preis and H Eugene Stanley have analysed the numbers of contracts traded in each individual transaction, the volume $v(t)$, and the behaviour of the inter-trade times $\tau(t)$ as well, in order to verify the universality of the results obtained for the volatility analysis, but these parts of their study are not included in this dissertation.

Anyway the results of these two analyses are consistent with the results of the volatility analysis.

CHAPTER 3

GEOMETRIC BROWNIAN MOTION

In finance one of the most common models to simulate the behaviour of stock prices is the Geometric Brownian Motion, that is a construction from a simpler stochastic process called Brownian Motion.

I have decided to test the model provided by Tobias Preis and H Eugene Stanley with Geometric Brownian Motion to see whether their result deals with the stochastic nature of stock prices or not.

3.1 BROWNIAN MOTION

The name Brownian Motion (BM) is due to Robert Brown, the botanist who first observed this kind of physical phenomenon, but the model was implemented by Norbert Wiener and that is why it is also known as Wiener Process.

A stochastic process $(B(t) : t \geq 0)$ is called Brownian Motion if it is characterised by the following conditions:

1. $B(0) = 0$;
2. $B(t)$ is continuous;

3. B(t) has independent increments, for all times $0 \le t_1 \le t_2 \le \ldots \le t_{n-1} \le t_n$ all increments $B(t_n) - B(t_{n-1})$, $B(t_{n-1}) - B(t_{n-2})$, ..., $B(t_2) - B(t_1)$, $B(t_1) - B(0)$ are independent each other;
4. All increments B(s) – B(t), where $s > t \ge 0$, are normally distributed with mean 0 and variance s – t.

The 1 point pdf of a Brownian Motion is [5]:

$$f(x,t) = \frac{1}{\sqrt{2\pi t}} exp\left(-\frac{x^2}{2t}\right) \qquad x \in R \quad (9)$$

and its transition pdf is:

$$f(x,y,t,s) = \frac{1}{\sqrt{2\pi(t-s)}} exp\left(-\frac{(x-y)^2}{2(t-s)}\right) \qquad (10)$$

Hence it is clear that a Brownian Motion is normally distributed B(t) ~ N(0,t).

3.2 GEOMETRIC BROWNIAN MOTION

A Geometric Brownian Motion (GBM) with drift parameter $\mu \in R$, volatility parameter $\sigma > 0$ and starting parameter S_0 (the initial price of the stock) is a stochastic process given by the following formula [6]:

$$S(t) = S_0 exp\big(\mu t + \sigma B(t)\big) \qquad (11)$$

where B(t) is a Brownian Motion.

In my analysis I have used $S_0 = 100.0$, $\sigma = 0.01$ and $\mu = \sigma^2/2$. The drift term is set like this in order to compensate for the noise induced by drift. Furthermore I have simulated daily prices only, so that these results are consistent with the onse regarding real data.

3.3 PREIS AND STANLEY'S MODEL APPLIED TO GBM

I have tried to apply the model provided by Tobias Preis and H Eugene Stanley to some paths of Geometric Brownian Motion.

I have used C++ both to generate the paths and to analyse the volatility close to their switching points (both these codes are attached at the end of the dissertation respectively as Appendix 1 and Appendix 2). My analysis is slightly different from Preis and Stanley's one, in fact I have used the local volatility described by Formula (3) while they have used the mean volatility described by Formula (4) for uptrends and Formula (6) for downtrends, but the aim of this analysis is the same and it should give the same kind of results.

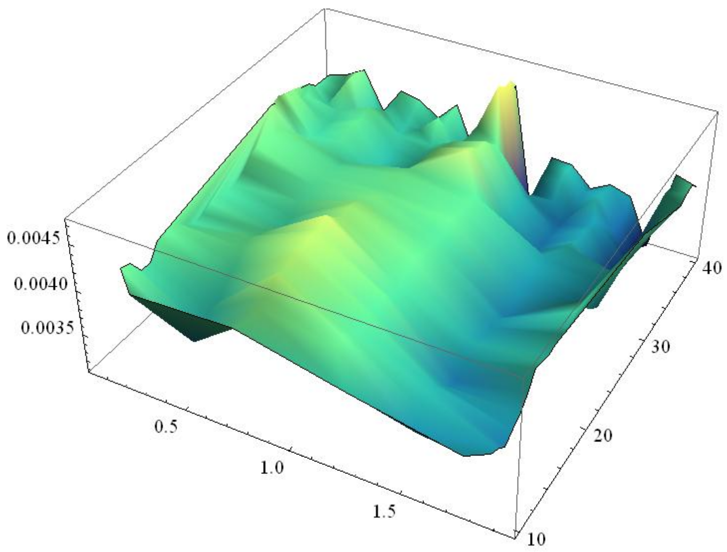
The outputs obtained with C++ are then plotted in 3D with Wolfram Mathematica and coloured graphs are given as well. In the coloured graphs low volatility is represented in blue, medium values are green and high volatility is represented in yellow.

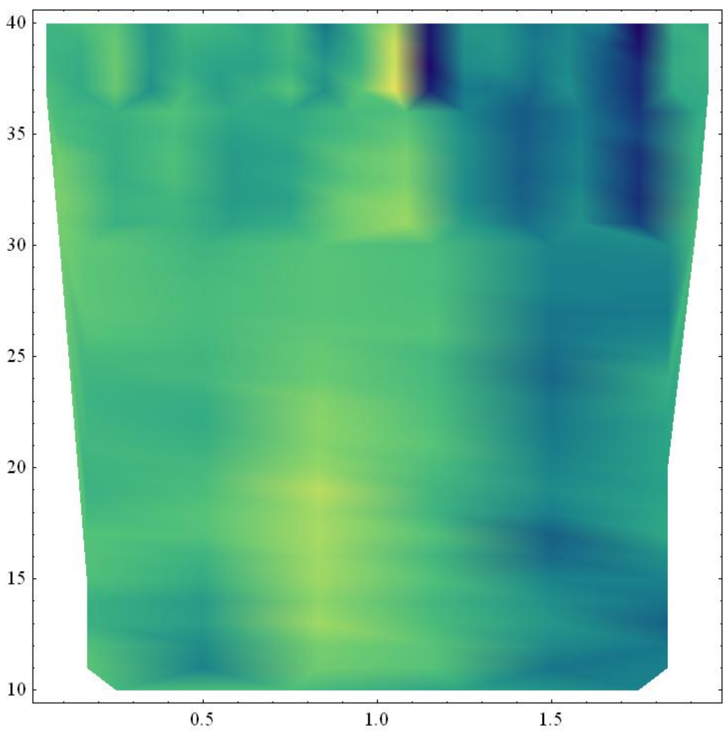
According to the outputs of my program, it is quite clear that there is a peak in the volatility for $\varepsilon = 1$ both for uptrends and for downtrends. The results are not as clear as the ones found by Preis and Stanley, but these outputs are not completely random and it is possible to observe a trend to have a peak close to the switching point, where $\varepsilon = 1$ (sometimes the peak is not just after 1 but very close to it, due to the size of the bins).

In particular, this peak becomes higher for larger data set, confirming that this characteristic is present in a stochastic process such as the Geometric Brownian Motion. In fact, it is possible to notice (Figure 01 and Figure 02) that for a small number of observations such as 15,957 the results are a little bit misleading (I have used 15,957 as size because it is the size of the larger number of closing prices that I have for real data, so it is possible to compare these outputs). The graph representing the downtrends (Figure 01) is not clear at all and it is not possible to observe any particular behaviour on it. For the uptrends (Figure 02) one can see that the volatility is higher when $\varepsilon = 0$ and $\varepsilon = 1$ as suggested by Preis and Stanley in their analysis, even though the difference is not that large (in this case volatility varies just from light blue to green).

FIGURE 01 - GBM DOWNTRENDS 3D

σ = 0.01; Δt = [10,40] ; n = 15957

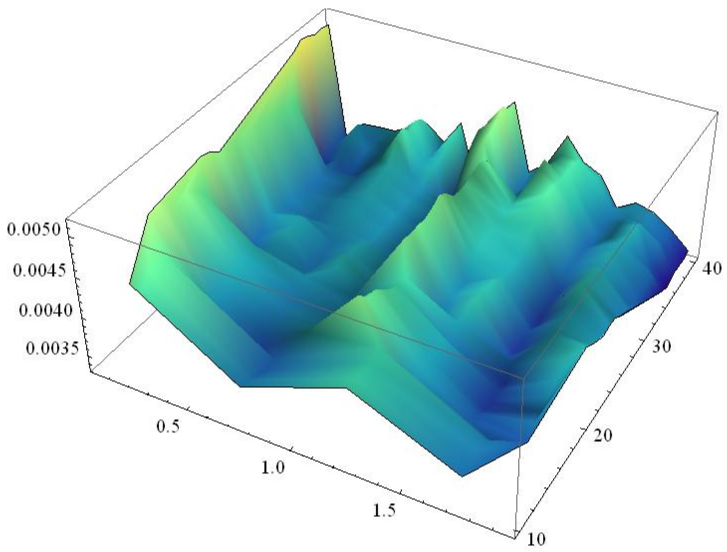


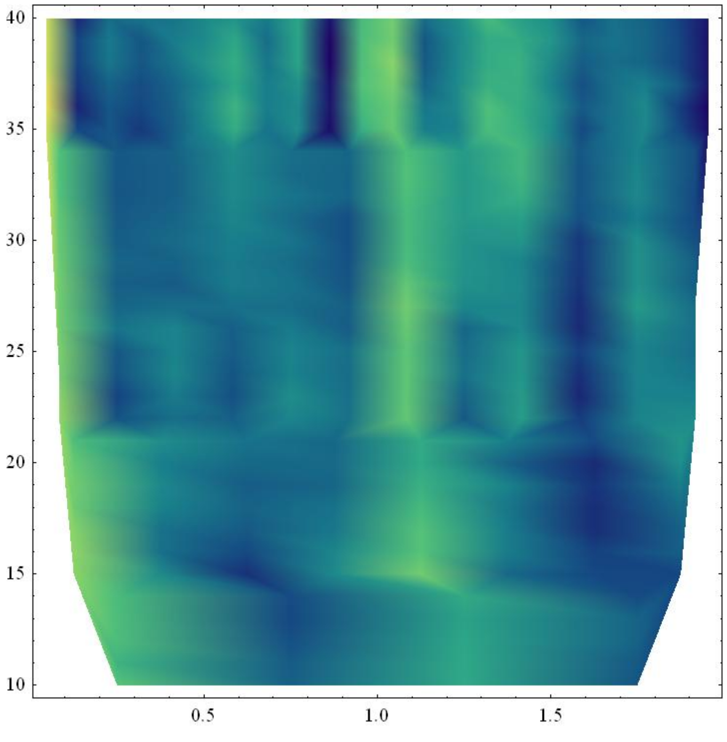


In the 3D picture the axis where the range goes from 0 to 2 represents the renormalised time scale ε, the axis with values from 10 to 40 represents Δt and the vertical axis represents the volatility.
In the 2D picture the x axis represents the renormalised time scale ε while the y axis represents Δt. In this case the volatility is given by the shades of different colours: blue represents low, green intermediate and yellow high values.
The parameters are the same for every figure, therefore this description holds for every other figure in this dissertation.

FIGURE 02 - GBM UPTRENDS 3D

σ = 0.01; Δt = [10,40] ; n = 15957





Incrementing the number of observations to 100,000 (Figure 03 and Figure 04) and running the program with more Δt (with n = 15,957 in fact the range of Δt is just from 10 to 40 due to the small size of the path, but for n = 100,000 it is possible to obtain decent results for a larger range of values of Δt, from 10 to 100), yellow peaks appear both for downtrends and for uptrends when ε is around 1.

FIGURE 03 - GBM DOWNTRENDS 3D

σ = 0.01; Δt = [10,100] ; n = 100000

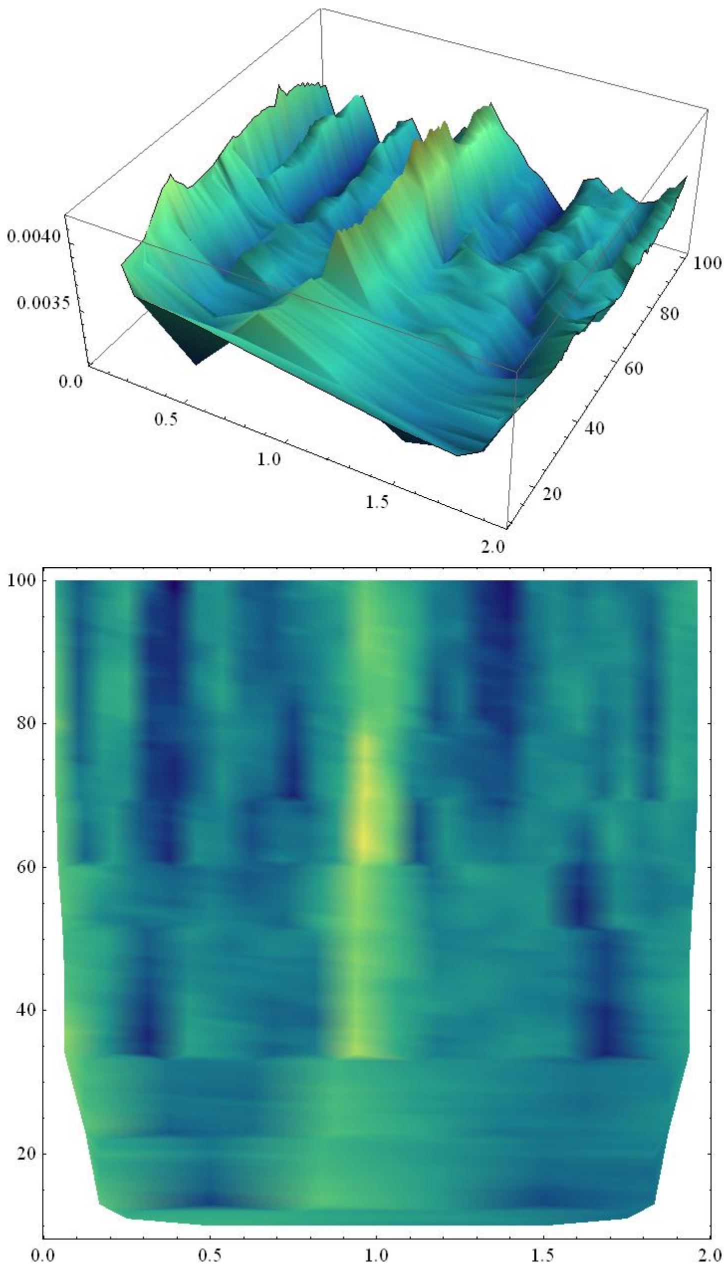
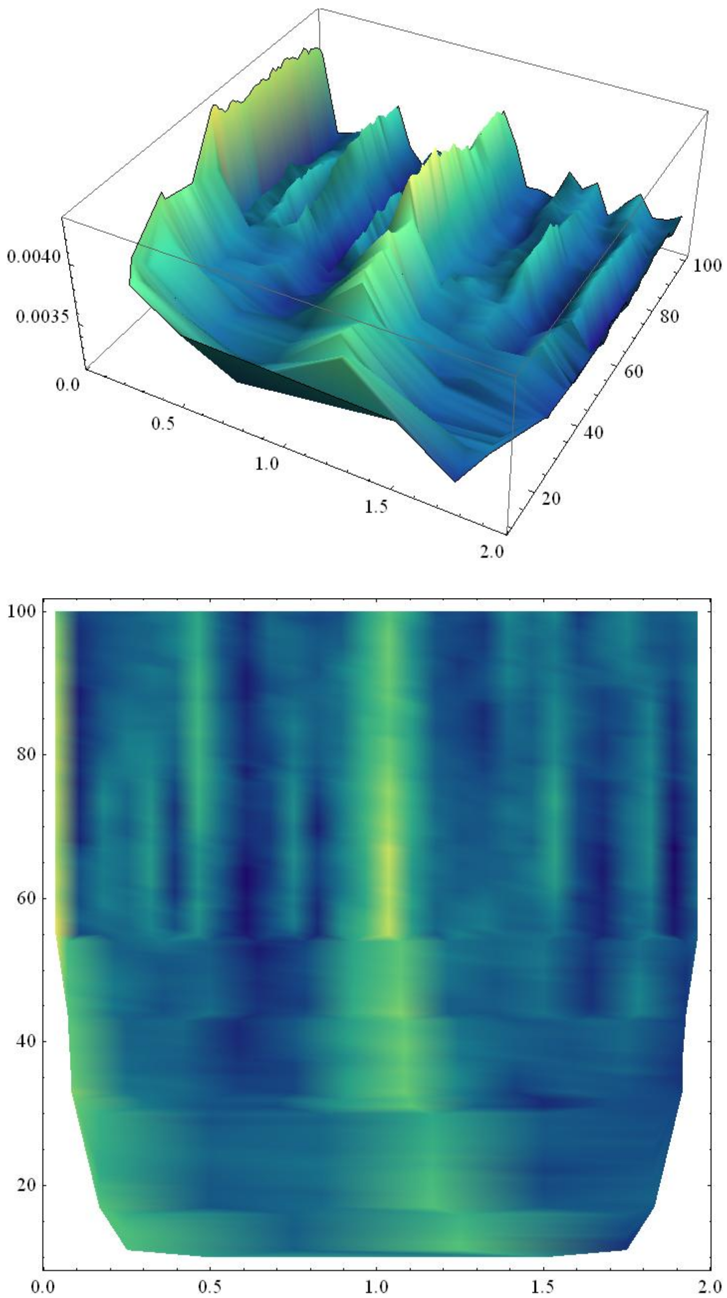


FIGURE 04 - GBM UPTRENDS 3D

σ = 0.01; Δt = [10,100] ; n = 100000



Incrementing the number of observations even further to 500,000 (Figure 05 and Figure 06) and keeping Δt range from 10 to 100, one can notice that peaks for $\varepsilon = 1$ are even higher than in the previous case both for downtrends and for uptrends.

According to these outputs then, the model provided by Preis and Stanley holds even for Geometric Brownian Motion; in fact there are huge peaks for $\varepsilon = 1$ even though my program is not as sophisticated as the one used in the original analysis for German DAX Future Contracts and S&P 500 Index and the number of data used in my analysis are not that large as well. Anyway, for a very small number of data such as 15,957 it is clear that this model does not hold anymore.

It is even possible to observe that results are somehow clearer for uptrends rather than for downtrends, in fact the gap between the values around $\varepsilon = 1$ and the rest of the values is wider for uptrends than for downtrends in all these 3 cases, but I do not have any theoretical explanations for this empirical observation.

So at this stage it seems that the characteristic to observe a peak in volatility when we are close to a switching point is due to the stochastic nature of processes such as stock prices, in fact this characteristic is observable for simulation such as Geometric Brownian Motion as well.

FIGURE 05 - GBM DOWNTRENDS 3D

σ = 0.01; Δt = [10,100] ; n = 500000

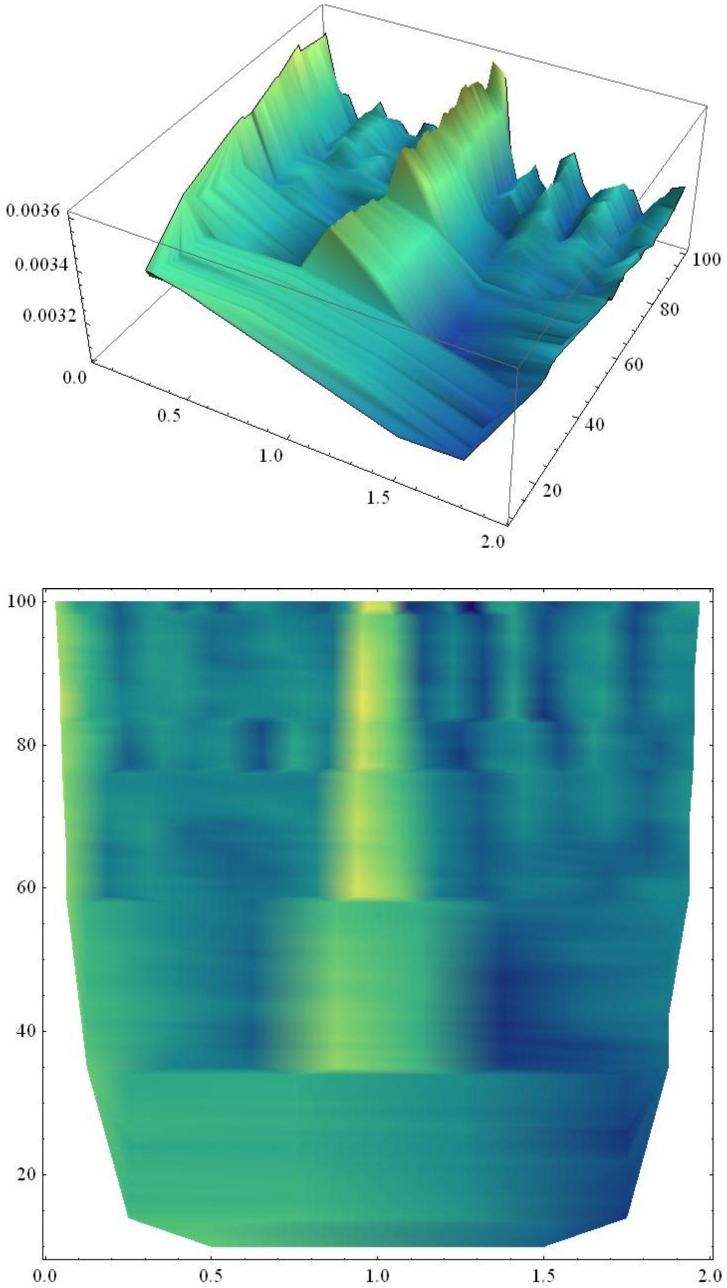
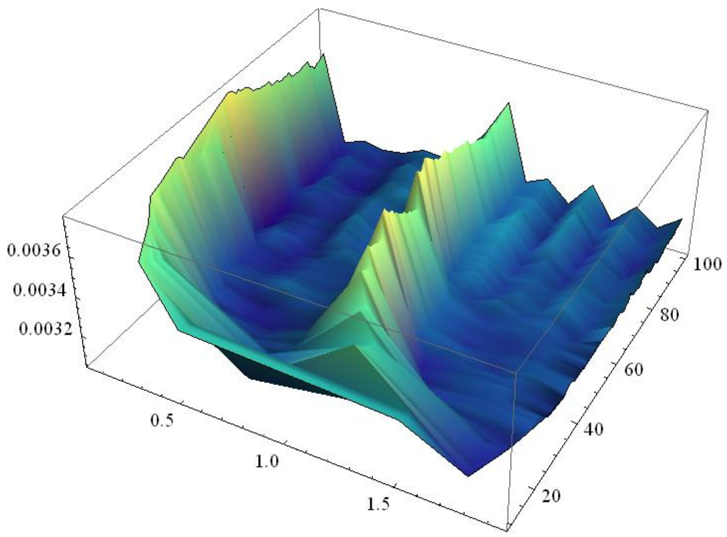
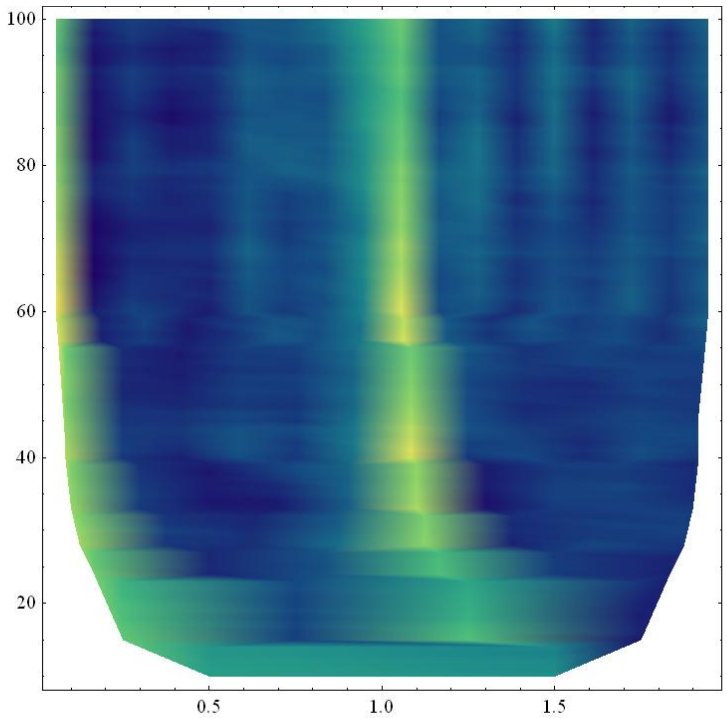


FIGURE 06 - GBM UPTRENDS 3D

σ = 0.01; Δt = [10,100] ; n = 500000





I have run the program for different values of sigma as well, in order to see if the same behaviour is observable or if it is due just to that particular kind of parameters. I have tried with σ equals to 0.02, 0.05 and 0.07 both with 15,957 transactions and with 100,000 transactions and the results are very similar to the ones I have described for $\sigma = 0.01$, therefore it seems that this model is not influenced by sigma.

In the next page it is possible to observe the graphs of the results obtained for $\sigma = 0.02$ and $n = 15,957$ (Figure 07 and Figure 08). Even in this case one can see that there is a peak around $\varepsilon = 1$ both for downtrends and for uptrends and that it is clearer for uptrends rather than for downtrends.

FIGURE 07 - GBM DOWNTRENDS 3D

σ = 0.02; Δt = [10,40] ; n = 15957

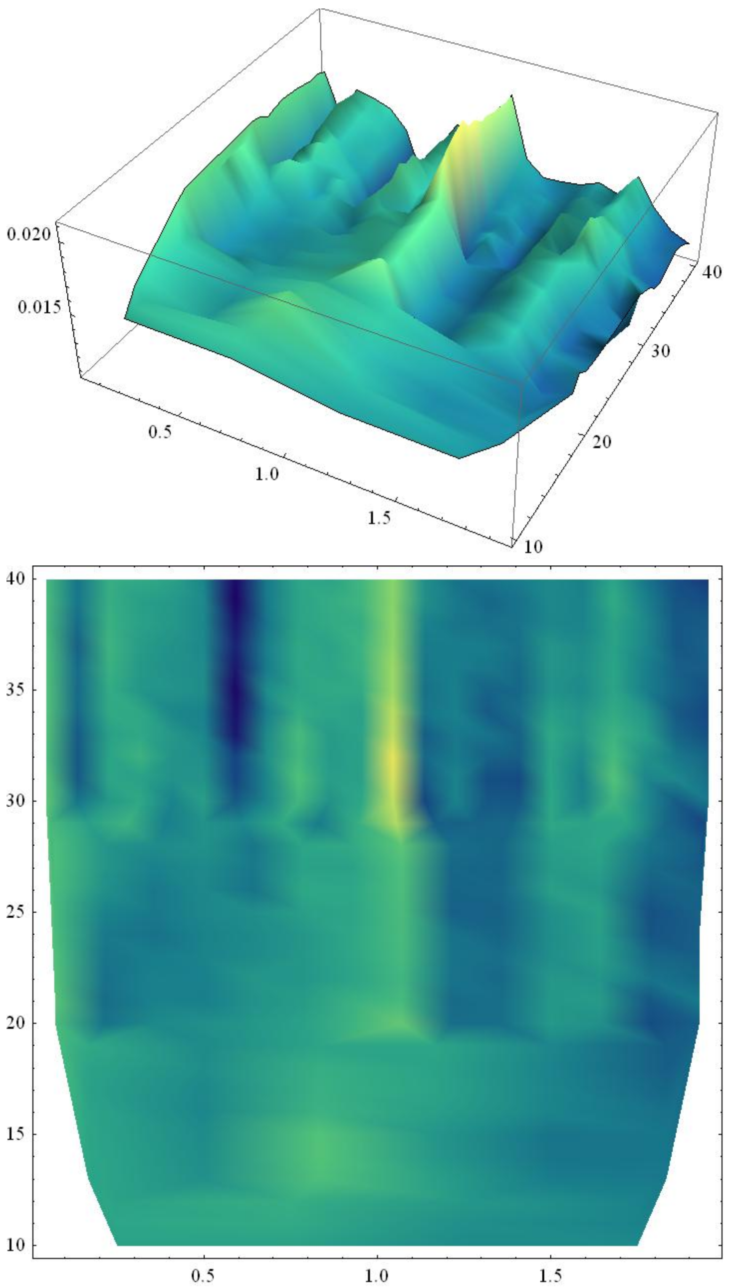
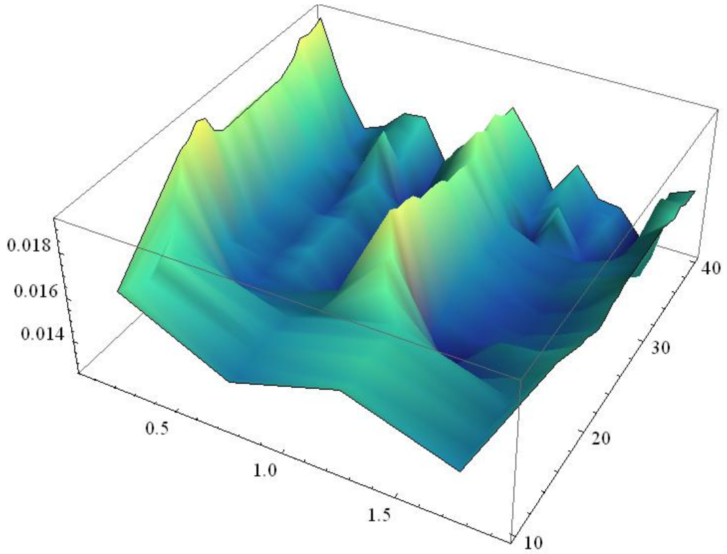
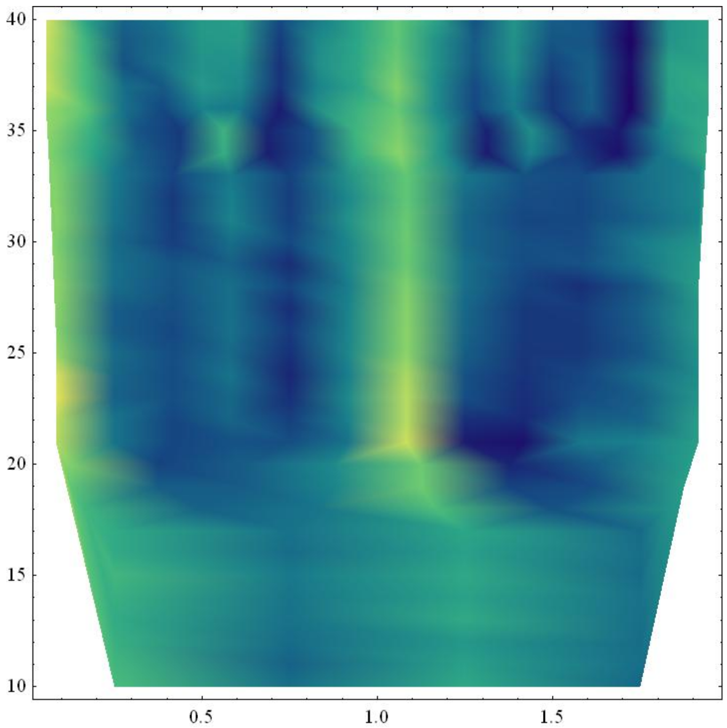


FIGURE 08 - GBM UPTRENDS 3D

σ = 0.02; Δt = [10,40] ; n = 15957





CHAPTER 4

REAL DATA

After testing Preis and Stanley's model with a simulation such as the Geometric Brownian Motion, I proceed to test this model with real stock prices. The analysis has been done using daily closing prices taken from Yahoo! Finance. In this analysis I have used both indices and single stock prices and I am going to explain the results obtained from different data one by one.

4.1 BARCLAYS (^BARC.L)

Barclays is one of the most famous banks in the UK and in the whole world and I have decided to use its series of closing prices in my analysis because I think that a bank like Barclays is a good benchmark, if we are dealing with bubbles and crashes.

It is traded on the London Stock Exchange. I have found a series of just 6,495 closing prices and I have run the program with Δt from 10 to 100.

It is possible to notice that Preis and Stanley's model holds for downtrends (Figure 09), for which there is a massive peak for $\varepsilon = 1$, while the graph representing uptrends (Figure 10) is quite confusing, even though a peak around $\varepsilon = 1$ is observable as well.

So, if, with Geometric Brownian Motion, the model holds both for downtrends and for uptrends and it seems to give larger peaks for uptrends, with a series of real prices, it seems to hold more for downtrends. At this point, I continue my analysis with some of the most famous stock indices, in order to validate or to refuse this empirical observation.

4.2 DAX (^GDAXI)

The DAX (Deutscher Aktien Index) is an index made by the 30 biggest German companies trading on the Frankfurt Stock Exchange. It is the index used as underlying for the series of future contracts which has been analysed by Tobias Preis and H Eugene Stanley.

The series that I have downloaded from Yahoo! Finance is composed by 5,704 closing prices and I have used Δt from 10 to 40. Again it is possible to see a clear peak for $\varepsilon = 1$ for downtrends (Figure 11), while for uptrends the peak is reached when $\varepsilon = 0$ (Figure 12) and then, around $\varepsilon = 1$ there is another peak but this time it is very small (in fact it is represented in light green).

FIGURE 09 - BARCLAYS DOWNTRENDS 3D

∆t = [10,100] ; n = 6495

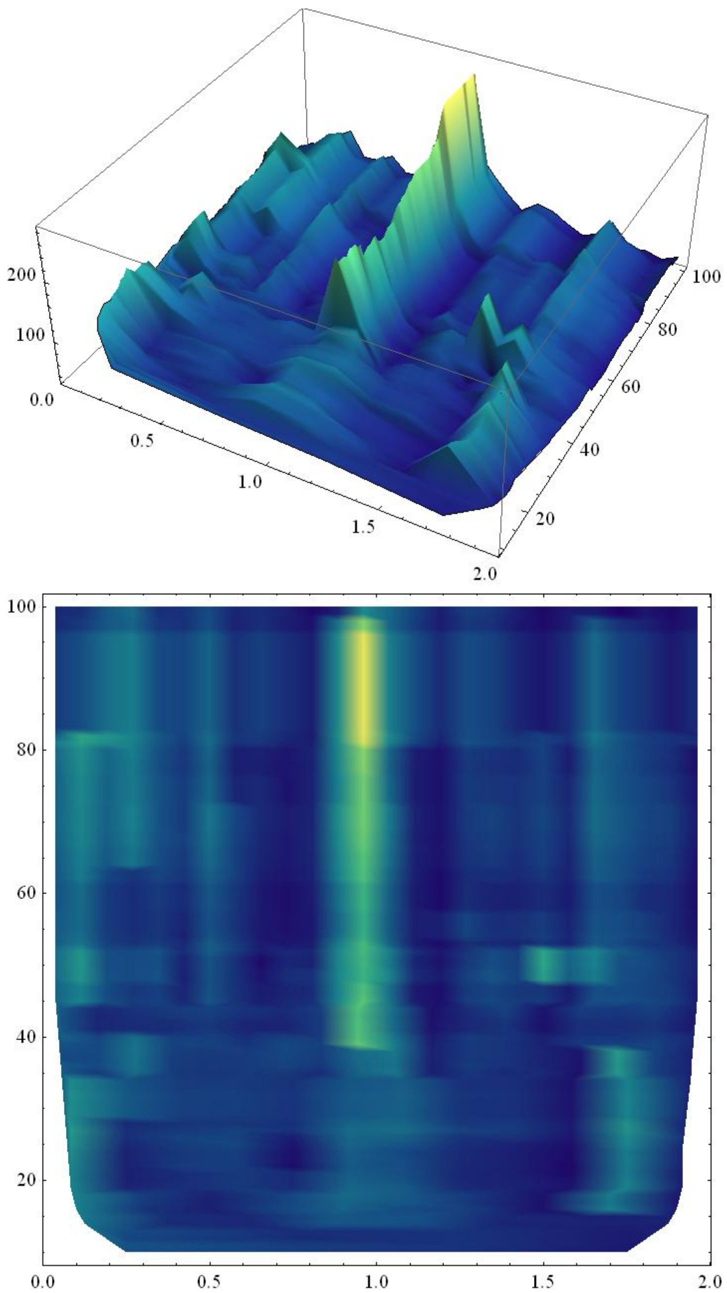


FIGURE 10 - BARCLAYS UPTRENDS 3D

Δt = [10,100] ; n = 6495

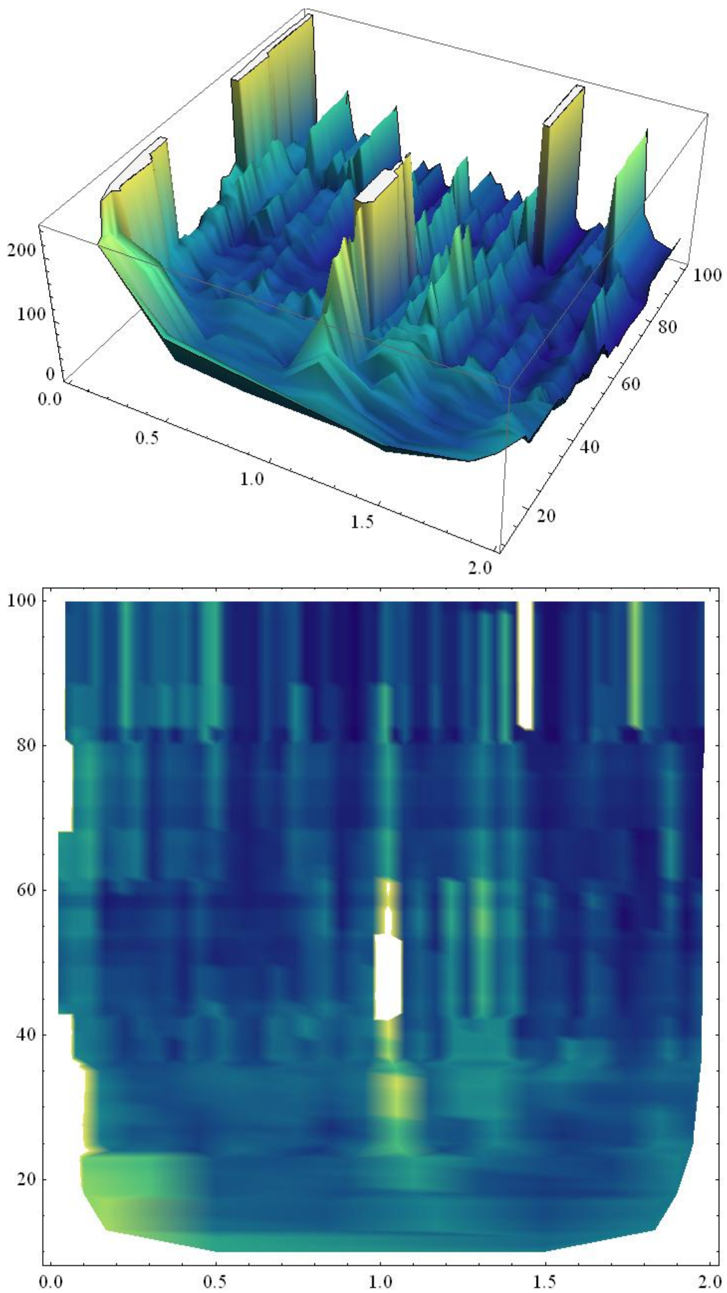
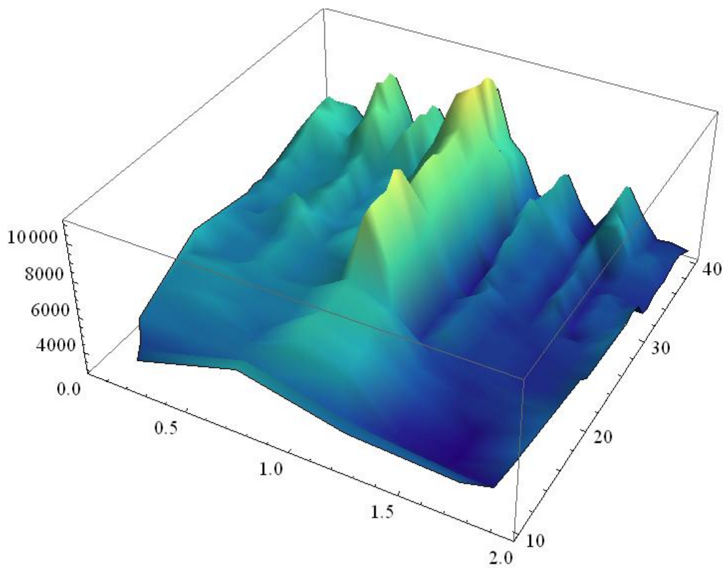


FIGURE 11 - DAX DOWNTRENDS 3D

$\Delta t = [10,40]$; n = 5704



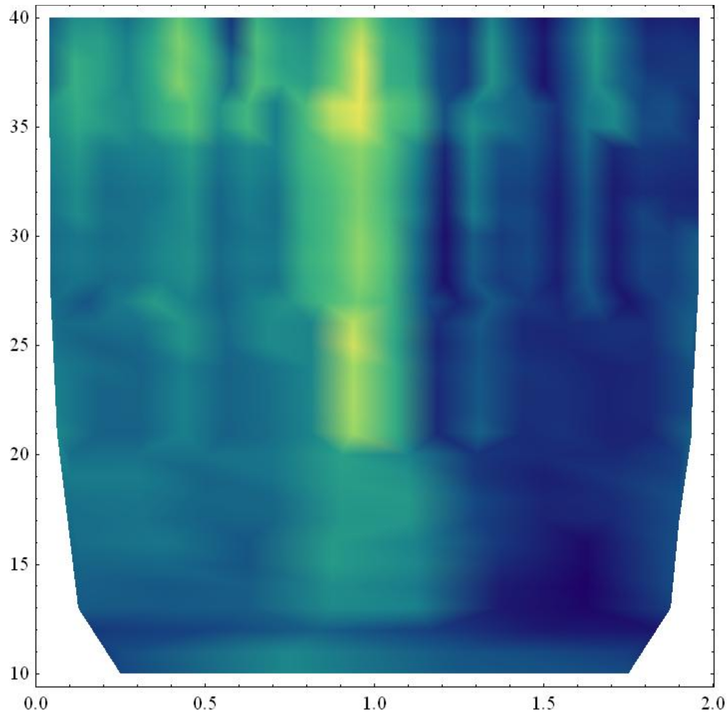
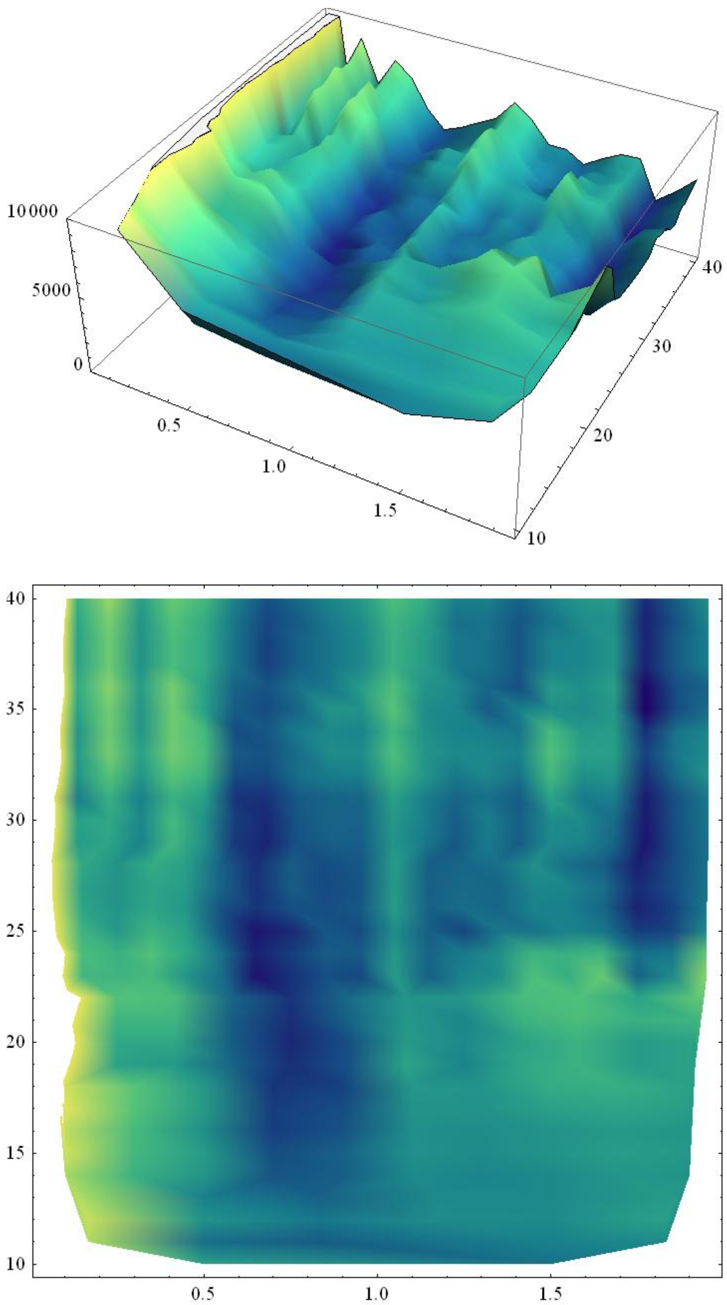


FIGURE 12 - DAX UPTRENDS 3D

Δt = [10,40] ; n = 5704



4.3 FTSE 100 (^FTSE)

The FTSE 100 is an index made by the best 100 companies listed on the London Stock Exchange.

In this case, for my analysis, I have used Δt from 10 to 40 on a series of 7,371 observations. Once again there is a yellow peak for $\varepsilon = 1$ for downtrends (Figure 13), while for uptrends (Figure 14) the graph is flat after the usual peak for $\varepsilon = 0$. So, this time, the model seems to hold just for downtrends.

4.4 NIKKEI 225 (^N225)

The Nikkei 225 is a stock index for the Tokyo Stock Exchange. The number 225 means that it is made by the biggest 225 Japanese companies traded on the Tokyo Stock Exchange.

The series that I found on Yahoo! Finance was composed by 7,235 closing prices and, as in the previous cases, I have used Δt from 10 to 40. The results (Figure 15 and Figure 16) are aligned with the previous ones and they confirm the empirical observation that Preis and Stanley's model holds for downtrends, but for uptrends the results are not that good.

FIGURE 13 - FTSE100 DOWNTRENDS 3D

Δt = [10,40] ; n = 7371

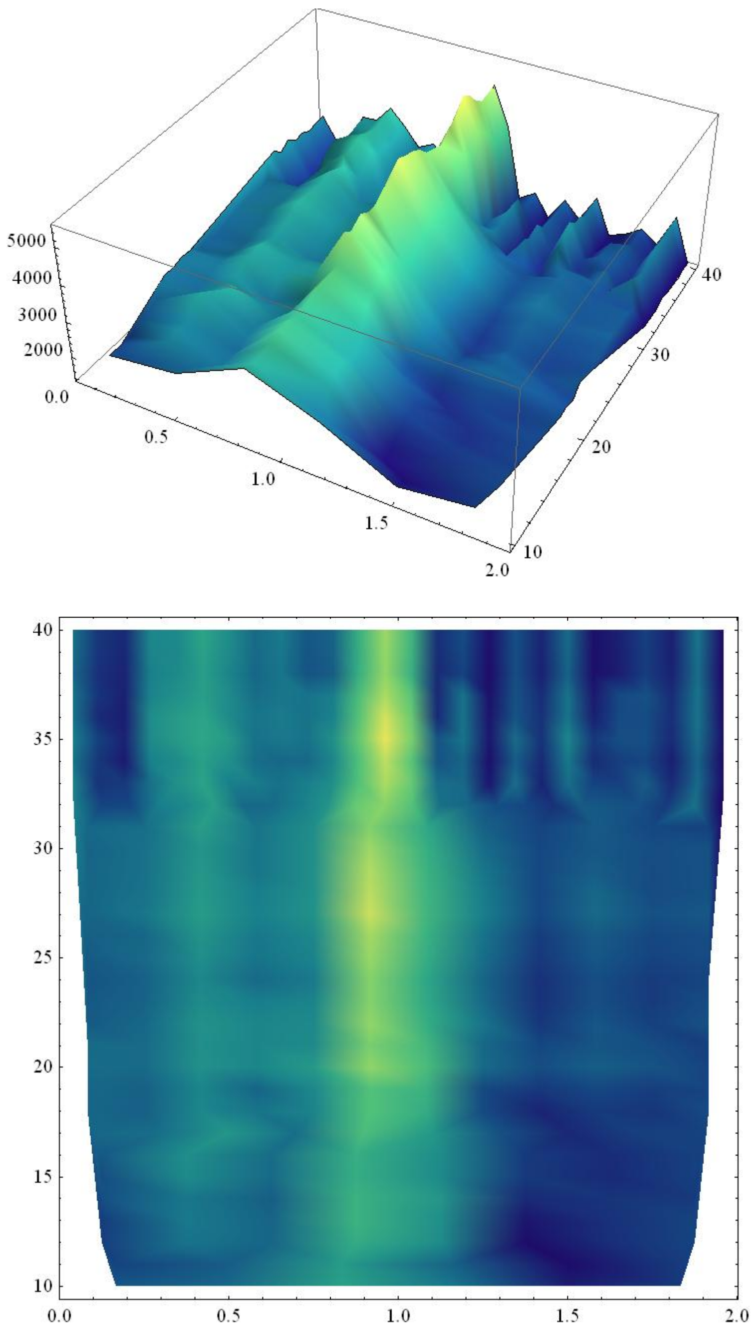
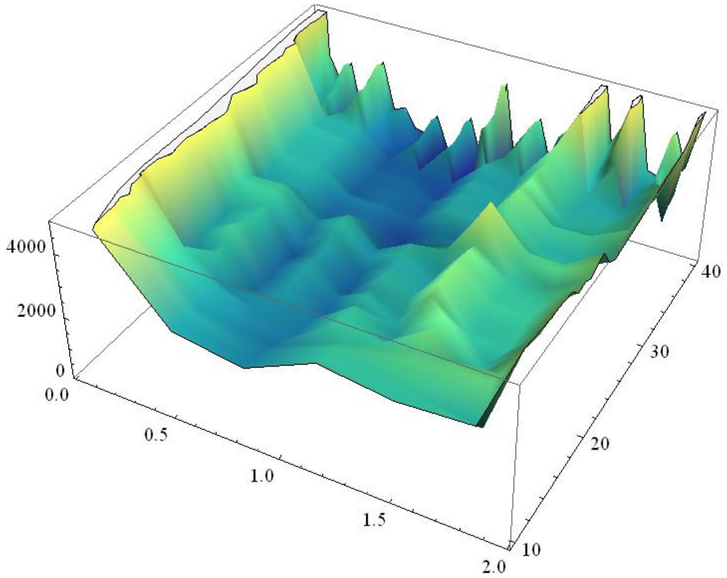


FIGURE 14 – FTSE100 UPTRENDS 3D

Δt = [10,40] ; n = 7371



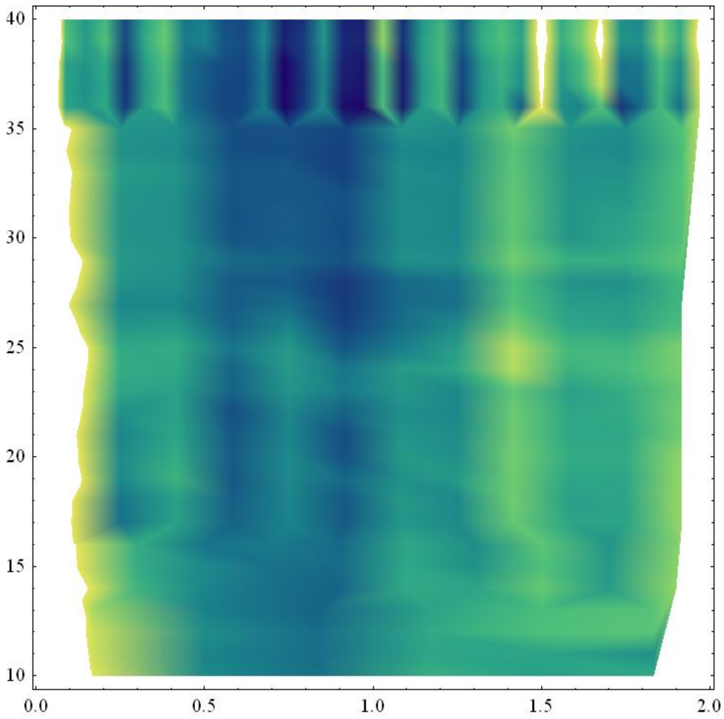
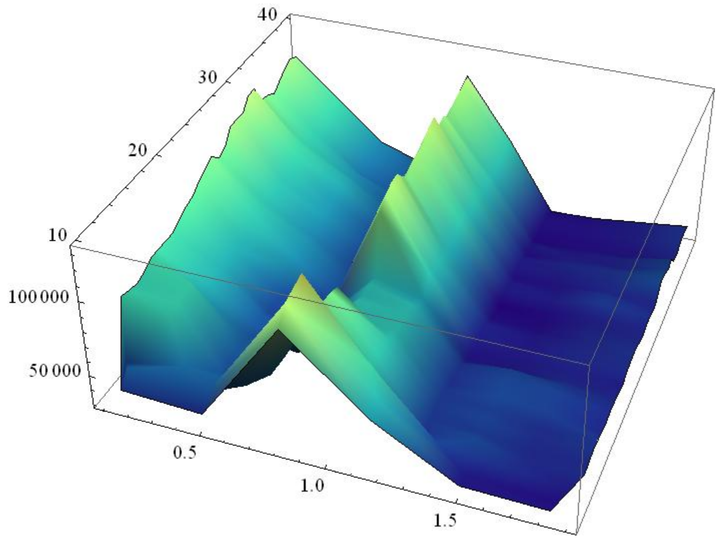


FIGURE 15 - NIKKEI 225 DOWNTRENDS 3D

$\Delta t = [10,40]$; $n = 7235$



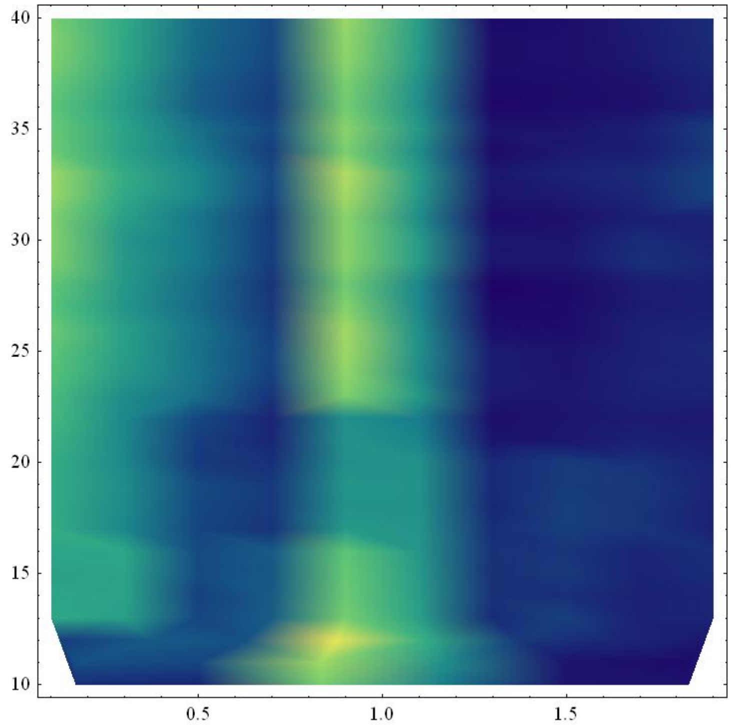
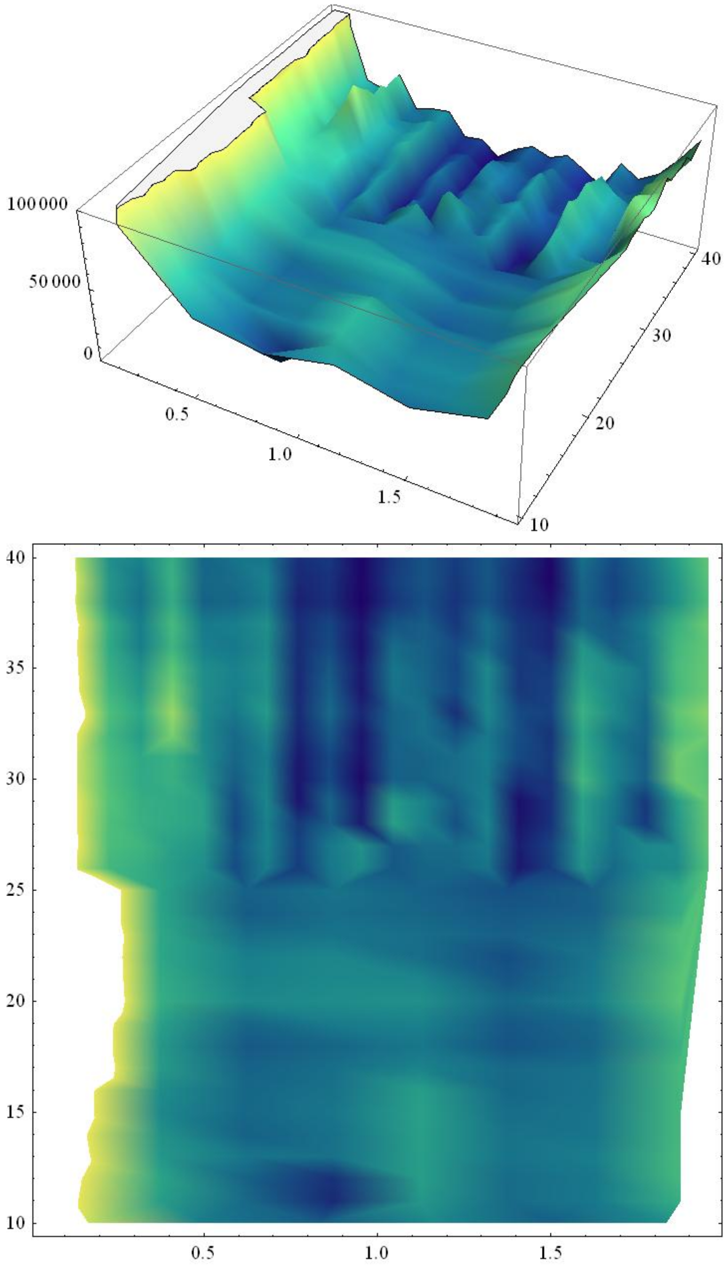


FIGURE 16 – NIKKEI 225 UPTRENDS 3D

$\Delta t = [10,40]$; n = 7235



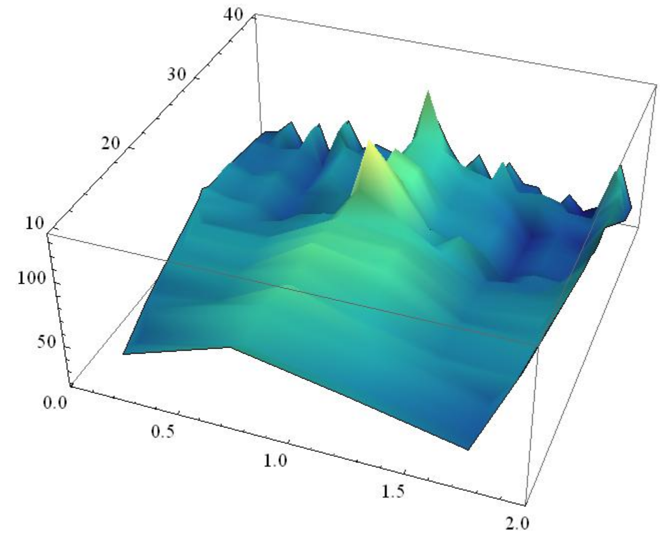
4.5 S&P 500 (^GSPC)

The S&P 500 (Standard & Poor’s 500) is a stock index based on the 500 best companies of the US market. It is the index used by Preis and Stanley in their analysis, but they have dealt with more data than me. In fact, I have used a series of 15,957 closing prices and I have run the program first with Δt from 10 to 40 and then with Δt from 10 to 100.

For $\Delta t = [10 , 40]$ the graphs (Figure 17 and Figure 18) give the same results obtained for the other indices, even though this time the peak for downtrends is not that high as well, while for $\Delta t = [10 , 100]$ (Figure 19 and Figure 20) the results are quite a mess and it is not possible to notice any particular trends both for up and downtrends.

FIGURE 17 - S&P500 DOWNTRENDS 3D

Δt = [10,40] ; n = 15957



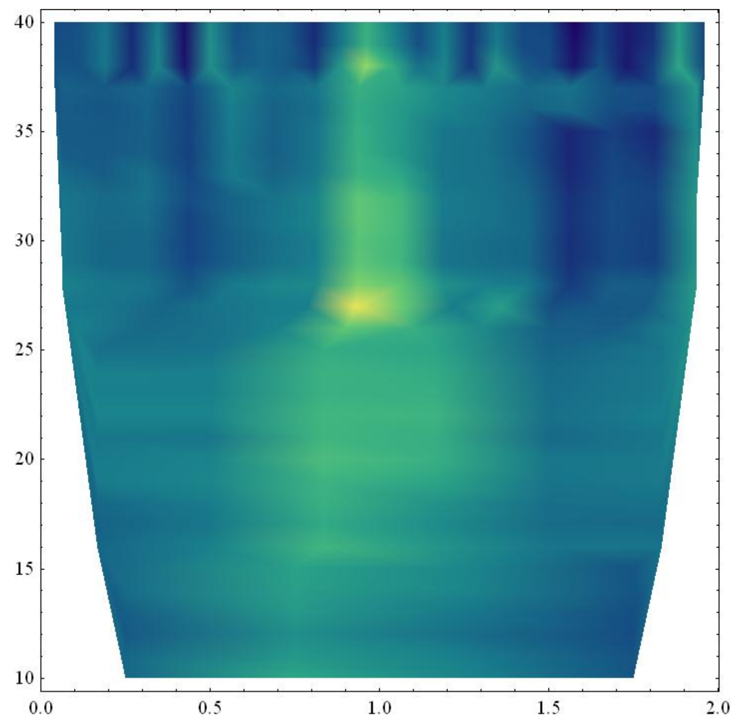
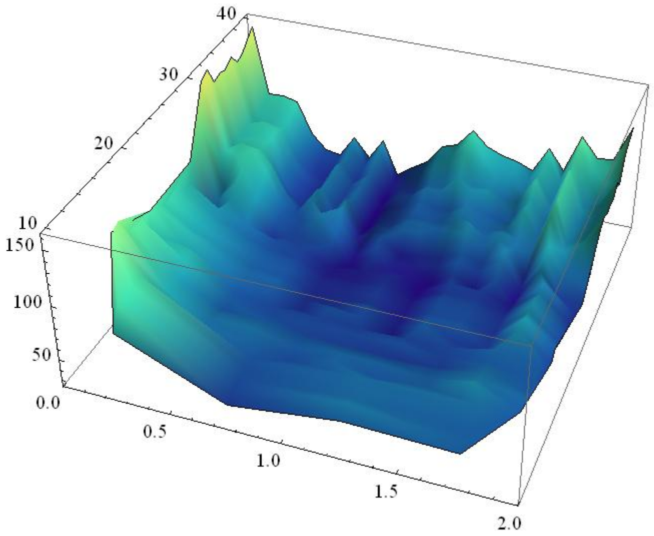


FIGURE 18 - S&P500 UPTRENDS 3D

Δt = [10,40] ; n = 15957



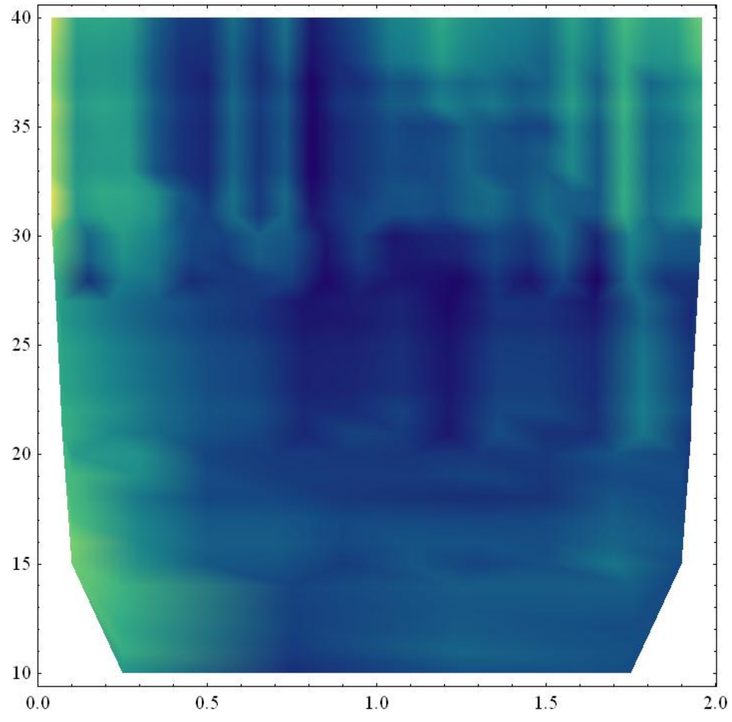


FIGURE 19 - S&P500 DOWNTRENDS 3D

Δt = [10,100] ; n = 15957

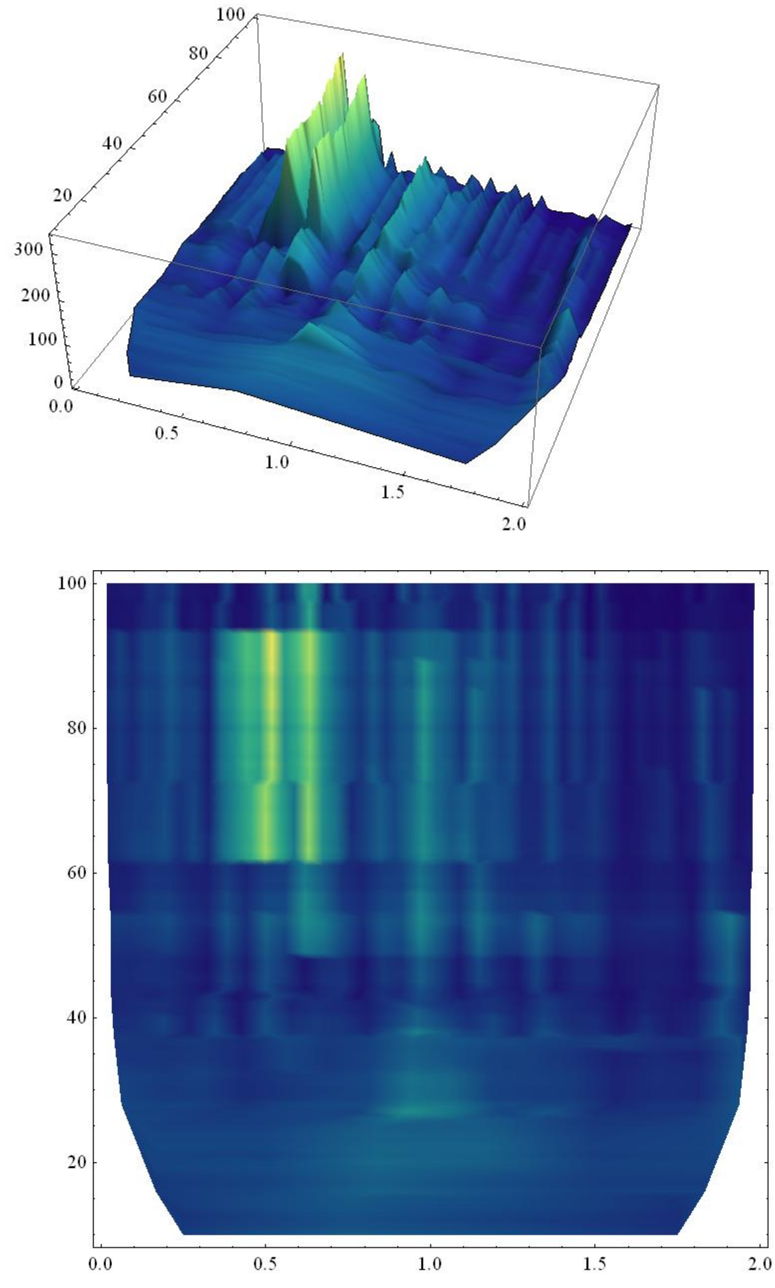
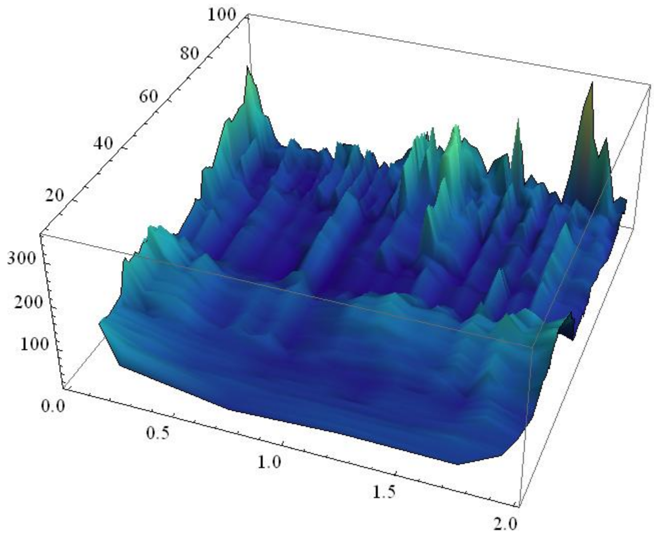
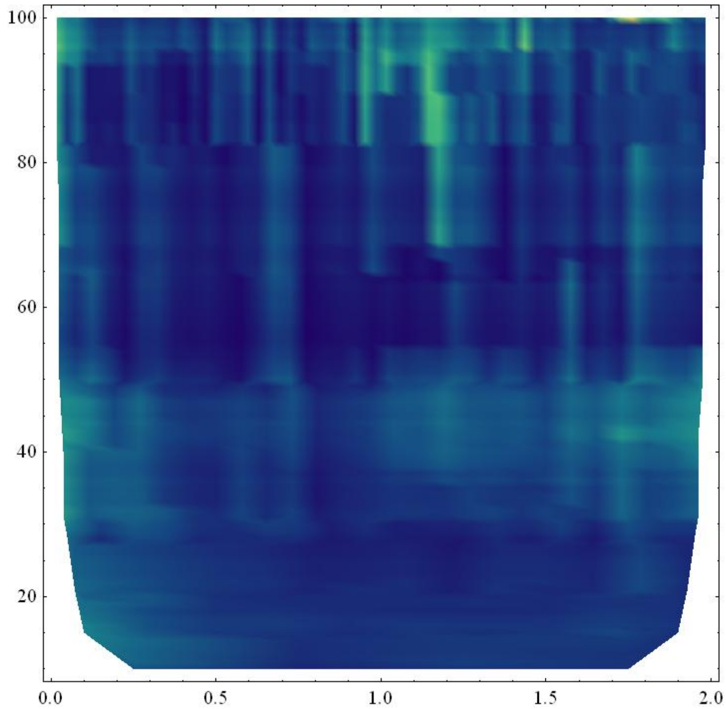


FIGURE 20 - S&P500 UPTRENDS 3D

Δt = [10,100] ; n = 15957





CHAPTER 5

CONSIDERATIONS AND CONCLUSIONS

5.1 ANALYSIS OF THE RESULTS

The main result obtained by Tobias Preis and H Eugene Stanley in their analysis of the German DAX future contracts and of the S&P 500 index is that crashes are not unlikely events, but they are just a consequence of increasing and decreasing trends in stock prices. Just after a switching point, traders sell stocks more frequently and in larger volumes because they get scared and they create a domino effect influencing each other.

Preis and Stanley have found very high peaks in volatility when ε is just after 1 both for downtrends and for uptrends, while in my analysis these peaks exist for Geometric Brownian Motion, but for real data they are observable just for downtrends, while for uptrends it is not possible to see massive peaks. This result can be a consequence of the smaller data set that I have used compared to the one used by Preis and Stanley, but, on the other hand, there is not a huge discrepancy between my results and the authors' ones; in fact, in their analysis, peaks are higher for downtrends rather than for uptrends and it can exist a theoretical explanation for this observation as well. Traders, in fact, do not consider uptrends and downtrends in the same way. In a downtrend, traders who have a long position (they are assumed to be more than traders in a short position) are facing a loss, while in an uptrend they are making money, so their behaviour in these two situations is very different. When they are losing money, they want to stop their losses and so they will sell their stocks soon. On the contrary, when they are making money, they want to maximise their gains, so, sophisticated traders will sell their stocks at some point, but less sophisticated traders will keep their stocks a little bit more, because they may think that prices will keep growing in the very near future. Therefore, for uptrends, traders do not have a unique behaviour and, as a consequence, results in this kind of analysis are less clear than for downtrends, in which traders tend to behave in a more standardised way.

Furthermore, I think that sophisticated traders have more short positions when a stock is facing a downtrend rather than when it is facing an uptrend and it can contribute to increase the gap between downtrends and uptrends. In fact, it is very strange for common investors to have a short position and it is more likely to see professional traders in this position and generally they know what they are doing, so I believe that the number of professional traders holding a short position is larger when stock price is falling. Anyway this is just a small consideration and we have got to bear in mind that this kind of position can be hold by hedgers as well, so it is not so easy to explain traders' behaviour in these situations.

This result highlights the limit of a model such as Geometric Brownian Motion in describing the behaviour of stock prices. In fact for Geometric Brownian Motion there is not any difference between uptrends and downtrends. This is the limit of this kind of processes applied to finance: they can be good but they cannot predict human beings' behaviour and stock prices are widely influenced by human beings.

Furthermore, it is not a surprise that Geometric Brownian Motion has some flaws. In fact, it assumes that the probability density of log-returns is Gaussian, but, as I mentioned in Chapter 2, Paramaswaram Gopikrishan and Vasiliki Plerou have found that returns on stock prices are actually better described by an "inverse quartic power law".

5.2 COMPARISON BETWEEN MINSKY'S THEORY AND PREIS AND STANLEY'S THEORY

Tobias Preis and H Eugene Stanley have given a strong conclusion to their work. In fact, in the abstract of their paper, one can read that "it is widely believed that switching phenomena require switches, but this is actually not true" [3]. It means that in order to see a crash, it is not necessary any trigger, because it is just a consequence of the nature of the evolution of stock prices. At a first view, one can think that this conclusion disagrees with the theoretical explanation of bubbles and crashes given by Markus K. Brunnermeier and Martin Oehmke and by Hyman Minsky, but it is actually not true.

To be more precise, in their paper, Brunnermeier and Oehmke have described a crisis as made by two phases: the run-up phase and the crisis phase. The switch from the run-up phase to the crisis phase is represented by the so-called Minsky moment, in which a triggering event happens and it generates panic among investors, making stock prices fall sharply. From this description it is then clear that it is necessary an event in order to see a crash, while Preis and Stanley have affirmed in their paper that a switch in the real world is not necessary to generate a switch in the evolution of stock prices. On the other hand, Brunnermeier and Oehmke have clearly said that the triggering event is not that important and sometimes a small event can lead to a huge crash. The domino effect among investors is what really generates a crash. From this point of view the conclusions of these two papers are very similar; the only small difference is that in Preis and Stanley's model the triggering event is not even necessary, in the sense that it can happen just in investors' minds, but what is very important for both these two theories is the domino effect, which is represented by the peak in volatility just after the switching point in Preis and Stanley's model.

To summarise, in my modest opinion, I think that the intuition of Tobias Preis and H Eugene Stanley is very good and it is in accord with the theoretical explanation of bubbles and crashes given by economists. Furthermore, this model is empirically confirmed also with smaller data sets like the ones that I have used in my analysis, even though, of course, results are clearer with massive data sets as the ones used by the authors in their paper.

APPENDIX 1

PROGRAM 1: GBM GENERATOR

Here is the code that I wrote in order to generate different paths of Geometric Brownian Motion.

In this program I used a method called Box-Muller Method to generate a random number with normal distribution N(0,1). It is possible to see this part in the function called BoxMullerMethod(double m, double s).

In the main, then, the user has to enter the volatility parameter (sigma) and the initial stock price (s). The drift term is set equal to (pow(sigma,2.0))/2.0 because of the noise, as I mentioned in Chapter 3, and dt is 1.0/252.0 because I wanted to consider daily prices.

The number of transactions (n) is set equal to 100,000, but I have run the program changing it to 15,957 and 500,000 as well, as I have explained before.

Hence, the program simply generates a path of a GBM creating new values of it with the following formula:

s = s + drift*s*dt + sigma*s*sqrt(dt)*a

where drift, *dt* and sigma are the parameters that I have described before and *a* is a random number generated by the function BoxMullerMethod(double m, double s).

The values of the path are then saved into a file using the fstream library.

CODE:

```
#define _USE_MATH_DEFINES
#include <iostream>
#include <cmath>
#include <cstdlib>
#include <ctime>
#include <fstream>
using namespace std;

double BoxMullerMethod(double m, double s) {
	double u1, u2, r, theta, x, y;
	u1 = (double) (rand()+1)/(RAND_MAX+1);
	u2 = (double) (rand()+1)/(RAND_MAX+1);
	r = sqrt(-2.0*log(u1));
	theta = 2.0*M_PI*u2;
	x = r*cos(theta);
	y = x*s + m;
	return y;
}

int main () {

	int i;
	double drift;
	double sigma;
	double s;
	double a;
	double dt = 1.0/252.0;
	int n = 100000;
	srand(time(NULL));

	ofstream fout;
	fout.open("file01.csv");
	if(!fout) {
		cout << "Error writing file" << endl;
		return -1;
	}

	cout << "Enter sigma(double): " ; cin >> sigma;
	cout << "Enter initial stock price(double): " ; cin >> s;
	drift = (pow(sigma,2.0))/2.0;
	fout << s << endl;

	for(i=1; i<n; i++) {
		a = BoxMullerMethod(0.0,1.0);
		s = s + drift*s*dt + sigma*s*sqrt(dt)*a;
		fout << s << endl;
	}
```

```
    fout.close();
    return 0;
}
```

APPENDIX 2

PROGRAM 2: PREIS AND STANLEY'S MODEL

Here is the program that I wrote in order to obtain my analysis. Every time a data set is read and a new file containing the results is created. The user has to enter the range of delta t for which he wants to obtain the output and he has to choose if he wants to analyse uptrends or downtrends. Then, the program finds local minima and local maxima and it keeps only the trends that are larger than the previous one and the following one. For these trends, hence, it analyses the volatility. This procedure is repeated with a loop for every delta t that is in the range selected by the user. The output of this program is then open with Wolfram Mathematica in order to generate a 3D graph and a coloured graph, as shown in Chapter 3 and Chapter 4.

CODE:

```
#include <iostream>
#include <cmath>
#include <ctime>
#include <cstdlib>
#include <fstream>
#include <vector>
#include <map>
using namespace std;

int main (int argc, char* argv[]) {
```

```
int i = 0;
int j = 0;
int dt;
int dt_lower;
int dt_upper;
int tmin;
int tmax;
int index_neg;
int index_pos;
int b;
int c;
int d;
int ee;
int g;
int h;
int count;

double a;
double e;
double f;
double min;
double max;
double sigma;
double e_bin = 0.0;
bool possible;

vector<int> v1;
vector<int> v2;
vector<int> v3;
vector<int> v4;
vector<int> pos_trend_v1;
vector<int> pos_trend_v2;
vector<int> neg_trend_v1;
vector<int> neg_trend_v2;
vector<double> s1;
vector<double> vec1;
vector<double> vec2;
vector<double> vec_e_bin;
vector<double> vec_sigma_bin;
typedef map<int, int> Map;
Map m1;

ifstream fin;
fin.open(argv[1]);
if(!fin) {
        cout << "Error opening file: " << argv[1] << endl;
        return -1;
}

ofstream fout;
fout.open(argv[2]);
if(!fout) {
```

```
		cout << "Error writing file: " << argv[2] <<endl;
		return -1;
	}

	while (fin >> a) {
		s1.push_back(a);
	}

	cout << "Enter first value of dt(integer): "; cin >> dt_lower;
	cout << "Enter last value of dt(integer): "; cin >> dt_upper;
	cout << "Enter 1 if you want uptrends or 2 if you want downtrends: "; cin >> b;


	if(b == 1) {

		for(dt=dt_lower; dt<=dt_upper; dt++) {

			for(i=0; i<s1.size(); i++) {
				possible = true;
				index_neg = 0;
				index_pos = 0;
				for(j=1; j<=dt; j++) {
					if(i != j) index_neg = i-j; index_pos = i+j;
					if(i-j <= 0) index_neg = i;
					if(i+j > s1.size()-1) index_pos = s1.size()-1;
					if(s1[i] < s1[index_neg] && s1[i] < s1[index_pos]) {
						min = s1[i];
						tmin = i;
					}
					else {
						possible = false;
						tmin = NULL;
					}
				}
				if(possible) {
					m1.insert(pair<int,int>(tmin,0));
				}
			}

			for(i=0; i<s1.size(); i++) {
				possible = true;
				index_neg = 0;
				index_pos = 0;
				for(j=1; j<=dt; j++) {
					if(i != j) index_neg = i-j; index_pos = i+j;
					if(i-j <= 0) index_neg = i;
					if(i+j > s1.size()-1) index_pos = s1.size()-1;
					if(s1[i] > s1[index_neg] && s1[i] > s1[index_pos]) {
						max = s1[i];
						tmax = i;
					}
					else {
						possible = false;
```

```
                    tmax = NULL;
                }
            }
            if(possible) {
                m1.insert(pair<int,int>(tmax,1));
            }
        }

        for(Map::iterator it = m1.begin(); it != m1.end(); it++) {
            c = it->first;
            it++;
            if(it == m1.end()) {
                break;
            }
            else {
                d = it->first;
                it++;
                if(it == m1.end()) {
                    break;
                }
                else if(it != m1.end()) {
                    ee = it->first;
                    if(d-c < ee-d) {
                        it--;
                        it--;
                        v1.push_back(c);
                        v2.push_back(it->second);
                        v3.push_back(d);
                        it++;
                        v4.push_back(it->second);
                        it--;
                    }
                    if(d-c > ee-d) {
                        it--;
                        v1.push_back(d);
                        v2.push_back(it->second);
                        v3.push_back(ee);
                        it++;
                        v4.push_back(it->second);
                        it--;
                        it--;
                    }
                    else if(d-c == ee-d) {
                        it--;
                        it--;
                    }
                }
            }
        }

        for(i=1; i<v1.size(); i++) {
            if(v1[i] == v1[i-1]) {
                v1.erase(v1.begin()+i);
```

```
                    v2.erase(v2.begin()+i);
                    v3.erase(v3.begin()+i);
                    v4.erase(v4.begin()+i);
                }
            }

            for(i=0; i<v1.size(); i++) {
                if(v2[i] == v4[i]) {
                    v1.erase(v1.begin()+i);
                    v2.erase(v2.begin()+i);
                    v3.erase(v3.begin()+i);
                    v4.erase(v4.begin()+i);
                }
            }

            for(i=0; i<v2.size(); i++) {
                if(v2[i] == 0) {
                    pos_trend_v1.push_back(v1[i]);
                    pos_trend_v2.push_back(v3[i]);
                }
                else if(v2[i] == 1) {
                    neg_trend_v1.push_back(v1[i]);
                    neg_trend_v2.push_back(v3[i]);
                }
            }

            if(pos_trend_v1.size() !=0) {
                h = 2*(pos_trend_v2[0]-pos_trend_v1[0]);
                for(i=0; i<pos_trend_v1.size(); i++) {
                    for(j=pos_trend_v1[i]; j<=(pos_trend_v2[i]+(pos_trend_v2[i]-
pos_trend_v1[i])); j++) {
                        if(j<1) continue;
                        if(j<s1.size() && j>=1) {
                            e = (double) (j - pos_trend_v1[i])/(pos_trend_v2[i] -
pos_trend_v1[i]);

                            sigma = pow((s1[j] - s1[j-1]),2.0);
                            vec1.push_back(e);
                            vec2.push_back(sigma);
                        }
                        if(j>=s1.size()) break;
                    }
                    g = 2*(pos_trend_v2[i]-pos_trend_v1[i]);
                    if(g < h) h = g;
                    else if(g >= h) continue;
                }

                f = (double) 1/h;
                for(i=0; i<h; i++) {
                    e_bin = (double) (1+(2*i))/h;
                    vec_e_bin.push_back(e_bin);
                    vec_sigma_bin.push_back(0.0);
                }
                for(i=0; i<vec_e_bin.size(); i++) {
```

```
                                        count = 0;
                                        for(j=0; j<vec1.size(); j++) {
                                                if(vec1[j] >= (vec_e_bin[i] - f) && vec1[j] < (vec_e_bin[i] +
f)) {

                                                        vec_sigma_bin[i] = vec_sigma_bin[i] + vec2[j];
                                                        count++;

                                                }
                                        }
                                        if(vec_sigma_bin[i] != 0.0) vec_sigma_bin[i] = (double)
vec_sigma_bin[i]/count;
                                }

                                for(i=0; i<vec_e_bin.size(); i++) {
                                        fout << vec_e_bin[i] << "," << dt << "," << vec_sigma_bin[i] << endl;
                                }
                        }

                        m1.clear();
                        v1.clear();
                        v2.clear();
                        v3.clear();
                        v4.clear();
                        pos_trend_v1.clear();
                        pos_trend_v2.clear();
                        neg_trend_v1.clear();
                        neg_trend_v2.clear();
                        vec1.clear();
                        vec2.clear();
                        vec_e_bin.clear();
                        vec_sigma_bin.clear();

                }

        }

        if(b == 2) {

                for(dt=dt_lower; dt<=dt_upper; dt++) {

                        for(i=0; i<s1.size(); i++) {
                                possible = true;
                                index_neg = 0;
                                index_pos = 0;
                                for(j=1; j<=dt; j++) {
                                        if(i != j) index_neg = i-j; index_pos = i+j;
                                        if(i-j <= 0) index_neg = i;
                                        if(i+j > s1.size()-1) index_pos = s1.size()-1;
                                        if(s1[i] < s1[index_neg] && s1[i] < s1[index_pos]) {
                                                min = s1[i];
                                                tmin = i+1;
                                        }
                                        else {
```

```
                possible = false;
                tmin = NULL;
            }
        }
        if(possible) {
            m1.insert(pair<int,int>(tmin,0));
        }
}

for(i=0; i<s1.size(); i++) {
        possible = true;
        index_neg = 0;
        index_pos = 0;
        for(j=1; j<=dt; j++) {
            if(i != j) index_neg = i-j; index_pos = i+j;
            if(i-j <= 0) index_neg = i;
            if(i+j > s1.size()-1) index_pos = s1.size()-1;
            if(s1[i] > s1[index_neg] && s1[i] > s1[index_pos]) {
                max = s1[i];
                tmax = i+1;
            }
            else {
                possible = false;
                tmax = NULL;
            }
        }
        if(possible) {
            m1.insert(pair<int,int>(tmax,1));
        }
}

for(Map::iterator it = m1.begin(); it != m1.end(); it++) {
        c = it->first;
        it++;
        if(it == m1.end()) {
            break;
        }
        else {
            d = it->first;
            it++;
            if(it == m1.end()) {
                break;
            }
            else if(it != m1.end()) {
                ee = it->first;
                if(d-c < ee-d) {
                    it--;
                    it--;
                    v1.push_back(c);
                    v2.push_back(it->second);
                    v3.push_back(d);
                    it++;
                    v4.push_back(it->second);
```

```
                                        it--;
                                }
                                if(d-c > ee-d) {
                                        it--;
                                        v1.push_back(d);
                                        v2.push_back(it->second);
                                        v3.push_back(ee);
                                        it++;
                                        v4.push_back(it->second);
                                        it--;
                                        it--;
                                }
                                else if(d-c == ee-d) {
                                        it--;
                                        it--;
                                }
                        }
                }
        }

        for(i=1; i<v1.size(); i++) {
                if(v1[i] == v1[i-1]) {
                        v1.erase(v1.begin()+i);
                        v2.erase(v2.begin()+i);
                        v3.erase(v3.begin()+i);
                        v4.erase(v4.begin()+i);
                }
        }

        for(i=0; i<v1.size(); i++) {
                if(v2[i] == v4[i]) {
                        v1.erase(v1.begin()+i);
                        v2.erase(v2.begin()+i);
                        v3.erase(v3.begin()+i);
                        v4.erase(v4.begin()+i);
                }
        }

        for(i=0; i<v2.size(); i++) {
                if(v2[i] == 0) {
                        pos_trend_v1.push_back(v1[i]);
                        pos_trend_v2.push_back(v3[i]);
                }
                if(v2[i] == 1) {
                        neg_trend_v1.push_back(v1[i]);
                        neg_trend_v2.push_back(v3[i]);
                }
        }

        if(neg_trend_v1.size() !=0) {
                h = 2*(neg_trend_v2[0]-neg_trend_v1[0]);
                for(i=0; i<neg_trend_v1.size(); i++) {
```

```
                                        for(j=neg_trend_v1[i]; j<=(neg_trend_v2[i]+(neg_trend_v2[i]-
neg_trend_v1[i])); j++) {
                                                if(j<1) continue;
                                                if(j<s1.size() && j>=1) {
                                                        e = (double) (j - neg_trend_v1[i])/(neg_trend_v2[i]
- neg_trend_v1[i]);

                                                        sigma = pow((s1[j] - s1[j-1]),2.0);
                                                        vec1.push_back(e);
                                                        vec2.push_back(sigma);
                                                }
                                        }
                                        g = 2*(neg_trend_v2[i]-neg_trend_v1[i]);
                                        if(g < h) h = g;
                                        else if(g >= h) continue;
                                }

                                f = (double) 1/h;
                                for(i=0; i<h; i++) {
                                        e_bin = (double) (1+(2*i))/h;
                                        vec_e_bin.push_back(e_bin);
                                        vec_sigma_bin.push_back(0.0);
                                }
                                for(i=0; i<vec_e_bin.size(); i++) {
                                        count = 0;
                                        for(j=0; j<vec1.size(); j++) {
                                                if(vec1[j] >= (vec_e_bin[i] - f) && vec1[j] < (vec_e_bin[i] +
f)) {

                                                        vec_sigma_bin[i] = vec_sigma_bin[i] + vec2[j];
                                                        count++;

                                                }
                                        }
                                        if(vec_sigma_bin[i] != 0.0) vec_sigma_bin[i] = (double)
vec_sigma_bin[i]/count;
                                }

                                for(i=0; i<vec_e_bin.size(); i++) {
                                        fout << vec_e_bin[i] << "," << dt << "," << vec_sigma_bin[i] << endl;
                                }
                        }

                        m1.clear();
                        v1.clear();
                        v2.clear();
                        v3.clear();
                        v4.clear();
                        pos_trend_v1.clear();
                        pos_trend_v2.clear();
                        neg_trend_v1.clear();
                        neg_trend_v2.clear();
                        vec1.clear();
                        vec2.clear();
                        vec_e_bin.clear();
```

```
                    vec_sigma_bin.clear();

            }
    }

    if(b != 1 && b != 2) {
            cout << "Error: the number entered is out of range!" << endl;
    }

    fin.close();
    fout.close();
    return 0;

}
```

BIBLIOGRAPHY

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