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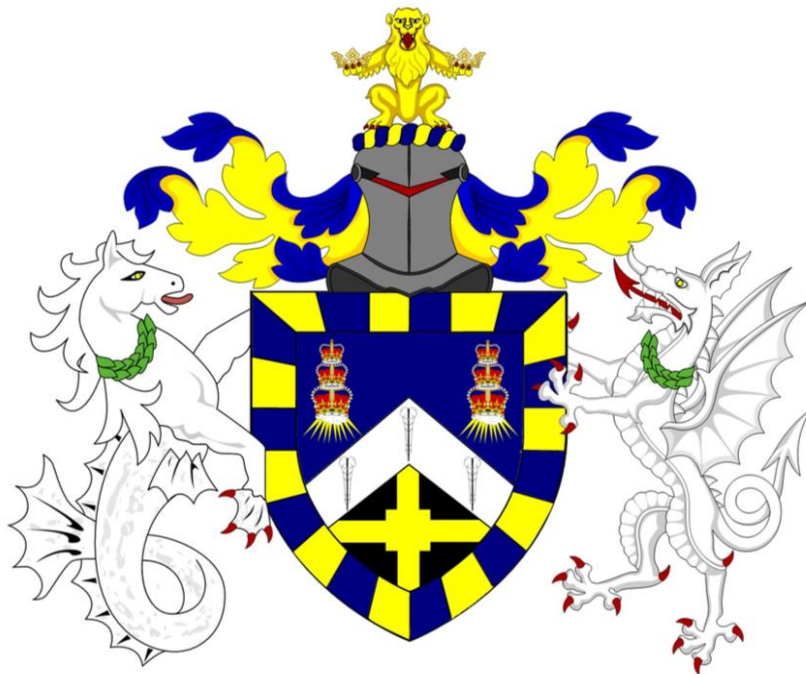
# **Analytic Signal Processing for Financial Time Series**

A study on the phase of an analytic signal

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A thesis presented for the degree of  
Master of Science in Mathematical Finance

School of Mathematical Sciences  
Queen Mary University of London

## Declaration of original work

This declaration is made on September 5, 2018

**Student Declaration:** I, Rumanah Sajid Tailor, hereby declare that the work in this thesis is my original work. I have not copied from any other students' work, work of mine submitted elsewhere, or from any other sources except where due reference or acknowledgement is made explicitly in the text, nor has any part been written for me by another person.

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## **Acknowledgement**

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## Abstract

The purpose of this thesis is to discover if there exist some information in the phase of an analytic signal, which would allow one to predict or indicate the occurrence of extreme events. In this thesis, we use a financial time series from the Nord Pool market, which contains data of the price at which electricity was traded at across 7 years from the 1<sup>st</sup> January 1999 to the 26<sup>th</sup> January 2007. The data contains seasonal changes, where we observe the prices of electricity to continuously fluctuate. Alongside these seasonal changes, unexpected extreme prices of electricity are observed. The aim is to find information in the phase, in the form of a phase slip, in the neighbourhood of these critical events; and deduce if they can allow us to predict massive unexpected changes in the price of a stored commodity.

This thesis will discover the formation of the Nordic power market and understand some of the characteristics which make electricity a unique commodity to trade. We will look at the Nordic Spot market operates and understand why electricity is a non-storable commodity.

During this thesis, we will lean on traditional concepts extracted from Electrical Engineering and Mathematics as well as, well known results brought to us by the pioneer Joseph Fourier in 1822, who is known for the establishment of what we call today a 'Fourier Transform'. We will explore the mathematical elements required to process a signal and extract the amplitude and phase of this particular financial time series.

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# 1.0 Introduction

## 1.1 What is a Signal?

The Oxford dictionary refers to a signal as “a gesture, action, or sound that is used to convey information or instructions” [Oxford Dictionary, 2018]. As humans we are endlessly surrounded by a sea of signals. From the occurrence of chemical reactions within us to the information we perceive daily. They convey information which alerts us “about the behaviour or attributes to some phenomenon” [Priemer, 1991]. Signals allow energy to be distributed across our nations, keeping us connected globally in various ways. Over many decades signals have been recorded, giving us access to historical information to analyse as well as the ability to predict future events using certain forecasting techniques. Many signals carry information which may be encoded with a message, whilst others may require the user to process or decode them in a certain way, to allow an understanding of the underlying information that they carry. Cumulating what is known thus far about the intricacies and uses of signals, it leads us to the question, why are signals extremely useful to us?

For many years, signals have been used throughout various fields. They have allowed us to stay connected with current time, from delivering live stock prices across our trading floors, to controlling our traffic systems, as well as monitoring heart rates in our hospitals. In any form of use, monitoring a signal always gives us some information. So, can we sum up exactly what a signal is?

Not exactly! To define a signal precisely will vary from field to field. As a common backbone across the fields, we may say that a signal is a “physical manifestation of information that changed with time and/or space” [Moura, 2009]. Understanding what a signal is from a mathematical point of view can be defined as “a real (or complex) valued function of one or more real variable(s)” [Anon, n.d.]. In later chapters, we shall look at different types of signals and their mathematical form. The main focus of the thesis will be around a specific type of signal known as a financial time series.

Financial time series are continuously brought to our attention [Taylor, 2008]. Information gathered from financial time series are published through various platforms such as Bloomberg, The Financial Times but also our local newspapers, radio and television screens. They give us a vast amount of information regarding the events occurring on our financial markets. For many professionals such as investors, brokers, analysts, traders and businessmen; it is essential to monitor the frequent behaviour of prices and to have the capability to predict the development of these prices in the future.

A time series in general is “a sequence of observations taken over a period of time, usually at equal intervals” [Clapham and Nicholson, 2014]. Thus, a financial time series can be defined as a sequence of observations of securities or derivatives on the stock market which has a value attached to it at certain time intervals.

Analysing a financial time series is concerned with identifying factors that influence the variation in price differences. Firstly, it is important to understand the behaviour of prices as they may become extremely volatile over a certain period. Understanding the behaviour of prices is a difficult concept. This is not something that can be concluded from ‘just’ studying the price values at certain times but further requires specific and targeted analysis of the information presented. Academics and professionals’ use their knowledge of statistical analysis and the benefits from the continuous advancement of technology, the ability to store and process large amount of information, to investigate the concept of price behaviour.

All financial time series contain an element of uncertainty [Tsay, 2006]. As humans we cannot predict the shape of the future markets by simply observing a time series alone. Alongside using our knowledge of price behaviour, it is crucial to monitor the continuous change in returns of a market. This can aid us in making better financial decisions and reduce the risk of future losses.

The uncertainty of price behaviour has opened many areas of research in finance. In recent years there has been advancement in understanding the underlying probability distribution of a financial time series and forecasting, the ability to estimate future prices using today's price. These developments have given us the capability to price options on underlying assets, make sound financial decisions and allowing us to manage any unwanted risk to our investment portfolio.

In this thesis, my aim is to use traditional techniques from the field of Electrical Engineering and Mathematics to process a financial signal which contains information about hourly electricity prices retrieved from the Nord Pool market, over a range of 7 years. I wish to analyse the phase and amplitude of the signal, in order to remove any seasonal changes, and discover if there is any information shown in the phase that would indicate the occurrence of any extreme events.

Now that we have aimed to understand what we define a signal as, we may now explore the concept of signal processing and understand where the technique widely used by investment banks and hedge funds has rooted from, and how can it be applied to finance.

## 1.1 What is Signal Processing?

The notion of *signal processing* might seem like an impenetrable concept. However, signal processing is used unconsciously by humans on a daily basis. Processing a signal may be considered as decoding information to extract something useful. When we listen to a sound signal we use our brain, as the 'operator' on the signal, to process the audio paths and extract key details. Essentially, any signal can be processed. We may even define signal processing as "acquiring, shaping and transforming data, cleaning it for the sake of improved analysis and extraction of useful information." [Flandrin, 2018]

Information Engineering and Mathematics are two disciplines that have allowed the advancement of signal processing. Development in digital technology and the rate at which we are able to process and store data, interwind with theory brought to us by the renowned French mathematician and physicist, Joseph Fourier (1768-1830); signal processing has become an extremely sought-after technique in many fields especially finance.

Signal processing can be tracked back to the findings of the pioneer Joseph Fourier (1768- 1830). During Fourier's research on heat propagation in the "The Analytical Theory of Heat", he developed a method known today as the 'Fourier Transform'. In 1822, Fourier proposed that any function of a variable, whether continuous or discontinuous, can be expanded in a series of sines and cosines of multiples of the variables [Gray, 2015]. The idea of being able to decompose a function in terms of simpler functions was already occasionally used by Leonhard Euler, while "he developed the concept of function in mathematical analysis, through which variables are related to each other and in which he advanced the use of infinitesimals and infinite quantities" [Boyer, 2018], as well as other 18<sup>th</sup> century mathematicians.

During the 19<sup>th</sup> Century, Fourier's research was a very controversial topic within the science community. Fourier's findings from heat propagation allowed him to present his ideas in several stages. Along the way, Fourier received a lot of negative feedback from the board of the Paris Academy of Science, where his work was later published. Fourier's mathematical analysis was distrusted by Joseph-Louis Lagrange (1736-1813) and Pierre-Simon Laplace (1749-1827), who were both on the

board of the Paris Academy of Science and are known to have made great contributions to the fields of Mathematics and Physics. Their rejections were due to the distrust in Fourier's expansion of functions as a trigonometric series, what we call a Fourier Series today; but also, Fourier discovered properties of trigonometric functions which were different to what they had stipulated. They believed that Fourier sums implied unusual properties, for example a discontinuous step like function had the ability to be rewritten in terms of a nice continuous function consisting of sines and cosines. Later in the thesis, we shall see examples of such transformations.

After several attempts of re-constructing his ideas, in 1822 Fourier published his research. In 1829, a German mathematician, Peter Gustav Lejeune Dirichlet (1805-1859), published a famous memoir supporting Fourier's ideas and showing that given some conditions for certain functions, the convergence of the Fourier Series holds. This began the wave of a new discovery which is known to be the stepping stone to key analysis today. In 1867 Lord Kelvin, a Scots-Irish mathematical physicist and engineer said, "Fourier's theorem is not only one of the most beautiful results of modern analysis, but it may be said to furnish an indispensable instrument in the treatment of nearly every recondite question in modern physics." [Jha, 2014] Little did he know that this concept had the ability to stretch far beyond the theory of heat propagation and into the world of finance!

The birth of signal processing was brought to us in 1965 by the American mathematicians, James Cooley (1926-2016) and John Tukey (1915- 2000), whilst working for the research division of IBM. Fourier's work had been put into practise by Cooley and Tukey, as they developed the "Fast Fourier Transform" (FFT). The FFT is an algorithmic method which allows one to take a time domain signal and transform in to a frequency domain signal. Most signals recorded from observed phenomena are discrete data points. To perform a Fourier transformation on a discrete signal requires a Discrete Fourier Transformation (DFT). The FFT is a fast-computational algorithm for the DFT. The purpose of such transformation is to identify whether the signal contains any cyclical components. The Fourier transformation deduces the frequencies present in the signal as well as the contribution they make to the signal. This leads to the question, how is this transformation and the idea of signal processing relevant to the world of finance?

To understand the application of signal processing to the financial industry, let us consider a very simple example from electrical engineering. Suppose we are given data which measures the voltage of the current at a time interval of a second. The data is presented in the following table [Nepal, 2015]:

Time (Seconds)	Voltage (Volts)
0	10
1	12
2	14
3	8
4	16
5	10
6	12
7	14
8	8
9	16
10	10
11	12
12	14
13	8
14	16



Earlier we defined a signal to be a “physical manifestation of information that changed with time and/or space” [Moura, 2009]. The data recorded is an example of a discrete signal. At each time interval, we are given the exact value of the voltage. Graphing the signal, we can visualise the fluctuations as shown below in figure 1.

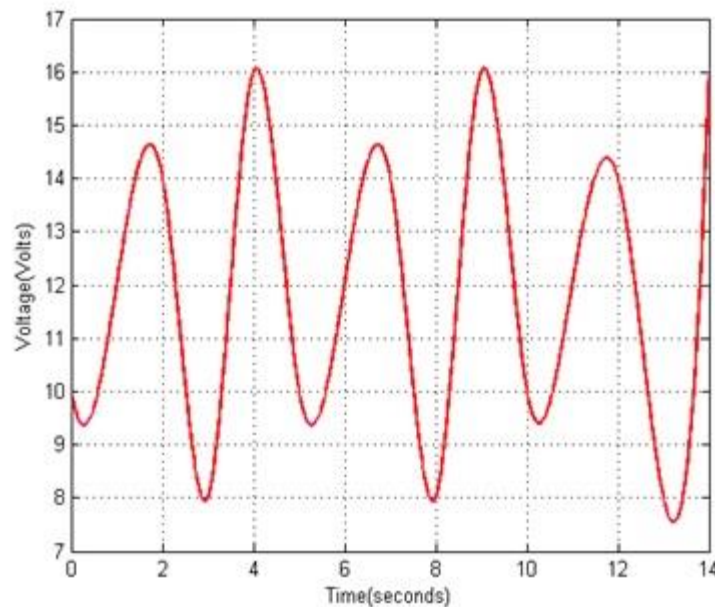


Figure 1: Fluctuations of voltage. Source Nepal, 2015.

However, the information presented tells us very little, which is where signal processing would be very helpful to an analyst. Now you may be thinking, how and why can I apply such a concept from engineering to the financial world? The answer is a simple yet profound one. Let us consider the dependent variable ‘voltage’ to be the ‘daily returns of a stock’ and the independent ‘time’ variable as the ‘day of the week’. Again, we may use the same method to produce a graph to show the fluctuations of the returns of the investing stock.

Day of the week	Daily returns of a stock (GBP, £)
0	10
1	12
2	14
3	8
4	16
5	10
6	12
7	14
8	8
9	16
10	10
11	12
12	14
13	8
14	16

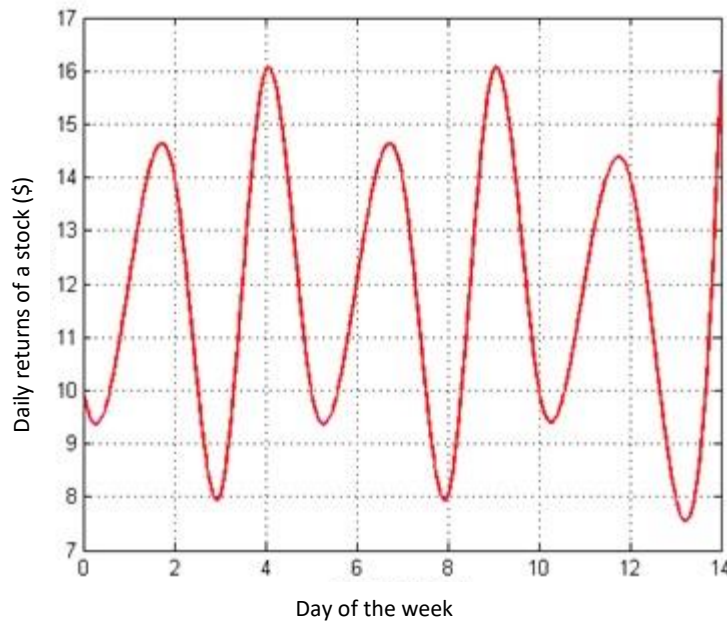


Figure 2: Fluctuations of stock return. Source Nepal, 2015.

As you can see from figure 2, no matter how the columns of the table are labelled, the method used to treat the numbers are the same. It can be said, the “crux of any signal is always in the numbers” [Nepal, 2015]. Financial data which is gathered over many years can be very difficult to comprehend, just like the voltage or current data presented. However, financial data can benefit from the same methods which are currently used in engineering.

In finance there are two prominent schools of thought [Nepal, 2015]; the fundamental analysis and technical analysis. The fundamental school believes in evaluating a securities performance by looking closely at the financial and economical impact the security has. For fundamental analyst, their end goal is to “produce a quantitative value that an investor can compare with a security’s current price” [Investopedia, 2018] in order to understand the value of their investment. Whereas the technical school of thought aims to study the historical price movements of a security and use the data to forecast future price movements. For the fundamental school there is very little use of signal processing as they avoid the use of data, such as daily price movements and rather take a more subjective view of the security. Meanwhile, the technical analyst uses a vast amount of historical data to find any trends or patterns as well as forecast future prices. The calculation required to perform such tasks can be directly applied from the field of engineering.

Major investment banks and hedge funds use signal processing as an investment strategy. Most investment strategies which include signal processing techniques are kept a secret, as banks and hedge funds are not required by government regulations to unravel these gems, which give their firms a competitive edge over their rivals. According to Robin Wigglesworth from the Financial times, Goldman Sachs’ is known to use signal processing as a successful tool for their quantitative analysts (also known as quants) to detect small but persistent signals, which they can feed into their trading algorithms. In 2017 the firm was able to manage \$91.8bn worth of investments using signal processing techniques [Wigglesworth, 2018]. In recent years there has been a rapid growth in algorithmic investing which is a trading method merely using some form of computer techniques. High-frequency trading is a branch of algorithmic trading which heavily relies on signal processing, as quants take advantage of the random fluctuations in the market to generate a profitable return on their investments. There

are many other investment strategies where signal processing can be used. These strategies are deeply rooted in mathematics and rely on a fast-computational algorithm such as the FFT to pass through all the data points of the signal and understand the information they carry about the markets or investment portfolios.

Techniques used in signal processing are not shared widely by firms which use them to their advantage. In this thesis, I wish to explore the mathematical element which is the building block to signal processing, as I aim to extract information from the frequency signal of the Nord Pool market electricity prices; by studying the phase of the frequency signal and deducing if any phase slips may give an indication to the occurrence of extreme events.

## 2.0 Background of Nord Pool Data Set

### 2.1 Nord Pool Market

In this thesis, we shall be experimenting on data provided by the Nord Pool market. The data is formed of electricity prices which have been recorded at an hourly time interval across 7 years, starting from the 1<sup>st</sup> January 1999 until the 26<sup>th</sup> January 2007. The data may be represented as a financial time series as shown below. Figure 3 illustrates the financial time series of the Nord Pool electricity prices which has been produced using Matlab, and the code can be found in the appendix of the thesis.

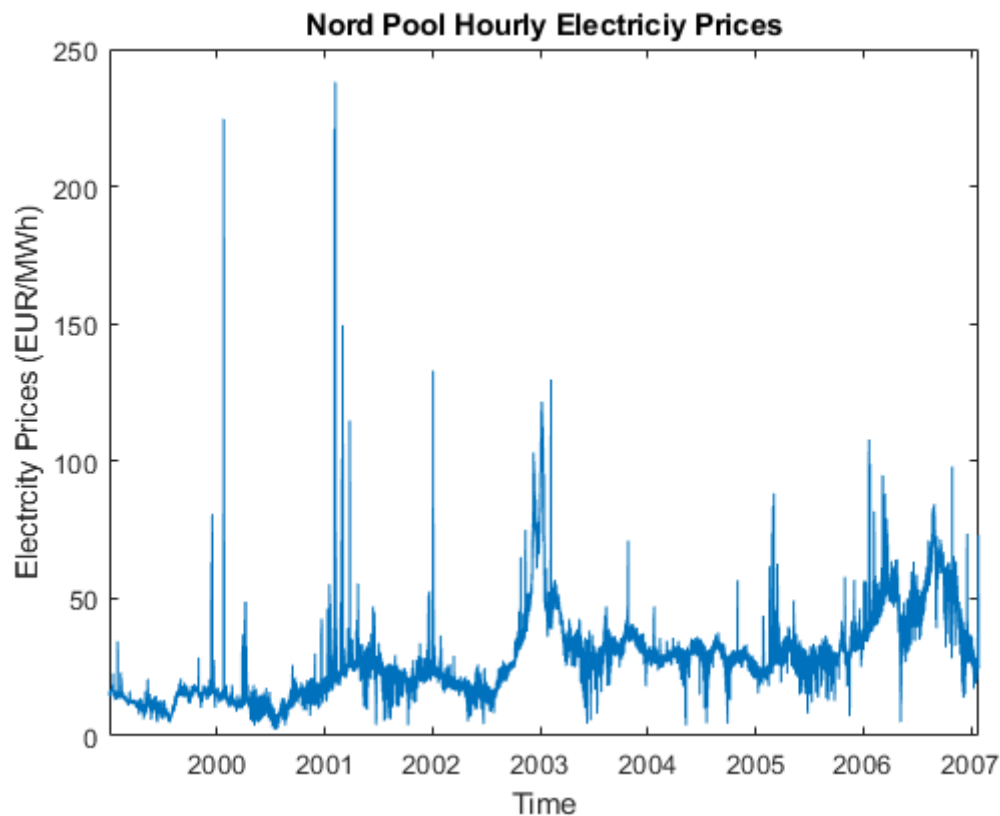


Figure 3: Nord Pool Hourly Electricity Prices. Published with MATLAB® R2017b

Throughout the course of this thesis, the term 'Nord Pool' occurs at various points, and may seem to be a cryptic term to those unfamiliar with it.

The Nordic electricity market, which is the leading power market in Europe, is known as Nord Pool. They were created in the 1990s after the Norwegian parliament decided to deregulate the market for

trading electricity, and create a single electricity market with a common power exchange in 1991. [Erzgräber et al., 2008]. Today, Nord Pool continue to trade and deliver power in the Nordic, Baltic (Estonia, Latvia and Lithuania) and UK day-ahead markets.

The Nordic electricity market are comprised of Denmark, Finland, Norway and Sweden [Bergman, 2003]. In terms of population, the Nordic countries, who trade in the Nord Pool market, are said to be small. However according to Bergman (2003), they are known to have a high level of consumption of electricity per capita. The 2001 electricity consumptions stretched as high as 393 TWh [Bergman, 2003], and in 2017 a total of 512 TWh of power was traded [NordPoolGroup, 2018].

During the 1990s the idea of creating a single power market was due to benefiting from eradicating the border tariffs on electricity between the countries. The single market is thought to be well integrated if and only if there is a single price of electricity or power which can be traded across the countries which trade on the Nordic market [Bergman, 2003]. The reforms of the Norwegian electricity market had spread across the borders. In 1996 a joint Norwegian-Swedish power exchange was established and grew to include Finland in 1998, and lastly Denmark in 2004 [Erzgräber et al., 2008]. To some trading on the Nord Pool market can seem quite complicated. Let us see how the electricity markets function and how power is traded on the Nord Pool power market.

When electricity is produced by companies, it is transmitted to a grid. Across the participating countries there are many providers who supply electricity. When a switch is turned on, it cannot be said where the power was generated. Producers buy and sell electricity on markets such as Nord Pool. Nord Pool is a market which allows one to buy and sell physical power contracts and financial contracts. When stating that 'one' can buy or sell such a contract, the pronoun 'one' typically refers to large players, such as power suppliers, traders, brokers, energy companies and large-scale consumers or enterprises, in the power market that trade directly on the Nord Pool exchange [The Norwegian Ministry of Petroleum and Energy, 2015]. Small consumers such as home owners do not trade or purchase electricity directly on the Nord Pool marketplace, as the transaction costs of such financial contracts can be quite costly. Rather they buy fixed price contracts from electricity suppliers which are called the end user market.

On the Nordic power exchange, a different story is told, and it is not so straightforward. The Nord Pool market consists of the exchange Nord Pool ASA, named in 1996 and Nord Pool Spot AS, named in 2002. Nord Pool ASA is the financial marketplace for those countries who trade on the Nord Pool market. It is currently owned by both Nordic (Statnett SF in Norway (50%) and Affärverket Svenska Kraftnät (50%)) [Erzgräber et al., 2008] and Baltic transmission operators. Now, Nord Pool Spot AS is the marketplace where physical power contracts are traded. A physical power contract is essentially a contract for the physical delivery of electricity for the following day. The price at which electricity is sold at on the market is determined each day at an hourly rate on the Nord Pool Spot exchange, for the following day. Just like any other commodity, the spot price, which is the current price at which the commodity trades at, of electricity will depend on the supply and demand which can be affected by several factors. These factors include economic growth of the country, production of the electricity, transmission costs and conditions between then Nordic regions and other exporting countries.

The spot market operates in the following manner. There is an auction where buyers and sellers can bid to either buy or sell power contracts, where each contract provides one hour of electricity, which covers the 24-hours of the following day. There is a deadline at which all the buy and sell orders are collected and gathered into a supply and demand curve for each hour of power which is delivered. Using these supply and demand curves, an equilibrium price is calculated, which is then known as the hourly spot price for electricity for that specific hour [Erzgräber et al., 2008]. This electricity spot

market is called the Elspot, which is the day-ahead market, where at 12:00 CET the spot price for each power contract is calculated. A day-ahead market is where one can buy or sell a financial contract which has a maturity date of the following day. The Elspot market is responsible for nearly 80% of the hourly consumed electricity trades for the Nord Pool markets [Knapik, 2017]. The Nord Pool Spot AS also consists of Elbas which is the continuous intraday market. An intraday market is a trading platform where the buyer or seller of the financial contract must complete all transactions on the same day before the market closes. This type of trading is also known as 'Day Time' trading. On the Nordic markets, Elbas is used to give traders the opportunity to trade in 'real time'. Trading on Elbas takes place around the clock until the hour before delivery. The Nordic markets and other similar markets are moving towards more of a clean energy source, which is known to be unreliable. Elbas is becoming more important with this transition into renewable energy especially with the arrival of wind turbines. This is because by nature wind turbines may not deliver a consistent amount of electricity unlike traditional methods, and thus production will fluctuate with relations to the consumption of electricity on the Elspot [Knapik, 2017]. With Elbas in place, it will allow the markets to cover any losses or gains in the production quantity of electricity by giving traders, brokers and other large consumers the opportunity to trade within the same day.

Nord Pool ASA also allows market makers to trade derivatives on electricity as the underlying asset. Electricity can also be traded as a method for a trader to hedge their portfolios against some risk without physically delivering the asset. It is permissible to write long-term futures and forward contracts on electricity which are mark-to-market daily, and can be traded on Elspot where players can take opportunities of the price differences between the 'system price' (or the equilibrium price) [Erzgräber et al., 2008] and the bid-ask spread on each of the contracts, depending on whether the market is in backwardation or in contango [Mork, 2006]. With forward contracts there is no settlement during the trading period or even before the expiry date of the forward. Nord Pool AS also has a clearing house which settles and clears any financial contracts. The clearing house controls the transaction between the buyer and seller, in addition to regulating any transaction in accordance to the Ministry of Finance regulations. The clearing house reduces any counterparty risk or defaults making the Nordic power market an efficient platform to trade in.

## **2.2 Characteristics of Electricity Prices**

When we trade commodities such as wheat or metal, using derivatives such as forwards or futures which are physically settled, those with insight into the industry are aware of what and how the underlying commodity is traded. However, when trading electricity, it may not be as intuitive as one may think. Is electricity considered to be a tangible good which can be physically stored in a battery, or transferred from one place to another, as opposed to a physical good which cannot be seen or touched? To understand how electricity is traded or even priced, it is essential and valuable to firstly recognise the unique characteristics of electricity and how one would define what electricity is.

The Oxford Dictionary defines electricity to be "a form of energy resulting from the existence of charged particles (such as electrons and protons), either statically as an accumulation of charge or dynamically as a current." [Oxford Dictionary, 2018]. In this thesis, let us consider electricity to be a form of energy which is produced by the flow of charged particles as described above. Today, in some areas of the world, electricity has become easily accessible via our distribution system set in place. However, it is important to recognise some of the key features which have shaped our current power markets and given us the ability to trade electricity using financial contracts.

Firstly, we must be aware of the inadequacy of transferring electricity across continents. For example, electricity produced in the United States is not transferred to the United Kingdom for consumption

due to the impracticalities of reaching cables across the North Atlantic Ocean! This feature prevents the creation of a global electricity market [Burger, Schindlmayr, Graeber, 2014]. Despite the difficulty with countries across oceans, we can build markets comprised of neighbouring countries, just like the Nordic power market, as well as trade in different markets which are not located in our home country.

Another vital characteristic which needs to be recognised is that electricity has a non-storable property, as explained by Burger, Schindlmayr, Graeber (2014). The inability to store electricity has been a main concern for traders, brokers and large enterprises as this characteristic results in high volatility of power prices in the spot market; especially when there is an imbalance of supply and demand for the commodity. This is the reason as to why the Nordic Spot AS has two types of trading platform, Elspot and Elbas. Gianfreda and Grossi (2012) discusses the same feature of electricity and deemed it to be a new commodity with unique characteristics. As electricity prices become highly volatile, it is essential for traders, investors, brokers and other business professionals to monitor the frequent behaviour of electricity prices, so that they can build models which gives them the ability to forecast the future prices, which becomes a key factor in risk management and pricing derivatives on the underlying commodity [Gianfreda and Grossi, 2012]. When forward contracts are traded as electricity as the underlying asset, according to Burger, Schindlmayr, Graeber (2014) the prices are less volatile as the production of power and dependent factors such as weather conditions mean that the demand for power is yet unknown.

When investing in many commodities, as an investor, it is ideal to comprehend the benefits from physically holding an asset which is not obtained from holding a futures contract on the asset. This benefit is known as the convenience yield [Boyle and McDougall, 2011]. The convenience yield,  $y$ , is backed out from the market spot prices by using the reverse operation on the following relationships:

$$F_0 = S_0 e^{(r+u-y)T}$$

Where,  $F_0$ - Forward or the future price

$S_0$ - Spot price of the commodity

$T$ - Time to delivery (in years)

$r$ - Risk-free rate annualised

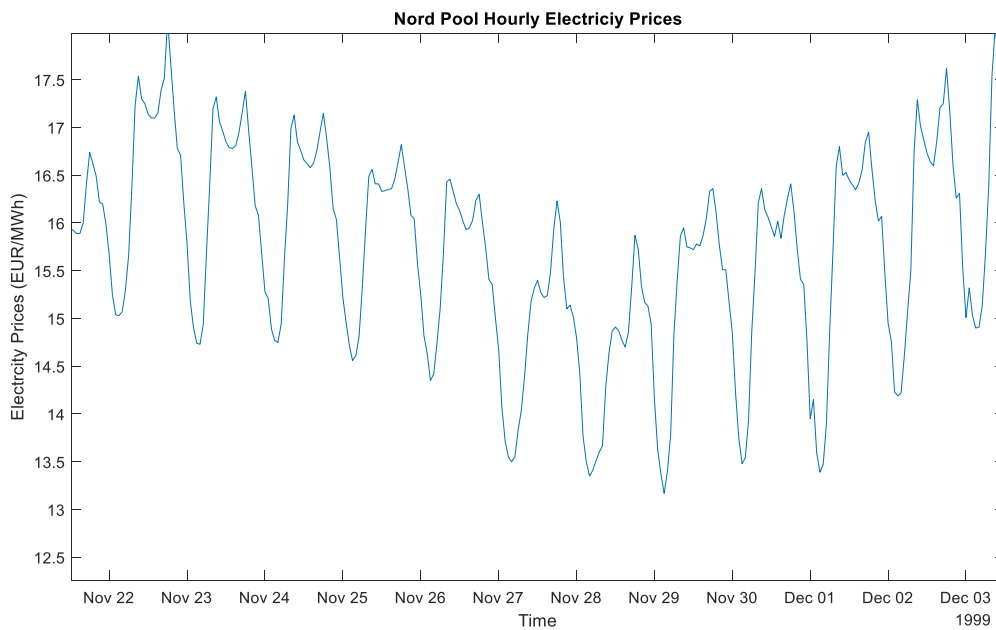
$u$ - Percentage storage cost of the commodity

$y$ - Convenience yield

According to German and Roncoroni (2006), since there is no technology to store power, the idea of having a convince yield for electricity does not make sense, as an investor cannot have any benefit from holding electricity and neither is there any storage costs attached to it. We also know that for any storable commodity the futures price should converge to the spot price, the closer we get to the delivery date. However, for electricity, as it is not a storable commodity this hypothesis does not hold. This means that for electricity, the spot price should reflect the properties of power, the fact that it cannot be stored, and neither can it be transported in a way to meet the demands of a global power market.

As mentioned earlier, the data set used in this thesis is from Nord Pool market and we shall assume that the electricity cannot be stored. As we study the financial time series from the 1<sup>st</sup> January 1999 to the 26<sup>th</sup> January 2007, we see random occurrence of extreme spikes, which can be explained by firstly the outcome of the inability to store electricity, but also by the demand for electricity at various times of the year. By comparing the financial time series to the different seasons of the year, we can

make the conclusion that during the winter periods of the Nord Pool regions, due to the extremely cold temperatures there is a higher demand for heating. This would result in a higher electricity demand by households within the region. Now looking to the summer periods, we see that there is a dip in the prices of electricity, which may explain that there is not a high demand during the summer periods. Studying figure 4, which is extracted from the Nord Pool financial time series, we can analyse the electricity prices to be increasing and decreasing at certain times during the day. Studying the price of electricity across the twelve days we notice a pattern which appears repeatedly. The price of electricity is at its lowest for the day around the hours of 00:00. As we begin the day, around early morning to midday, we see that the price of electricity starts to increase, with a small drop between 12:00 and mid-afternoon. We again see a peak in the price around mid-afternoon to early evening before the price of electricity drops again for the cycle to repeat.



*Figure 4: Repetition in Nord Pool's electricity prices between 22 Nov 1999 and 03 Dec 1999. Published with MATLAB® R2017b*

Looking at these extreme spikes which occur at random times in figure 3, it can be useful to investors, traders and large market players, to be able to predict the occurrence of these spikes in order to maximise their investment portfolios. In this thesis our goal is to decompose the financial time series containing electricity prices traded on the Nord Pool market, using methods which will be explained in later chapters; to extract information from the frequency signal and study if the occurrence of any phase slips may give any indication to the existence of any if these extreme events.

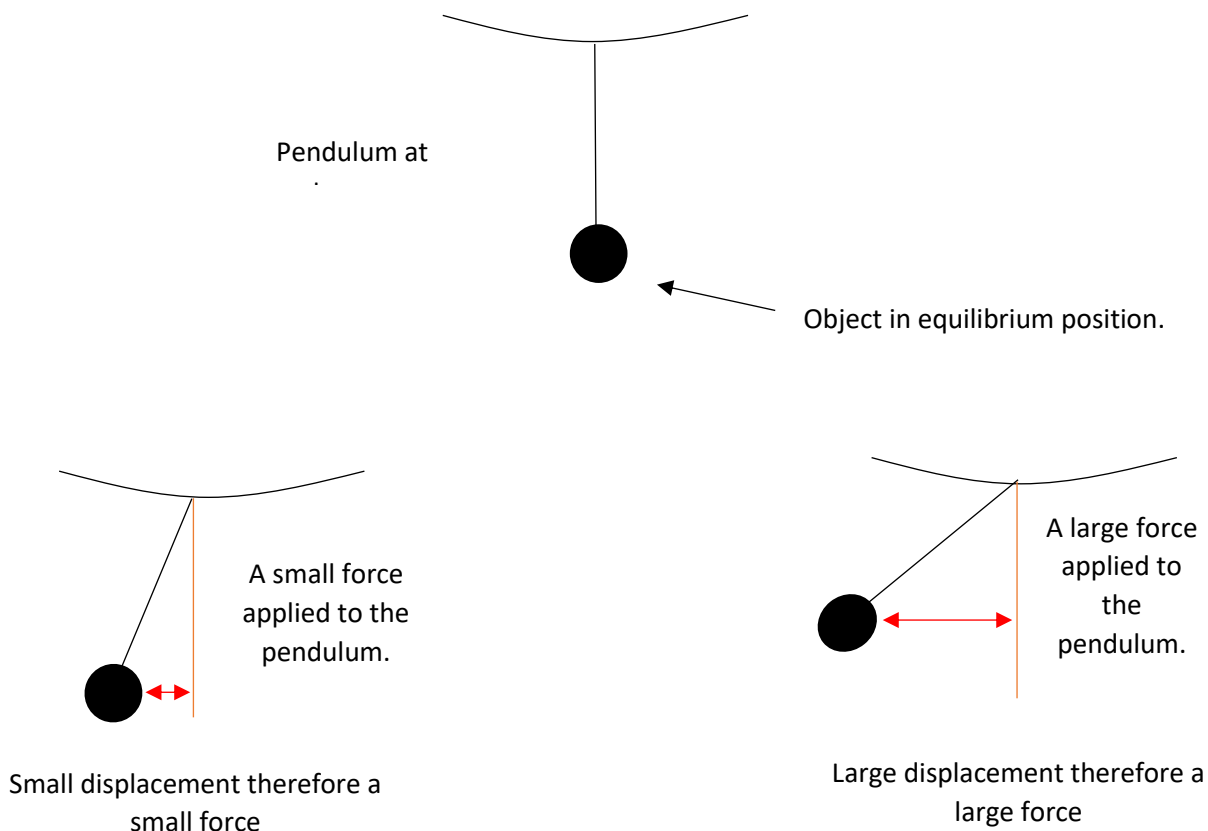
### 3.0 Mathematical Background of Signal Processing

In this section of the thesis, I shall explore the mathematical elements which form the building block to signal processing. It is an ideal place to begin by clearly defining what a signal is mathematically and the different components needed to construct a signal.

#### 3.1 Simple Harmonic Functions: Amplitude and Phase

In many areas of Mathematics and Physics, the term ‘Simple Harmonic Motion’ surfaces on many occasions. Let us go back in time to recall the idea of Simple Harmonic Motion (SMH) and further explore how the concept can be defined mathematically.

Going back to high school, in our physics courses we always tend to study the motion of an object which oscillated to and fro, either side of an equilibrium position. The equilibrium position is defined to be the midpoint of the object’s motion. As the object moves from side to side, the distance from the midpoint is measured and called the displacement of the object. When an object moves back and forth there is always a restoring force pulling or pushing the object. This restoring force makes the object accelerate towards the equilibrium position, which is the centre of mass of the object. A simple example to consider is a swinging pendulum. When the pendulum is at rest, the object is in equilibrium position, as shown in the figure 5 below. Now, when a small force is applied to the pendulum, we measure a smaller displacement from the equilibrium position than when a large force is applied which is clearly seen in the diagram.



*Figure 5: Swinging pendulum in motion.*



The swinging of a pendulum is the simplest form of an oscillatory motion which is in Simple Harmonic Motion. We may define Simple Harmonic Motion to be an oscillation in which the acceleration of an object is directly proportional to its displacement from its equilibrium position and is directed towards the equilibrium. It is possible for one to draw graphs to show the displacement, velocity and acceleration of an oscillating object which is in simple harmonic motion. Such graphs will consist of functions which include sine and cosine waves as demonstrated below in figure 6.

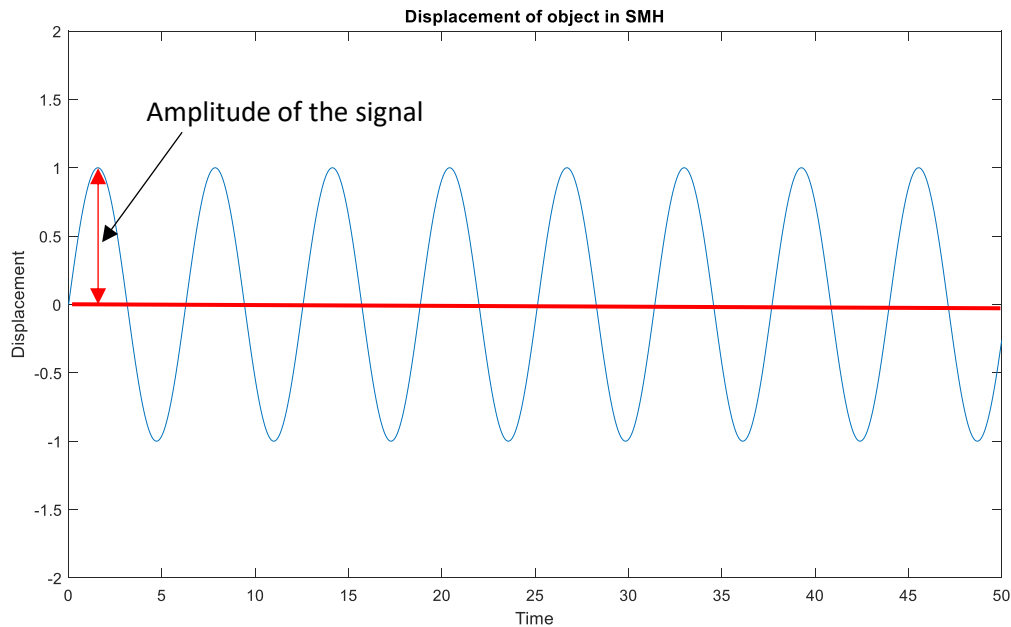


Figure 6: Displacement of object in SMH. Published with MATLAB® R2017b

Taking this concept from physics we may now define a simple harmonic function mathematically. A function  $x(t)$  is called a simple harmonic function (or harmonic oscillation) if

$$x(t) = a \cos(\phi(t)) \text{ or } x(t) = a \sin(\phi(t))$$

And

$$\phi(t) = \omega t$$

Where we can define,

$a$ - The amplitude or the argument of the trigonometric function and  $a \in \mathbb{R}, a > 0$

$\phi(t)$ - The phase shift

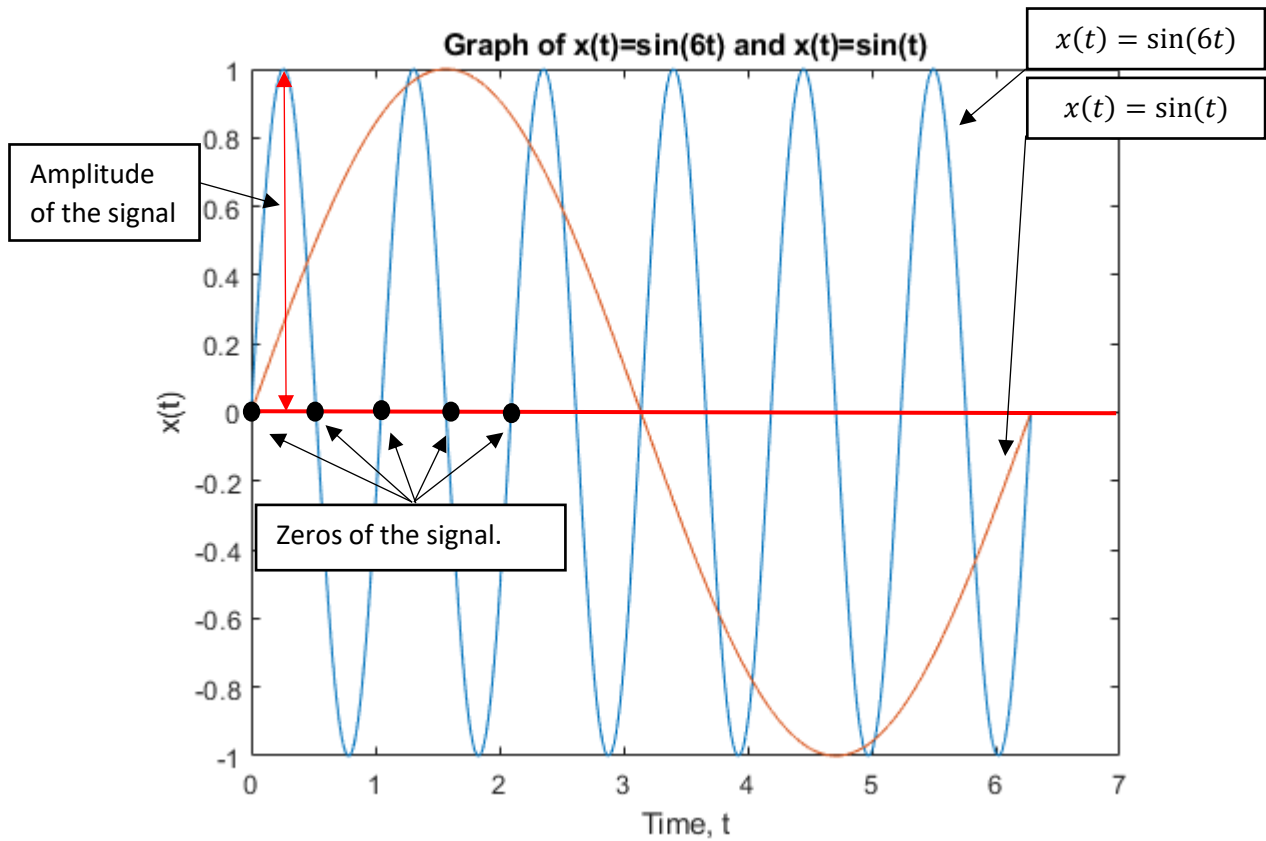


Figure 7: Graph of  $x(t) = \sin(6t)$  and  $x(t) = \sin(t)$ . Published with MATLAB® R2017b

The amplitude is defined to be the height of the peaks and the phase determines the number of cycles in the same distance along the x-axis. If the phase increases by a factor of  $\pi$  then the function  $x(t)$  moves from one zero to the next. The zeros of the function are the points where the curve crosses the x-axis. The phase of the function tells us about the distribution of the zeros, maxima and minima. Figure 7 above illustrates the amplitude, phase and the zeros of a signal. We may see that  $x(t) = \sin(6t)$  has 6 times as many cycles in the same distance along the x-axis than  $x(t) = \sin(t)$ , which helps us identify the phase of the signal.

In Physics and Engineering, a signal is simply a function which is composed of sine and cosine waves. A simple harmonic function is a general signal. In this thesis, we shall use the terms function and signal interchangeably.

Now that we have defined what a simple harmonic function is, our aim is to decompose a general signal  $x(t)$  and find the instantaneous amplitude and phase of the signal. Let us work with the function  $x(t)$  such that,

$$x(t) = A(t) \cos(\phi(t))$$

There is no unique way to decompose such a signal. By this we mean that, if we compare  $x(t)$  to our definition of a simple harmonic function, we could either let  $A(t) = a$  and  $\phi(t) = \omega t$  or we may even consider  $A(t) = a \cos(\omega t)$  and  $\phi(t) = 0$ . Either method would have been suitable to decompose the signal, however in our case let us define  $A(t)$  to be the amplitude and  $\phi(t)$  to be the phase of the signal.

To produce an algorithm to find the amplitude  $A(t)$  and the phase  $\phi(t)$  of such a signal, let us consider the following example. Let,

$$x(t) = A(t) \cos(\phi(t)) \text{ where } A(t) = a \text{ and } \phi(t) = \omega t$$

$$\Rightarrow x(t) = a \cos(\omega t), \text{ where } a > 0$$

Just like in the definition of a simple harmonic function. Now, to make our computation easier we wish to express cosine in terms of complex exponential. We know from our first-year calculus course, the relationship between cosine, sine and complex exponentials, which is brought to us by Euler's formula:

$$e^{\pm i\theta} = \cos(\theta) \pm i \sin(\theta) \text{ for any } \theta \in \mathbb{R}$$

Thus,

$$\cos(\theta) = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

$$\sin(\theta) = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$$

We may now express  $x(t)$  in terms of a complex exponential Fourier polynomial. A Fourier polynomial is simply a Fourier series which is the expansion of a periodic function  $f(x)$  in terms of an infinite sum of simple function such as sine and cosine. Here we have taken a step further and used Euler's formula to further expand the signal in terms of complex exponentials. Thus, we get that

$$x(t) = a \left( \frac{e^{i\omega t} + e^{-i\omega t}}{2} \right)$$

$$\Rightarrow x(t) = \frac{a}{2} e^{i\omega t} + \frac{a}{2} e^{-i\omega t}$$

To define a new time dependent complex signal, we wish to only consider the positive frequency part of  $x(t)$ . To be precise, we wish to have twice the positive frequency. The new time dependent complex signal (or also referred to as an analytic signal) will be denoted by  $z(t)$ , where

$$z(t) = a e^{i\omega t} \text{ where } t \in \mathbb{R}, a \in \mathbb{R}, a > 0$$

The new analytic signal  $z(t)$  is in polar form. Let  $Z \in \mathbb{C}$  (a complex number), where  $Z = r e^{i\theta}$ . Then if

- $Z \neq 0$
- $|Z| = r$  (the absolute value of  $Z$ )
- $\arg(Z) = \theta$

$Z$  is said to be in polar form.

Now that we have defined what a polar form is, we may use properties of complex numbers to find the absolute value and the complex argument of  $z(t)$ . We find the absolute value of  $z(t)$  by:

$$|z(t)| = |a e^{i\omega t}|$$

$$\Rightarrow |z(t)| = |a| |e^{i\omega t}| \quad (\text{As } a \text{ is a real number})$$

$$\Rightarrow |z(t)| = |a| |\cos(\omega t) + i \sin(\omega t)| \quad (\text{Expanding } z(t) \text{ using Euler's formula})$$

$$\Rightarrow |z(t)| = |a| \sqrt{\cos^2(\omega t) + \sin^2(\omega t)} \quad (\text{Using identity } \cos^2 x + \sin^2 x = 1)$$

$$\Rightarrow |z(t)| = |a| \sqrt{1}$$

$$\Rightarrow |z(t)| = |a| \quad (\text{assuming } a > 0)$$

$$\Rightarrow |z(t)| = a$$

$$\Rightarrow |z(t)| = A(t)$$

As  $z(t)$  is in polar form, we know that  $\arg(z(t)) = \omega t = \phi(t)$ . Thus, we have found the amplitude of the signal,  $A(t) = a$ , and the complex argument  $\arg(z(t)) = \omega t$  which is the phase of the signal. We can further analyse that

$$x(t) = \text{Re}(z(t))$$

This relationship will become important in our next section. Using the simple harmonic function, we can now generate the general algorithm of how to decompose a signal to find the amplitude and phase. The general algorithm is:

- Given a signal  $x(t)$ , rewrite the signal in terms of a complex exponential Fourier polynomial.
- Define a new analytic signal,  $z(t)$  ensuring to keep twice the positive frequency part only.
- To find the amplitude  $A(t)$ ,  $A(t) = |z(t)|$ .
- To find the phase  $\phi(t)$ ,  $\phi(t) = \arg(z(t))$

### 3.2 An Analytic Signal Example

Let us illustrate the general idea of extracting the amplitude and phase of a signal, by applying the algorithm described above to an expression containing two harmonic functions. Let us start with the signal  $x(t)$  which is defined as,

$$x(t) = 0.5 \cos(0.9t) + 1.5 \cos(1.1t), \text{ where } t \in \mathbb{R}$$

Where  $t$  is defined to be the time dependent variable.

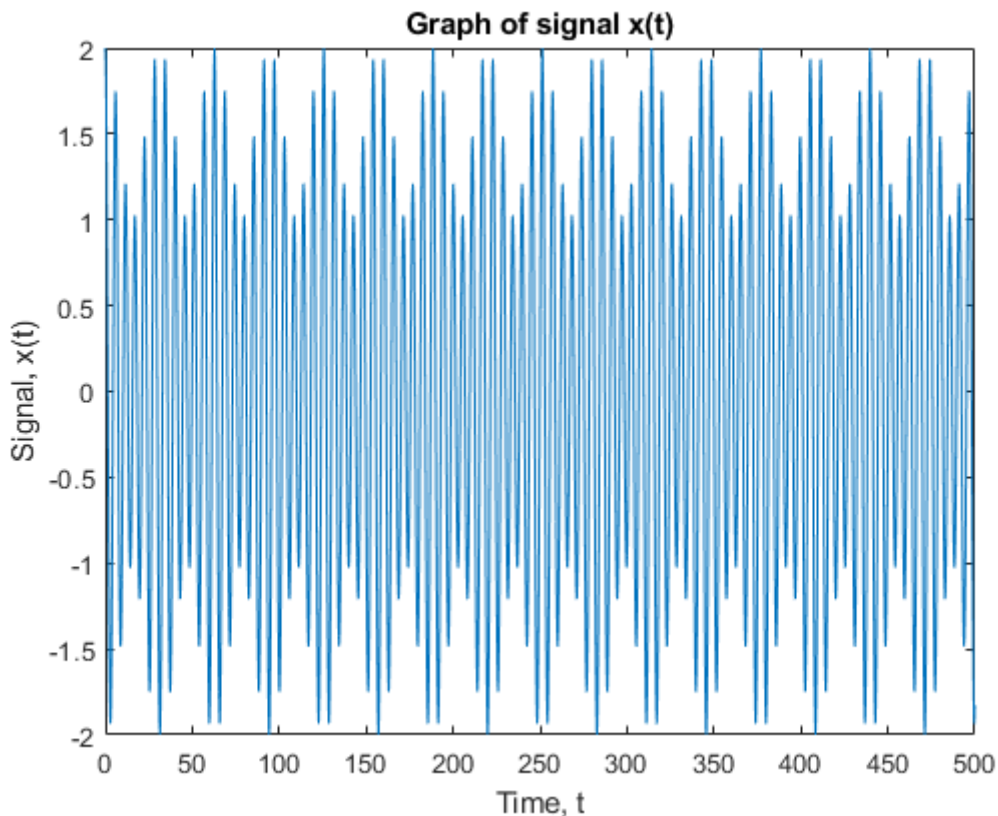


Figure 8: Graph of signal  $x(t)$ . Published with MATLAB® R2017b

The signal  $x(t)$  has been plotted on a graph using Matlab and show in figure 8 above. The code can be found in the appendix of the thesis. Functions used in Matlab code will be explained as we go along the thesis.

Now, using the algorithm above, our first step is to write the signal  $x(t)$  in terms of a complex exponential Fourier polynomial. Before we do so, let us define each component of  $x(t)$  in terms of a complex exponential. We know that by using Euler's formula we can express cosine as:

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}, x \in \mathbb{R}$$

Thus,

$$\cos(0.9t) = \frac{e^{i0.9t} + e^{-i0.9t}}{2}, t \in \mathbb{R}$$

$$\cos(1.1t) = \frac{e^{i1.1t} + e^{-i1.1t}}{2}, t \in \mathbb{R}$$

Substituting each of the above terms into the signal  $x(t)$  gives us,

$$\begin{aligned} x(t) &= 0.5 \left( \frac{e^{i0.9t} + e^{-i0.9t}}{2} \right) + 1.5 \left( \frac{e^{i1.1t} + e^{-i1.1t}}{2} \right) \\ \Rightarrow x(t) &= \frac{0.5}{2} e^{i0.9t} + \frac{0.5}{2} e^{-i0.9t} + \frac{1.5}{2} e^{i1.1t} + \frac{1.5}{2} e^{-i1.1t} \end{aligned}$$

Now that we have written the signal in terms of a Fourier polynomial, our aim is to define a new analytic signal by only keeping the positive frequency part. The new signal is denoted as  $z(t)$  and is defined as,

$$z(t) = 0.5e^{i0.9t} + 1.5e^{i1.1t}, t \in \mathbb{R}$$

As we mentioned, the relationship  $x(t) = \text{Re}(z(t))$  will become important. We may now show why this relationship holds. In order to demonstrate this, we must use Euler's formula to rewrite the terms  $e^{i0.9t}$  and  $e^{i1.1t}$  as

$$e^{i0.9t} = \cos(0.9t) + i \sin(0.9t)$$

And,

$$e^{i1.1t} = \cos(1.1t) + i \sin(1.1t)$$

Substituting the two terms into  $z(t)$  gives us

$$\begin{aligned} z(t) &= 0.5[\cos(0.9t) + i \sin(0.9t)] + 1.5[\cos(1.1t) + i \sin(1.1t)] \\ \Rightarrow z(t) &= \underbrace{[0.5 \cos(0.9t) + 1.5 \cos(1.1t)]}_{\text{Real part}} + i \underbrace{[0.5 \sin(0.9t) + 1.5 \sin(1.1t)]}_{\text{Imaginary part}} \end{aligned}$$

If you look at  $z(t)$ , you will notice it is of the form of a complex number, where  $z(t)$  consists of a real part namely  $0.5 \cos(0.9t) + 1.5 \cos(1.1t)$  and an imaginary part  $0.5 \sin(0.9t) + 1.5 \sin(1.1t)$ . Thus,

$$\text{Re}(z(t)) = 0.5 \cos(0.9t) + 1.5 \cos(1.1t) = x(t)$$

We have shown that  $x(t) = \text{Re}(z(t))$  holds.

Now going back to the next step of the algorithm, we may denote the instantaneous amplitude to be  $A(t) = |z(t)|$  and the phase of the signal to be  $\phi(t) = \arg(z(t))$ . We defined earlier that the new analytic signal will hold the form

$$z(t) = A(t)e^{i\phi(t)}, t \in \mathbb{R}$$

Hence, using the property which we just showed,  $x(t) = \text{Re}(z(t))$ , we have that

$$\begin{aligned} x(t) &= \text{Re}(A(t)e^{i\phi(t)}) \quad (\text{Using Euler's formula to expand}) \\ \Rightarrow x(t) &= A(t)\cos(\phi(t)) \end{aligned}$$

Our aim is to explicitly find  $A(t)$  and  $\phi(t)$ . We can easily compute the amplitude,  $A(t)$  and the phase,  $\phi(t)$  from  $z(t) = 0.5e^{i0.9t} + 1.5e^{i1.1t}$ . Before we compute the amplitude, we must prove a property of complex numbers which will be used in our calculations. This proposition was extracted from Beheshti (2016).

**Proposition 1:**

Given two complex numbers  $z, w \in \mathbb{C}$ . Then

$$|zw| = |z||w|$$

Proof:

Consider

$$|(zw)^2| = (zw)(\overline{zw}) \quad (\text{By property of complex numbers, } |z|^2 = z\bar{z})$$

Where,  $\overline{zw}$  is the complex conjugate of  $zw$ .

$$\Rightarrow |(zw)^2| = (zw)(\overline{zw}) \quad (\text{By closure property of complex numbers, } \mathbb{C})$$

$$\Rightarrow |(zw)^2| = (z\bar{z})(w\bar{w}) \quad (\text{By commutativity of complex numbers, } \mathbb{C})$$

$$\Rightarrow |(zw)^2| = |z|^2|w|^2 \quad (\text{By property of complex numbers, } |z|^2 = z\bar{z})$$

$$\Rightarrow |(zw)^2| = |z||w| \quad \square$$

We may now use the above, proposition 1, to compute the amplitude  $A(t)$ :

$$A(t) = |z(t)| = |0.5e^{i0.9t} + 1.5e^{i1.1t}|$$

$$\Rightarrow A(t) = \left| 1.5e^{i1.1t} \left( 1 + \frac{0.5}{1.5}e^{i(1.1-0.9)t} \right) \right|$$

$$\Rightarrow A(t) = \underbrace{|1.5e^{i1.1t}|}_{(1)} \underbrace{\left| 1 + \frac{0.5}{1.5}e^{i(1.1-0.9)t} \right|}_{(2)} \quad (\text{Using Proposition 1})$$

①

②

Let us compute each part separately:

①

$$\begin{aligned}
 |1.5e^{i1.1t}| &= |1.5(\cos(1.1t) + i \sin(1.1t))| \quad (\text{Using Euler's Formula}) \\
 &\Rightarrow 1.5|\cos(1.1t) + i \sin(1.1t)| \\
 &\Rightarrow 1.5\sqrt{\cos^2(1.1t) + \sin^2(1.1t)} \quad (\text{Using definition of modulus of a complex number}) \\
 &\Rightarrow |1.5e^{i1.1t}| = 1.5 \quad (\text{using the identity } \cos^2 x + \sin^2 x = 1)
 \end{aligned}$$

②

$$\begin{aligned}
 \left| 1 + \frac{0.5}{1.5} e^{i(1.1-0.9)t} \right| &= \left| 1 + \frac{0.5}{1.5} (\cos((1.1-0.9)t) + i \sin((1.1-0.9)t)) \right| \\
 &\Rightarrow \left| \underbrace{1 + \frac{0.5}{1.5} \cos((1.1-0.9)t)}_{\text{Real part}} + i \underbrace{\frac{0.5}{1.5} \sin((1.1-0.9)t)}_{\text{Imaginary part}} \right| \\
 &\Rightarrow \left| 1 + \frac{0.5}{1.5} e^{i(1.1-0.9)t} \right| = \sqrt{\left( 1 + \frac{0.5}{1.5} \cos((1.1-0.9)t) \right)^2 + \left( \frac{0.5}{1.5} \sin((1.1-0.9)t) \right)^2}
 \end{aligned}$$

Where the last step was calculated by applying the definition of the modulus of a complex number. We may now define the amplitude,  $A(t)$  to be:

$$A(t) = 1.5 \sqrt{\left( 1 + \frac{0.5}{1.5} \cos((1.1-0.9)t) \right)^2 + \left( \frac{0.5}{1.5} \sin((1.1-0.9)t) \right)^2}$$

We may verify that  $A(t)$  is indeed the amplitude of the signal  $x(t)$ , by plotting both the amplitude and the signal on the same graph as shown in figure 9 below:

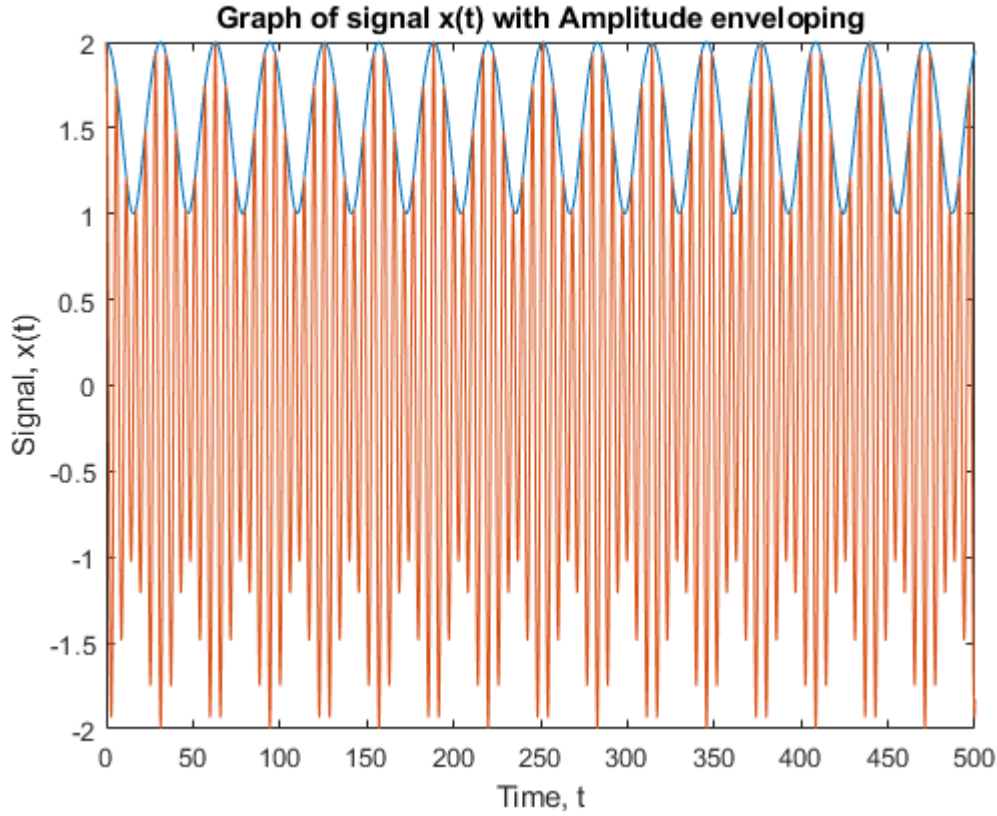


Figure 9: Graph of signal  $x(t)$  with Amplitude enveloping. Published with MATLAB® R2017b

As you can see from the graph in figure 9, the amplitude  $A(t)$  which is highlighted in blue, is a perfect envelope of the signal  $x(t)$ ; which is shown on the graph as the orange oscillating curve. An envelope of an oscillating signal is a smooth curve which outlines the extremes of the signal. Here the amplitude is a perfect envelope for the signal.

Before we move further to find the phase  $\phi(t)$ , we need to prove another property of complex numbers. This proposition was extracted from Beheshti (2016).

**Proposition 2:**

Let  $z, w \in \mathbb{C}$  both be non-zero. Then,

$$\arg(zw) = \arg(z) + \arg(w) \text{ (modulo } 2\pi)$$

Proof:

Let  $\theta = \arg(z)$  and  $\varphi = \arg(w)$ . Then

$$\begin{aligned} zw &= |z|(\cos(\theta) + i \sin(\theta))|w|(\cos(\varphi) + i \sin(\varphi)) \\ \Rightarrow zw &= |z||w|[(\cos(\theta)\cos(\varphi) - \sin(\theta)\sin(\varphi)) + i(\sin(\theta)\cos(\varphi) + \sin(\varphi)\cos(\theta))] \\ \Rightarrow zw &= |z||w|[\cos(\theta + \varphi) + i \sin(\theta + \varphi)] \end{aligned}$$

As the final equality is in modulus-argument form  $|zw| = |z||w|$  as proved earlier in proposition 1, and  $\theta + \varphi = \arg(z) + \arg(w)$  which is the argument of  $zw$ .  $\square$



We may now use proposition 2 in finding the phase  $\phi(t)$  of the signal. By using the definition of phase, we calculate  $\phi(t)$  as:

$$\begin{aligned}
 \phi(t) &= \arg(z(t)) = \arg(0.5e^{i0.9t} + 1.5e^{i1.1t}) \\
 \Rightarrow \phi(t) &= \arg\left(1.5e^{i1.1t} \left(1 + \frac{0.5}{1.5}e^{i(1.1-0.9)t}\right)\right) \\
 \Rightarrow \phi(t) &= \arg(1.5e^{i1.1t}) + \arg\left(1 + \frac{0.5}{1.5}e^{i(1.1-0.9)t}\right) \quad (\text{By Proposition 2}) \\
 \Rightarrow \phi(t) &= 1.1t + \arg\left(1 + \frac{0.5}{1.5}e^{i(1.1-0.9)t}\right)
 \end{aligned}$$

The last term of the phase is bounded between  $-\frac{\pi}{2}$  and  $\frac{\pi}{2}$ , and the real part of the argument is positive.

We may plot the phase  $\phi(t)$  on a graph using a special command in Matlab called 'unwrap'. By just plotting  $\phi(t)$ , as the  $\arg$  function operates modulo  $2\pi$ , this function will look like a sawtooth as it produces jumps every time the phase increases by  $\pi$ . The unwrap command will correct the phase angle by adding multiples of  $\pm 2\pi$  when the absolute value of the phase jumps between consecutive elements are greater than or equal to the 'jump tolerance',  $\pi$ . If the phase is less than  $\pi$ , then the unwrap command will choose the correct value of the phase [Uk.mathworks.com, 2018]. Figure 10 below shows the phase  $\phi(t)$  which has been plotted without using the unwrap command. You can clearly see the jumps which are made every time the phase increases by  $\pi$ .

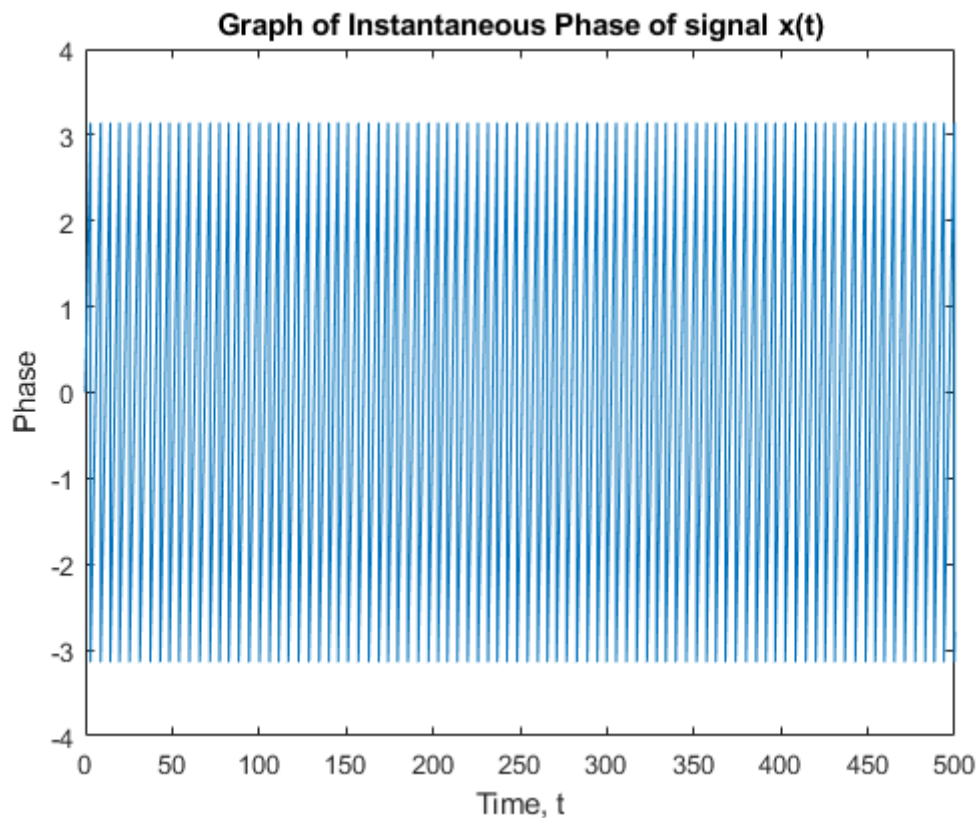


Figure 10: Graph of Instantaneous Phase without using unwrap command. Published with MATLAB® R2017b

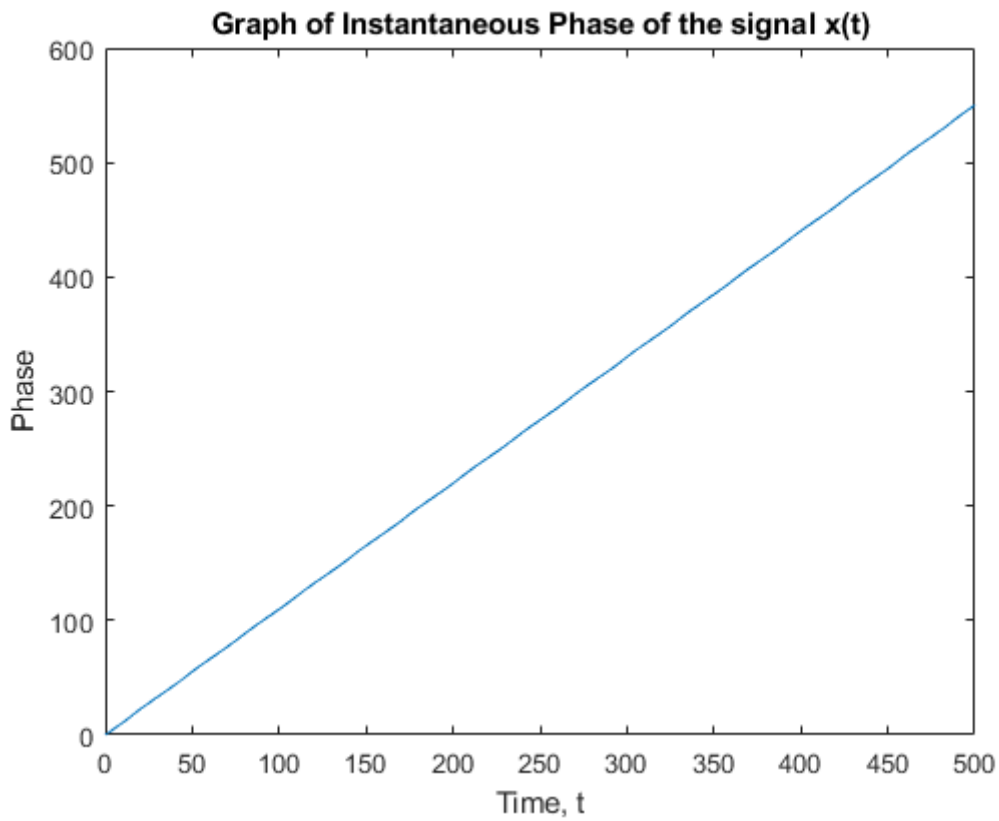


Figure 11: Graph of Instantaneous Phase using unwrap command. Published with MATLAB® R2017b

Figure 11 shows the phase  $\phi(t)$ , which has been plotted using Matlabs unwrap command. Analysing the phase from figure 11, we may simply conclude that the phase is a straight line. However let us zoom into the graph and carefully observe small bumps as shown in figure 12 below.

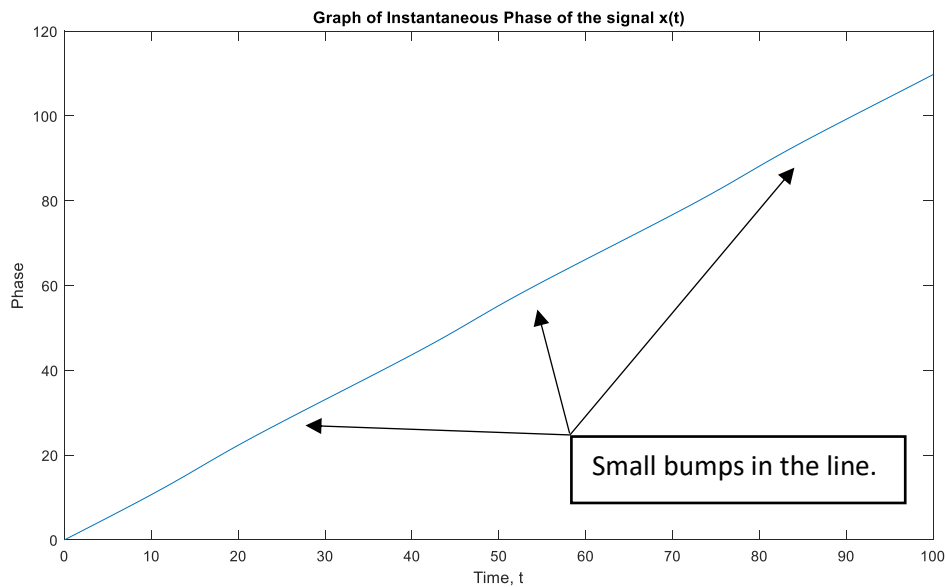


Figure 12: Graph of Instantaneous Phase using unwrap command. Published with MATLAB® R2017

Looking carefully the phase is not a complete straight line and contains small bumps. These small bumps are due to the phase angle repeating every  $2\pi$ . However, as  $x(t)$  is a periodic function, we do not see any extreme values as we would when working with real data. Thus, we do not see any specific phase slips, which is identified by a sudden change in the phase angle, to indicate these extreme events.

### 3.3 Zero Frequency Part

All the signals we have looked at so far have had a vanishing time average. Let us understand how we can deal with a constant offset using the following example. Let us consider a signal  $x(t)$  which is defined as:

$$x(t) = c + a \cos(\omega t), \text{ where } c \text{ is a constant such that } c \in \mathbb{R}$$

Thus, we may rewrite  $x(t)$  in terms of a Fourier polynomial:

$$x(t) = c + \frac{a}{2} e^{i\omega t} + \frac{a}{2} e^{-i\omega t}$$

The constant  $c \in \mathbb{R}$  is a constant which happens to be the average of the function  $\bar{x}$ :

$$\bar{x} = \frac{1}{T} \int_0^T x(t) dt$$

Where,  $T$  is the period of the signal. Now, this may seem bizarre at first but in fact  $\bar{x}$  is the term corresponding to the zero-frequency part of the Fourier polynomial. To understand where this statement appears from, simply consider

$$x(t) - c = x(t) - \bar{x}$$

What we have done here is simply subtract the time average from the signal. In this thesis, we will always assume the time average has been subtracted so that the signal has a zero average.

### 3.4 Fourier and Hilbert Transform

So far, we have implemented our algorithm to simple harmonic functions in order to extract the amplitude and the phase of a signal. To implement our algorithm to a general function  $x(t)$ , we must rewrite all the terms of the function  $x(t)$  in terms of complex exponentials as such:

$$x(t) = a_1 \cos(\omega_1 t) + a_2 \cos(\omega_2 t) = \frac{a_1}{2} e^{i\omega_1 t} + \frac{a_1}{2} e^{-i\omega_1 t} + \frac{a_2}{2} e^{i\omega_2 t} + \frac{a_2}{2} e^{-i\omega_2 t}$$

To write any general signal, whether continuous or discontinuous, in terms of complex exponentials we must use the Fourier transform. As explained earlier, the Fourier Transform proposed by Joseph Fourier in 1822, is a method which allows one to take any continuous or discontinuous function and expand the function in terms of simpler functions, sine and cosine (or in our case further expand each sine and cosine function in terms of complex exponentials using Euler's formula).

The Fourier coefficients of the signal under a Fourier transform are defined as,

$$\hat{x}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t'} x(t') dt'$$

Thus, the signal  $x(t)$  can be written as,

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega t} \hat{x}(\omega) d\omega$$

Now,  $x(t)$  is defined in terms of complex exponentials of the form,  $e^{i\omega t}$ , where  $\omega$  are the frequencies which can be both positive and negative. Our aim was to construct an analytic signal by only considering the positive frequencies and setting all negative frequencies to 0. We may introduce the new signal,  $z(t)$ , by integrating only over the positive frequencies. Thus,  $z(t)$  can be defined as:

$$z(t) = 2 \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{i\omega t} \hat{x}(\omega) e^{-\varepsilon\omega} d\omega$$

At the start of the new signal, we have the term 2 to account for only the positive frequency part of the signal, as we mentioned earlier, and to be more precise we want twice the positive frequency part.

Now you may have noticed the expression  $e^{-\varepsilon\omega}$  in the signal  $z(t)$ . It may not be apparent as to where this term appeared from. The expression has been introduced to simplify some of the subsequent steps in our calculations. As we are integrating over an infinite domain, to ensure that the integrals converge, we must introduce a small exponential cut off which will vanish once we take the limit  $\varepsilon \rightarrow 0$ .

Now let us express the new signal  $z(t)$  in terms on the original signal  $x(t)$  by simply substituting the expression  $\hat{x}(t)$  into  $z(t)$ . This can be done in the following manner:

$$\begin{aligned} z(t) &= 2 \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{i\omega t} \hat{x}(\omega) e^{-\varepsilon\omega} d\omega \\ \Rightarrow z(t) &= 2 \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{i\omega t} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t'} x(t') dt' e^{-\varepsilon\omega} d\omega \\ \Rightarrow z(t) &= 2 \frac{1}{2\pi} \int_0^{\infty} d\omega \int_{-\infty}^{\infty} x(t') e^{i\omega(t-t')-\varepsilon\omega} dt' \\ \Rightarrow z(t) &= \frac{1}{\pi} \int_{-\infty}^{\infty} x(t') \underbrace{\int_0^{\infty} e^{(i(t-t')-\varepsilon)\omega} d\omega}_{\text{Integration by substitution}} dt' \end{aligned}$$

Integration by substitution

Let us first calculate the sub-integral using the method of integration by substitution. Let

$$u = (i(t - t') - \varepsilon)\omega$$

Then,

$$\begin{aligned} \frac{du}{d\omega} &= i(t - t') - \varepsilon \\ \Rightarrow d\omega &= \frac{du}{i(t - t') - \varepsilon} \end{aligned}$$

Now, let us find the new limits of integration. When  $\omega \rightarrow \infty$ ,  $u \rightarrow -\infty$  and when  $\omega \rightarrow 0$ ,  $u \rightarrow 0$ . Our integral now becomes:

$$\int_0^{-\infty} e^u \frac{du}{i(t - t') - \varepsilon} = - \int_{-\infty}^0 e^u \frac{du}{i(t - t') - \varepsilon}$$

$$\begin{aligned}
&= -\frac{1}{i(t-t')-\varepsilon} \int_{-\infty}^0 e^u du \\
&= -\frac{1}{i(t-t')-\varepsilon} e^u \Big|_{-\infty}^0 \\
&= -\frac{1}{i(t-t')-\varepsilon} [e^0 - \lim_{u \rightarrow -\infty} e^u] \\
&= -\frac{1}{i(t-t')-\varepsilon} [1 - 0] \\
&= -\frac{1}{i(t-t')-\varepsilon}
\end{aligned}$$

Where we have used the results  $e^0 = 1$  and  $\lim_{u \rightarrow -\infty} e^u = 0$ .

Now we can input our results into  $z(t)$  as such:

$$z(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} x(t') \frac{-1}{i(t-t')-\varepsilon} dt'$$

Now, we may separate the fraction  $-\frac{1}{i(t-t')-\varepsilon}$  as such:

$$-\frac{1}{i(t-t')-\varepsilon} = \frac{-i(t-t')+\varepsilon}{(i(t-t')-\varepsilon)^2} = \frac{\varepsilon}{(t-t')^2+\varepsilon^2} + i \frac{t-t'}{(t-t')^2+\varepsilon^2}$$

Alongside using the fact  $i^2 = -1$ .

Thus, our signal  $z(t)$  can be expressed as,

$$\begin{aligned}
z(t) &= \frac{1}{\pi} \int_{-\infty}^{\infty} \left( \frac{\varepsilon}{(t-t')^2+\varepsilon^2} + i \frac{t-t'}{(t-t')^2+\varepsilon^2} \right) x(t') dt' \\
\Rightarrow z(t) &= \frac{1}{\pi} \left[ \int_{-\infty}^{\infty} \frac{\varepsilon}{(t-t')^2+\varepsilon^2} x(t') dt' + \int_{-\infty}^{\infty} i \frac{t-t'}{(t-t')^2+\varepsilon^2} x(t') dt' \right] \\
\Rightarrow z(t) &= \int_{-\infty}^{\infty} \frac{1}{\pi} \frac{\varepsilon}{(t-t')^2+\varepsilon^2} x(t') dt' + \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{t-t'}{(t-t')^2+\varepsilon^2} x(t') dt'
\end{aligned}$$

Let us introduce the abbreviations,  $\delta_\varepsilon$  which is a function, and  $P$  which is the principal value distribution. Each of the abbreviations are defined as,

$$\delta_\varepsilon(x) = \frac{1}{\pi} \frac{\varepsilon}{x^2+\varepsilon^2}, P\left(\frac{1}{x}\right) = \frac{x}{x^2+\varepsilon^2}$$

We may rewrite  $z(t)$  in terms of the two new functions:

$$z(t) = \int_{-\infty}^{\infty} \delta_\varepsilon(t-t') x(t') dt' + \frac{i}{\pi} \int_{-\infty}^{\infty} P\left(\frac{1}{t-t'}\right) x(t') dt'$$

For each of the new functions we must consider the limit  $\varepsilon \rightarrow 0$ . Let us begin by looking at the first integral of  $z(t)$ . Considering the limit  $\varepsilon \rightarrow 0$

$$\lim_{\varepsilon \rightarrow 0} \delta_\varepsilon(x) = \lim_{\varepsilon \rightarrow 0} \frac{1}{\pi} \frac{\varepsilon}{x^2+\varepsilon^2} = 0$$

When  $x = 0$ , the function  $\delta_\varepsilon(x)$  has a very sharp peak, which means that in the first integral of  $z(t)$ , only the value of  $x(t')$  when  $t = t'$  will matter. Thus, the limit as  $\varepsilon \rightarrow 0$  will become:

$$\begin{aligned}\lim_{\varepsilon \rightarrow 0} \int_{-\infty}^{\infty} \delta_\varepsilon(t - t') x(t') dt' &= x(t) \int_{-\infty}^{\infty} \delta_\varepsilon(t - t') dt' \\ &= x(t) \int_{-\infty}^{\infty} \frac{1}{\pi} \frac{\varepsilon}{(t - t')^2 + \varepsilon^2} dt' \\ &= \frac{x(t)}{\pi} \varepsilon \int_{-\infty}^{\infty} \frac{1}{(t - t')^2 + \varepsilon^2} dt'\end{aligned}$$

Using integration by substitution we may calculate the value of the integral. Let  $u = t - t'$ . Thus,

$$\begin{aligned}\frac{du}{dt'} &= -1 \Rightarrow du = -dt' \\ \Rightarrow \int_{-\infty}^{\infty} \frac{1}{(t - t')^2 + \varepsilon^2} dt' &= - \int_{-\infty}^{\infty} \frac{1}{u^2 + \varepsilon^2} du\end{aligned}$$

As you can see the limits will remain the same. Now to solve the integral we need to use integration by substitution again alongside the standard result:

- $\int \frac{1}{x^2 + 1} dx = \arctan(x) + C$
- $\lim_{x \rightarrow \infty} \arctan(x) = \frac{\pi}{2}$
- $\lim_{x \rightarrow -\infty} \arctan(x) = -\frac{\pi}{2}$

Now, letting  $v = \frac{u}{\varepsilon}$ , we get  $du = \varepsilon dv$ . The limits will not change. Our integral now becomes:

$$\begin{aligned}- \int_{-\infty}^{\infty} \frac{1}{u^2 + \varepsilon^2} du &= - \int_{-\infty}^{\infty} \frac{\varepsilon}{\varepsilon^2 v^2 + \varepsilon^2} dv \\ &= - \frac{1}{\varepsilon} \int_{-\infty}^{\infty} \frac{1}{v^2 + 1} dv \\ &= - \frac{1}{\varepsilon} \arctan(v) \\ &= - \frac{1}{\varepsilon} \arctan\left(\frac{u}{\varepsilon}\right) \\ &= - \frac{1}{\varepsilon} \arctan\left(\frac{t - t'}{\varepsilon}\right)\end{aligned}$$

Now inputting our calculations thus far we have:

$$\begin{aligned}\lim_{\varepsilon \rightarrow 0} \int_{-\infty}^{\infty} \delta_\varepsilon(t - t') x(t') dt' &= \frac{x(t)}{\pi} \varepsilon \int_{-\infty}^{\infty} \frac{1}{(t - t')^2 + \varepsilon^2} dt' \\ &= \frac{x(t)}{\pi} \varepsilon \left( - \frac{1}{\varepsilon} \arctan\left(\frac{t - t'}{\varepsilon}\right) \Big|_{-\infty}^{\infty} \right) \\ &= - \frac{x(t)}{\pi} \arctan\left(\frac{t - t'}{\varepsilon}\right) \Big|_{-\infty}^{\infty}\end{aligned}$$

$$\begin{aligned}
&= \frac{x(t)}{\pi} \left[ \lim_{\varepsilon \rightarrow \infty} \left( -\arctan \left( \frac{t-t'}{\varepsilon} \right) \right) - \lim_{\varepsilon \rightarrow -\infty} \left( -\arctan \left( \frac{t-t'}{\varepsilon} \right) \right) \right] \\
&= \frac{x(t)}{\pi} \left[ \frac{\pi}{2} - -\frac{\pi}{2} \right] \\
&= x(t)
\end{aligned}$$

The first integral of  $z(t)$  has now been calculated. In the second integral, the expression  $P\left(\frac{1}{x}\right)$  is  $\frac{1}{x}$  if  $x \neq 0$  while  $P\left(\frac{1}{x}\right)$  tends to 0 in a small interval around  $x = 0$ . The nonintegrable singularity has been removed which leaves us with the new time dependent complex signal to hold the form:

$$z(t) = x(t) + \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{x(t')}{t-t'} dt'$$

The imaginary part of the integral transform of  $x(t)$  is called the Hilbert transform. In Matlab, we may produce the signal  $z(t)$  using a single command called 'Hilbert'. The Hilbert command takes a given signal, applies a Fourier transform, set all the negative frequencies to 0, then applies the inverse Fourier transform, which give us our required signal to extract the amplitude  $A(t)$  and phase  $\phi(t)$  in such a manner:

$$A(t) = |z(t)|, \quad \phi(t) = \arg(z(t))$$

Where the amplitude and phase a related to the signal by:

$$x(t) = \text{Re}(z(t)) = A(t)\cos(\phi(t))$$

As  $z(t) = A(t)e^{i\phi(t)}$ .

## 4.0 Signal Processing using Matlab

### 4.1 Signal Processing of a Simple Harmonic Function

In this section of the thesis, I shall be illustrating the algorithm established in section 3.1 to process the financial time series of Nord Pool electricity prices, by implementing the Matlab command 'Hilbert'.

Before we attempt to implement the Hilbert command to the Nord Pool Data set, we shall first test the command on a simple harmonic signal,  $x(t)$  which is defined as,

$$x(t) = 0.5 \sin(0.9t) + 1.5\sin(1.1t)$$

to explain the output delivered by the Hilbert command.

The Hilbert command in Matlab is used to extract the analytic signal from a discrete time data sets of a finite block of data [Uk.mathworks.com, 2018]. In other words, the Hilbert function will take the finite data set, perform a Fourier transform on the data set, set all the negative frequencies to 0, and then finally apply the inverse Fourier transform on the data set; all within one function. The Matlab syntax for the Hilbert function is

$$x = \text{hilbert}(xr)$$

Where,

$x_r$ - a real data sequence

$x$ - the return which is an analytic signal

Using the output of the Hilbert function, our aim is to find the instantaneous amplitude and phase of the analytic signal, which is the new time dependent complex signal which has been explained in section 3.1.

Due to the Hilbert function performing numerous steps at once, we shall explain the method used to produce our results and the Matlab code used at each stage of the algorithm.

Using a similar example to which we studied by hand in section 3.2, let the signal  $x(t)$  be defined as:

$$x(t) = 0.5 \sin(0.9t) + 1.5 \sin(1.1t)$$

The period of this harmonic function will be defined between 0 and 500 on an interval of 0.0001. The signal  $x(t)$  has been plotted on a graph using Matlab as shown in figure 13.

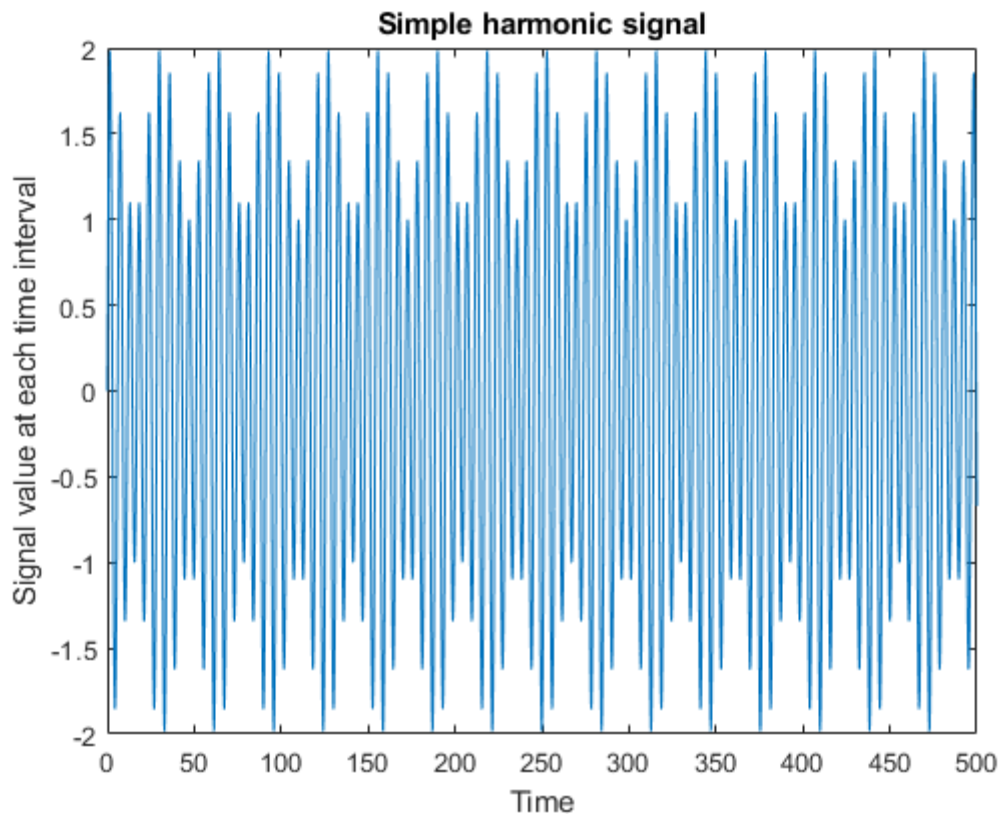


Figure 13: Simple harmonic signal. Published with MATLAB® R2017b

As we have identified the simple harmonic signal, we can continue to the next step and apply Matlab's Hilbert command to the signal in order to extract the analytic signal,  $z(t)$ . As the analytic signal elements are from the complex number system,  $\mathbb{C}$ , we cannot directly plot  $z(t)$  on a graph like we would have done with a real valued signal.

Firstly, let us verify the components of the signal  $z(t)$  are elements of  $\mathbb{C}$  (complex numbers) and compare the elements to the components of the real signal  $x(t)$ . This can be done using the following commands in Matlab:



```

%Let us show an example of the application of the Hilbert command
%on a discrete-time signal x(t), where t is the time dependent variable

%First we shall define the time period on which the simple harmonic
%function will be defined on
t=0:1e-4:500; %1e-4 is Matlab code for 0.0001

x=0.5*sin(0.9*t)+1.5*sin(1.1*t);

%implementing Matlab's Hilbert command in order to extract the analytic
%signal which will be defined by z

z=hilbert(x);

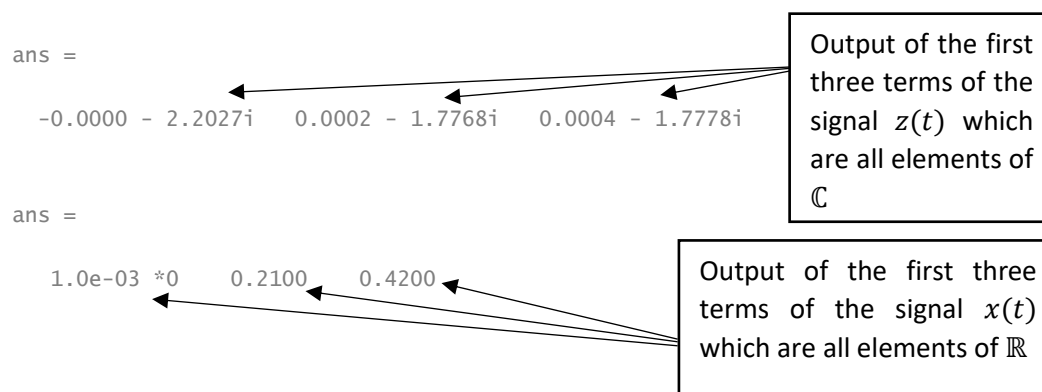
%Let us check the output of signal z are complex numbers
z(1:3) %This command is asking Matlab to display first 3 values of signal z

%We may compare this to the first 3 values of the original real valued
%signal
x(1:3)

```

Figure 14: Matlab code to produce an analytic signal of  $x(t)$  using Hilbert command. Published with MATLAB® R2017b

The results produced in Matlab's command window is:



The Hilbert function has indeed produced an analytic signal with components from  $\mathbb{C}$ . The Matlab commands, *abs* and *angle* may now be used to find the absolute value (the amplitude) and the phase of the signal  $z(t)$  respectively.

Let us begin with finding the amplitude  $A(t)$ , using the following Matlab code:

```

%Let us show an example of the application of the Hilbert command
%on a discrete-time signal x(t), where t is the time dependent variable

%First we shall define the time period on which the simple harmonic
%function will be defined on
t=0:1e-4:500; %1e-4 is Matlab code for 0.0001

x=0.5*sin(0.9*t)+1.5*sin(1.1*t);

```

```

%implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(x);

%Let us now use the Matlab command abs to plot the amplitude of the signal
figure
plot(t,x) %Plotting the original signal to check whether the amplitude envelopes it.
hold on
plot(t,abs(z))
hold off
title('Amplitude of the signal x(t)')
xlabel('Time')
ylabel('Signal value at each time interval')

```

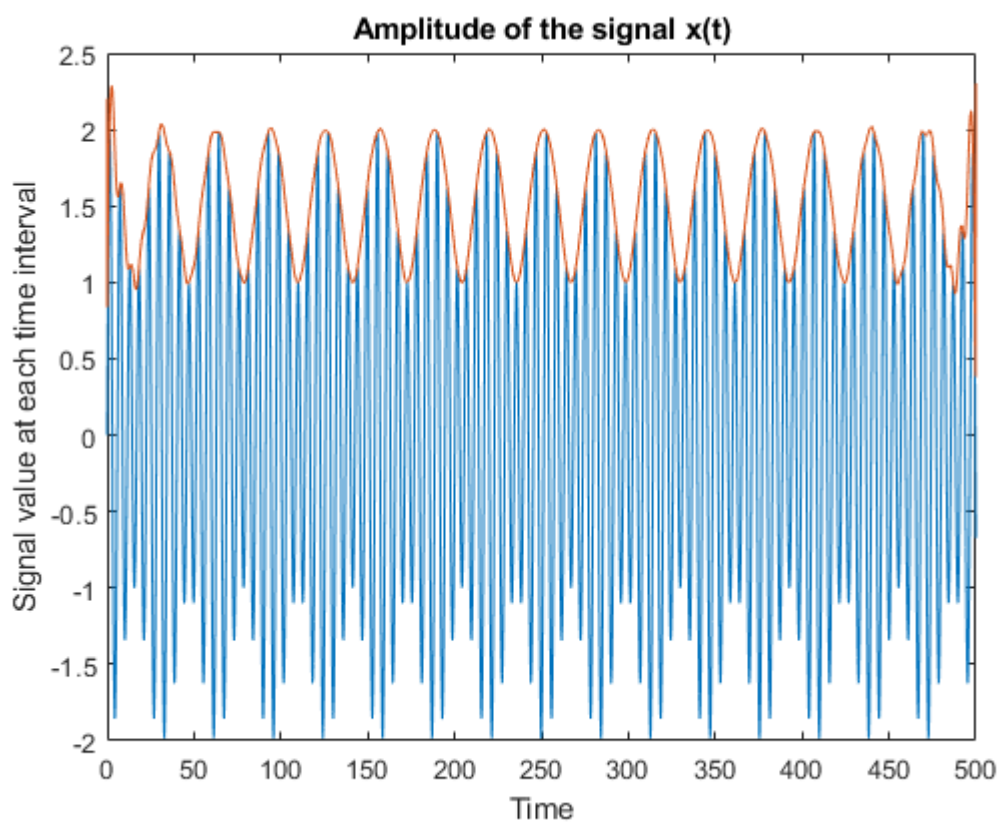


Figure 15: Amplitude of the signal extracted using the Hilbert command. Published with MATLAB® R2017b

As you can see we have extracted the amplitude which perfectly envelopes the signal by clearly highlighting the extremes of the signal as shown by the orange curve in figure 15.

As we have found the amplitude of the signal,  $x(t)$ , we may continue to extract the phase of the signal and plot it on a graph using the following Matlab code:

```

%Let us show an example of the application of the Hilbert command
%on a discrete-time signal x(t), where t is the time dependent variable

%First we shall define the time period on which the simple harmonic
%function will be defined on

```

```

t=0:1e-4:500; %1e-4 is Matlab code for 0.0001

x=0.5*sin(0.9*t)+1.5*sin(1.1*t);

%implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(x);

%Let us now plot the instantaneous phase of the signal using Matlabs angle
%command.
figure
plot(t, angle(z))
title('Instantaneous Phase')
xlabel('Time, t')
ylabel('Phase')

```

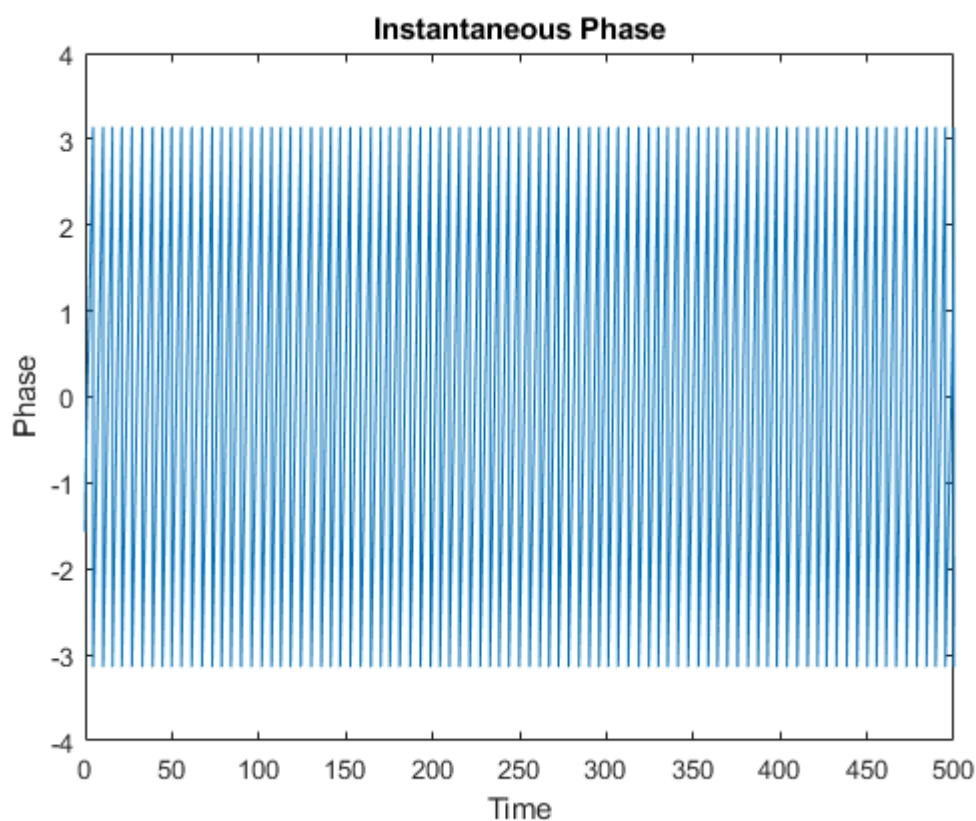


Figure 16: Instantaneous Phase. Published with MATLAB® R2017b

As you can see from figure 16, the phase angle jumps in absolute value whenever the angle increases by  $\pi$ . Using the unwrap command in Matlab, which was mention in section 3.2, we may ‘untangle’ the phase angle and plot it in a manner which can be analysed. The Matlab code below demonstrates who to use the unwrap command:

```

%Let us show an example of the application of the Hilbert command
%on a discrete-time signal x(t), where t is the time dependent variable

```

```

%First we shall define the time period on which the simple harmonic
%function will be defined on
t=0:1e-4:500; %1e-4 is Matlab code for 0.0001

x=0.5*sin(0.9*t)+1.5*sin(1.1*t);

%implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(x);

%plotting the phase using Matlab's unwrap command
figure
plot(t, unwrap(angle(z)))
title('Instantaneous Phase')
xlabel('Time')
ylabel('Phase')

```

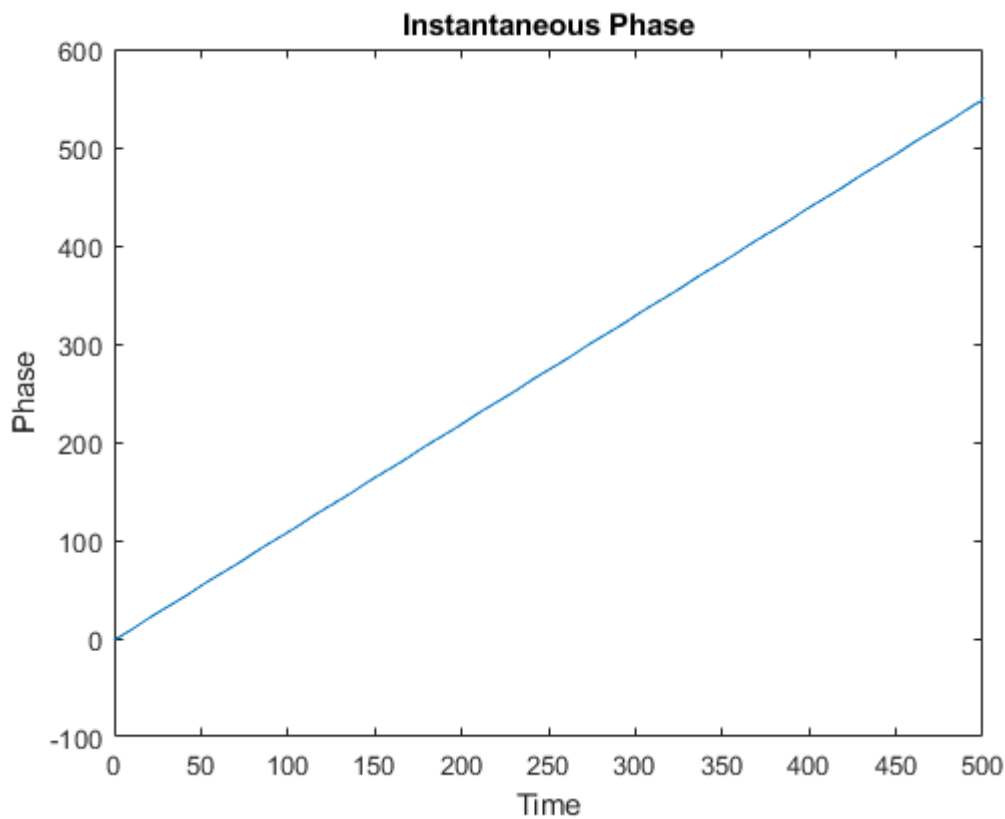


Figure 17: Instantaneous Phase using unwrap command. Published with MATLAB® R2017b

Analysing figure 17, you first assume this is simply a straight line. However, if we zoom into the graph as shown below in figure 18, you may notice that the instantaneous phase is not exactly a straight line. These small bumps are due to the phase angle repeating every  $2\pi$ . However, as  $x(t)$  is a periodic function, we do not see any extreme values as we would when working with real data. Thus, we do

not see any specific phase slips, which is identified by a sudden change in the phase angle, to indicate these extreme events.

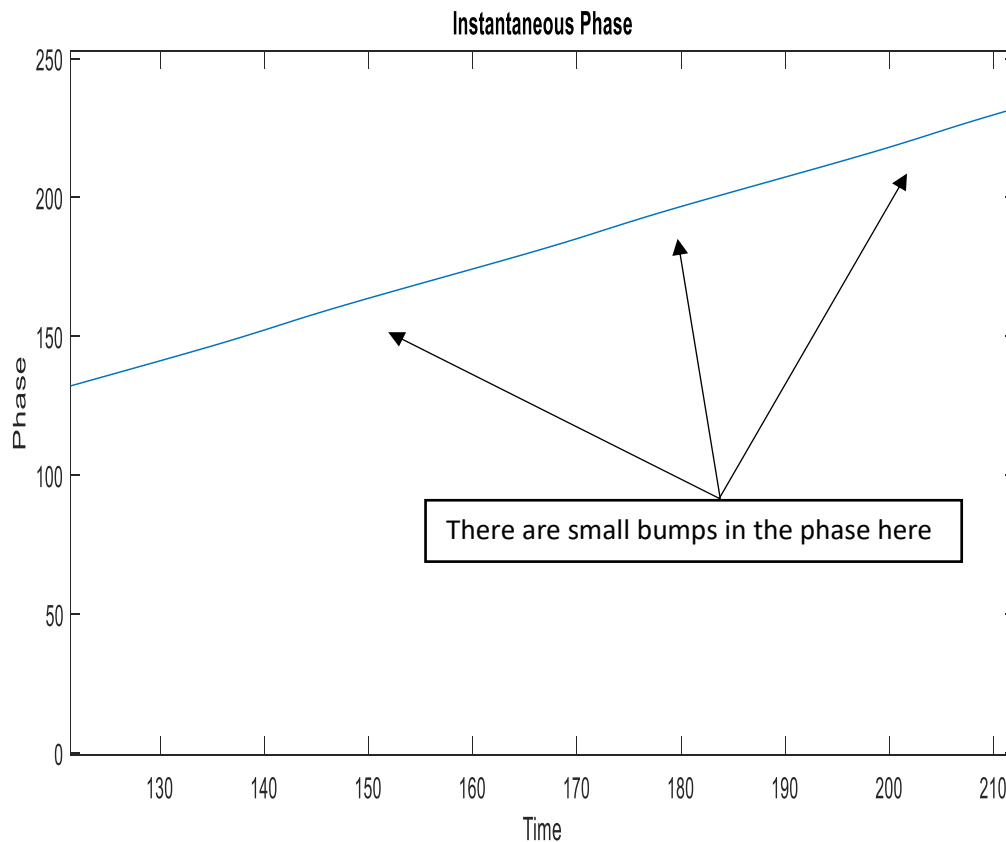


Figure 18: Instantaneous Phase using `unwrap` command on a smaller scale. Published with MATLAB® R2017b

## 4.2 Signal Processing with Nord Pool Electricity Prices

Reflecting on what we have done thus far, by successfully demonstrating how to use the Hilbert command in Matlab and applying it to a simple harmonic function, to extract the amplitude and the phase of the signal. Let us now apply this method to a real discrete-time data set to extract the same information from the signal. Using the Nord Pool data set for electricity prices, which have been recorded at an hourly interval across seven years, starting from the 1<sup>st</sup> January 1999 at 00:00 till the 26<sup>th</sup> January 2007; we may decode the signal in the following manner.

To start processing the signal, we must first import the data from the text file (.txt) into a variable in Matlab. To import that data, follow these steps:

- Under 'Home' tab, click on 'Import data' icon.
- Next, search the text file under which the data is stored in, for example 'NordPoolElectricityData.txt'
- Matlab will automatically recognise the layout of the data.
- Insure to import the data in a 'Column vector' form.
- Next, rename the variable. The electricity prices have been stored as NordPoolElecPrices.
- Once the above steps have been completed, click the 'Import Selection' button to complete the importing of the data set.

To know if you have successfully imported that data, in the right-hand column under the tab 'Workspace', the variable has been named and been given a specific value. This verifies that the data has been imported and stored in named variable.

As the data has been successfully imported, we may plot the financial time series on a graph. Figure 19.1 shows us the Nord Pool electricity prices which have been recorded at hourly intervals. The time variable has been stored as dates ranging from 1<sup>st</sup> January 1999 to 26<sup>th</sup> January 2007. There are exactly 70,752 data points. The appendix contains the related Matlab code used to produce the financial time series.

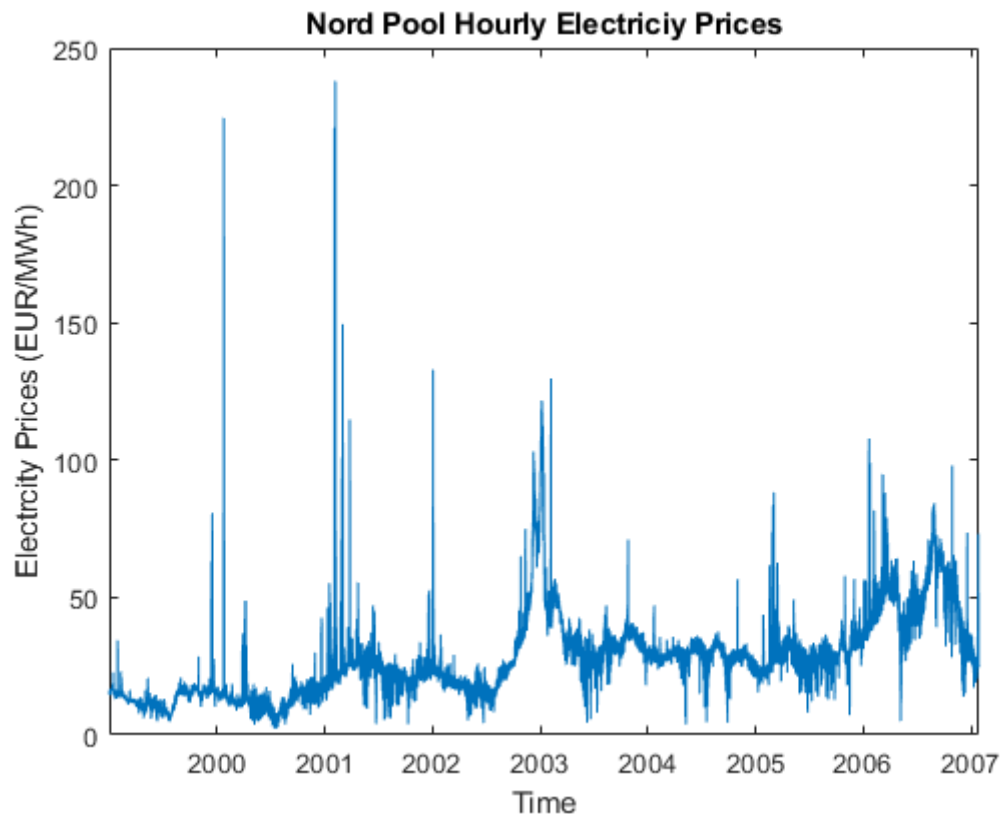


Figure 19.1: Nord Pool Hourly Electricity Prices. Published with MATLAB® R2017b

Studying figure 19.1 shows us the extreme data points contained in the signal. By adjusting the time series on a finer scale, we can observe some properties of the Nordic electricity prices. The greatest price witnessed at which Nordic electricity was traded at was 238.00 EUR/MWh on the 5<sup>th</sup> February 2001, at 09:00 as shown in figure 19.1 above and in more detail in figure 19.2 below. We must make awareness of the fact that the discrete time series is a non-stationary, non-periodic time series. However, when we study the time series on a finer scale, we can begin to see a form of periodicity in the electricity prices. Inspecting figure 19.3, between 22<sup>nd</sup> November 1999 to 3<sup>rd</sup> December 1999, we can analyse the electricity prices to be increasing and decreasing at certain times during the day. Analysing the price of electricity across the twelve days we notice a pattern which appears repeatedly. The price of electricity is at its lowest for the day around the hours of 00:00. As we begin the day, around early morning to midday, we see that the price of electricity starts to increase, with a small drop between 12:00 and mid-afternoon. We again see a peak in the price around mid-afternoon to early evening before the price of electricity drops again for the cycle to repeat. We can see a similar pattern again in March 2004 as illustrated in figure 19.4. Let us take a closer look at the prices of electricity between 10<sup>th</sup> March 2004 at 00:00 till 12<sup>th</sup> March 2004 at 00:00, as shown in figure 19.5.

The maximum and minimum prices of electricity can be clearly seen with indication to the time at which they occur at each day. We can observe that the power prices are at a maximum just before midday and reach a minimum between 00:00 and the early hours of the morning.

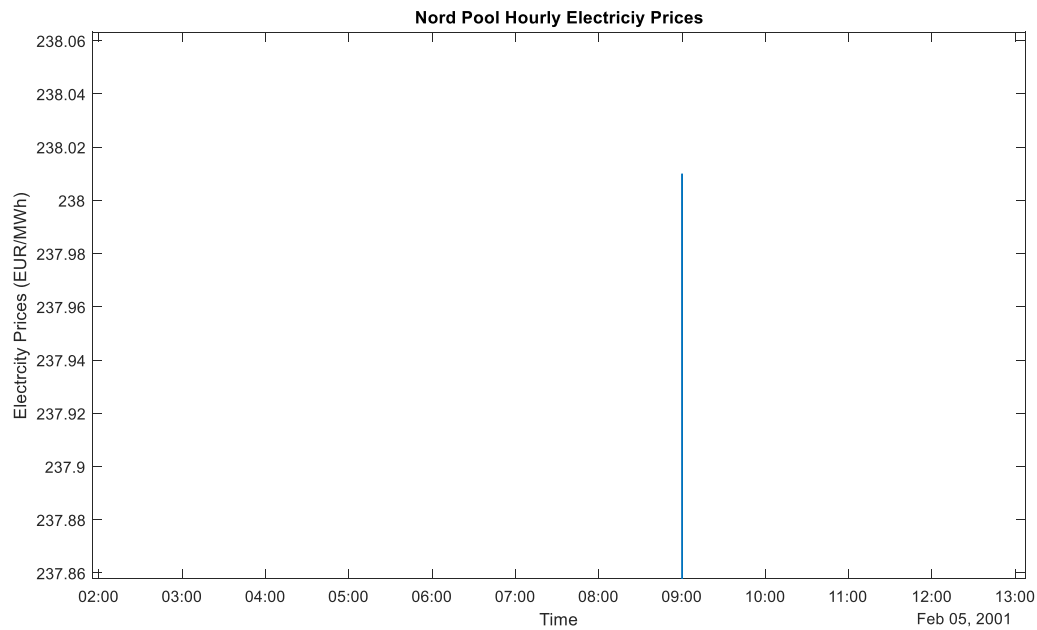


Figure 19.2: Nord Pool's highest price at which electricity is traded at. Published with MATLAB® R2017b

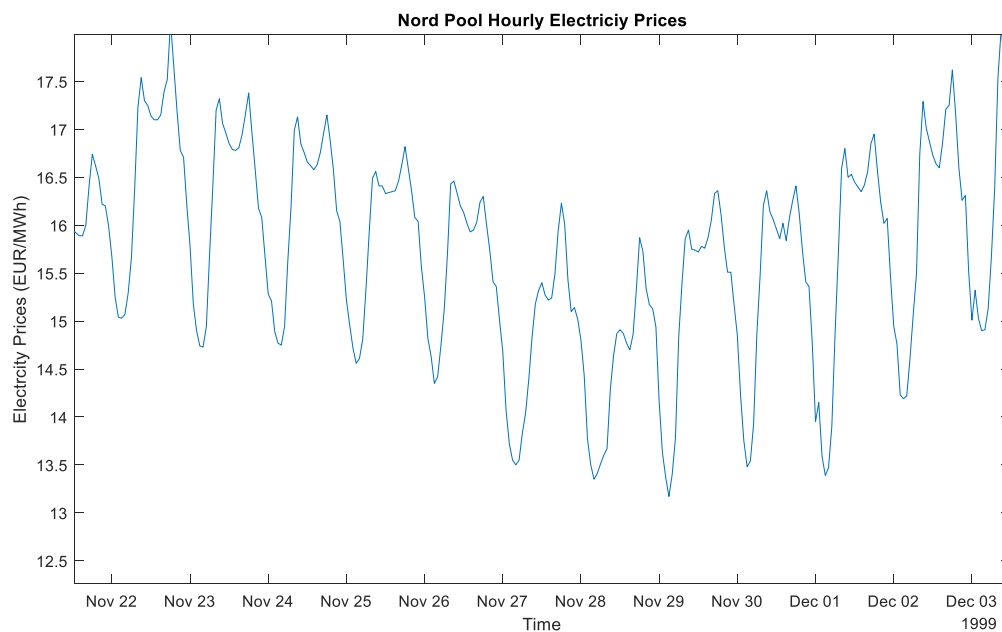


Figure 19.3: Repetition in Nord Pool's electricity prices between 22 Nov 1999 and 03 Dec 1999. Published with MATLAB® R2017b

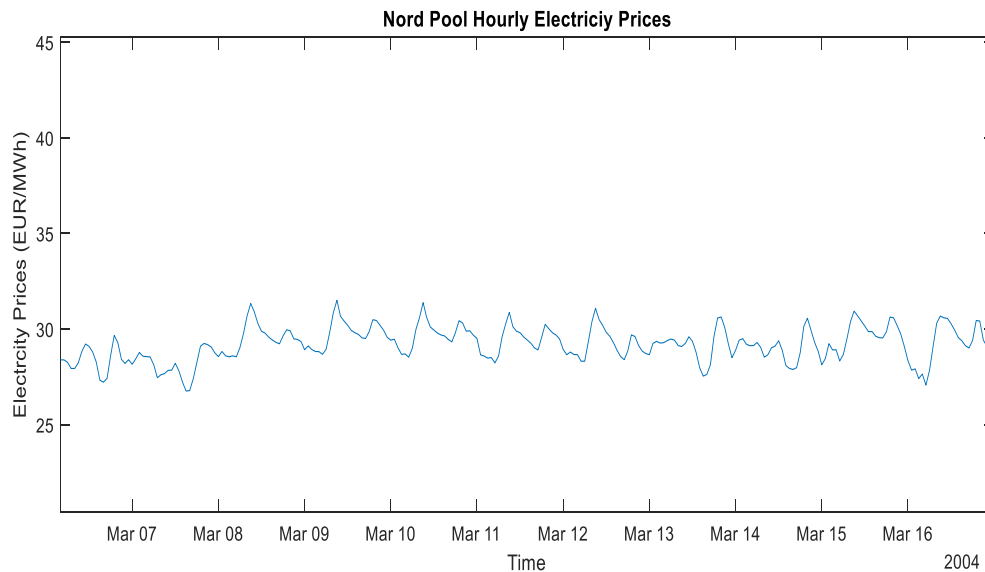


Figure 19.4: Repetition in Nord Pool's electricity prices in March 2004. Published with MATLAB® R2017b

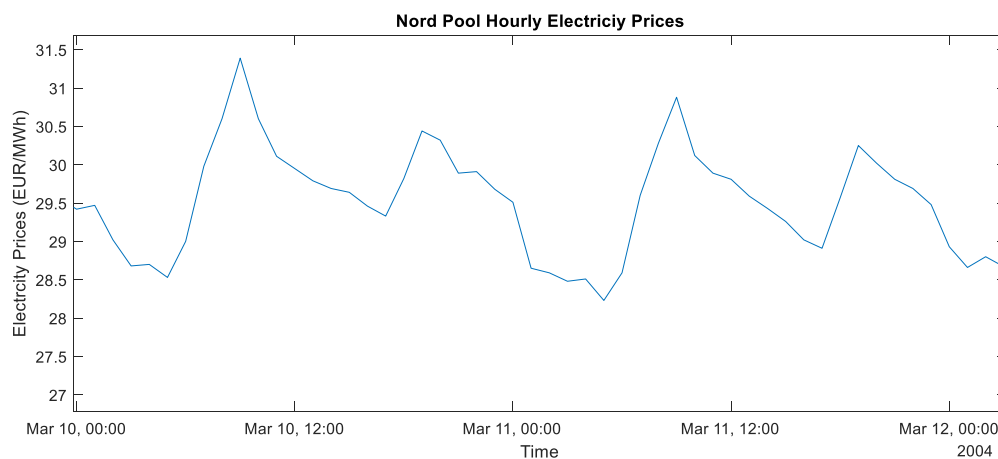


Figure 19.5: Identifying the maximum and minimum prices of Nord Pool electricity between 10<sup>th</sup> March and 12<sup>th</sup> March 2004. Published with MATLAB® R2017b

Analysing the time series, we have been able to identify patterns in the data set, as well as the occurrence of extreme data point of where Nordic electricity prices have increased at an unexpected rate. The purpose of this thesis is to find evidence in the phase of the signal, which may or may not provide information regarding the occurrence of these extreme events. Once analysing the phase of the time series, we hope to see if there exist any phase slips, which is identified by a sudden change in the phase angle, which will indicate the occurrence of any extreme events.

Let us now begin by decomposing the signal in order to define the analytic signal,  $z(t)$  using the Hilbert command in Matlab. In Matlab, to create the time dependent signal, we must save the time variable as shown below in the Matlab code. Just like before, we know that the analytic signal  $z(t)$  has complex elements. We may verify this as shown in the Matlab code and compare the signal elements to the original time series data.

```
%Let us begin by creating the Nord Pool electricity prices as a financial
%time series by creating a time variable.
```



```

%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows. We shall save the transposed variable as 'Time'.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z
z=hilbert(NordPoolElecPrices);
%Let us check the output of the signal z are complex numbers
z(1:3) %This command is asking Matlab to display first 3 values of signal z

%We may compare this to the first 3 values of the original real valued
%signal(which is the first 3 prices from the data set)
NordPoolElecPrices(1:3)

```

Figure 20: Matlab code to produce an analytic signal of the financial time series using Hilbert command. Published with MATLAB® R2017b

The results produced in Matlab's command window is:

```

ans =
    15.2900 +61.7409i
    15.1400 +55.5470i
    14.9000 +55.1734i

```

← Output of the first three terms of the signal  $z(t)$  which are all elements of  $\mathbb{C}$

```

ans =
    15.2900
    15.1400
    14.9000

```

← Output of the first three terms of the signal  $x(t)$  which are all elements of  $\mathbb{R}$

As we have verified that the Hilbert command has indeed produced an analytic signal with complex elements, we may now use Matlab commands, *abs* and *angle* to find the absolute value (the amplitude) and the phase of the signal  $z(t)$  respectively.

Let us begin with finding the amplitude:

```

%Let us begin by creating the Nord Pool electricity prices as a financial
%time series by creating a time variable.
%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows. We shall save the transposed variable as 'Time'.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z
z=hilbert(NordPoolElecPrices);

%Let us now use the Matlab command abs to plot the amplitude of the complex
%signal
figure
plot(Time,NordPoolElecPrices)%Plotting the original signal to check whether
%the amplitude envelopes it.
hold on
plot(Time,abs(z))
hold off
title('Amplitude of the Nord Pool electricity prices')

```

```
xlabel('Time')
ylabel('Electricity Prices (EUR/MWh)')
```

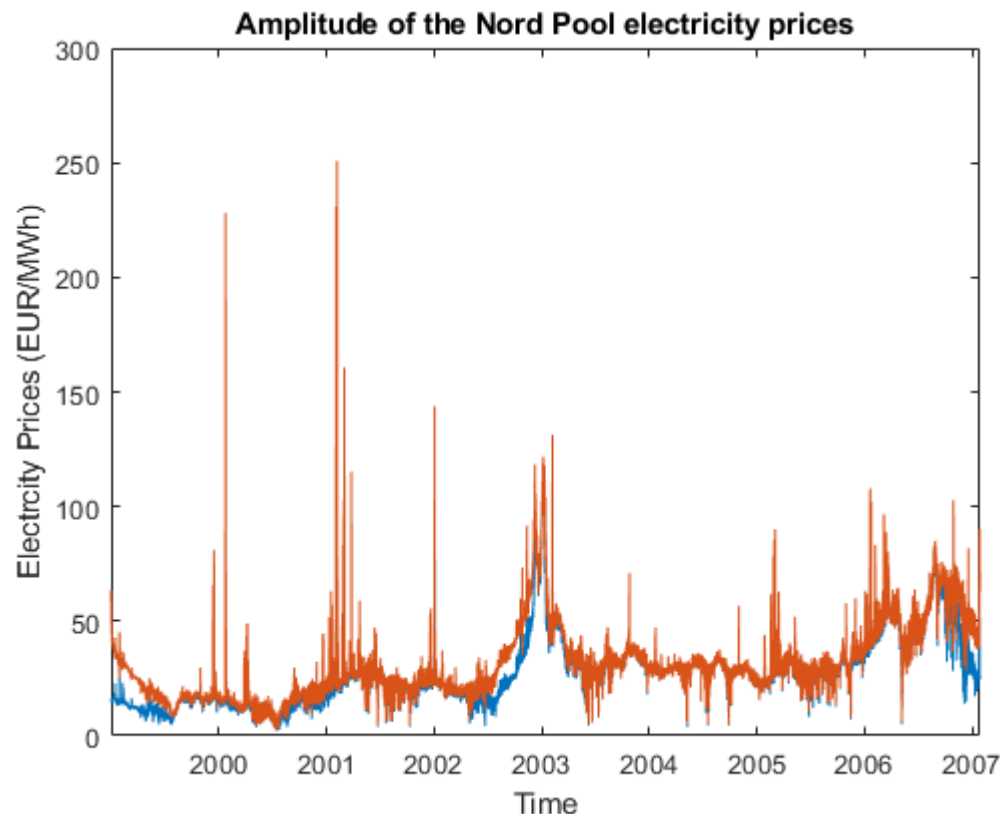


Figure 21: Amplitude of the Nord Pool electricity prices signal using Hilbert command. Published with MATLAB® R2017b

We mentioned earlier that the financial time series was non-periodic. In figure 21, the amplitude is highlighted in orange, while the signal is emphasised by the blue curve. Due to the real data set not having periodicity, the amplitude is not a perfect envelope of the signal. This is due to the real data set having many extreme points, and an envelope is a smooth curve which outlines the extremes of the signal. However, the main purpose of the amplitude has been achieved, which is to highlight the extreme value of the financial time series.

Let us now continue to extract the phase of the signal using the following Matlab command:

```
%Let us begin by creating the Nord Pool electricity prices as a financial
%time series by creating the time variable.
%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows. We shall save the transposed variable as 'Time'.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(NordPoolElecPrices);

%Let us now plot the instantaneous phase of the complex signal using
```

```

%Matlabs angle command
figure
plot(Time, angle(z))
title('Instantaneous Phase of complex signal')
xlabel('Time')
ylabel('Phase')

```

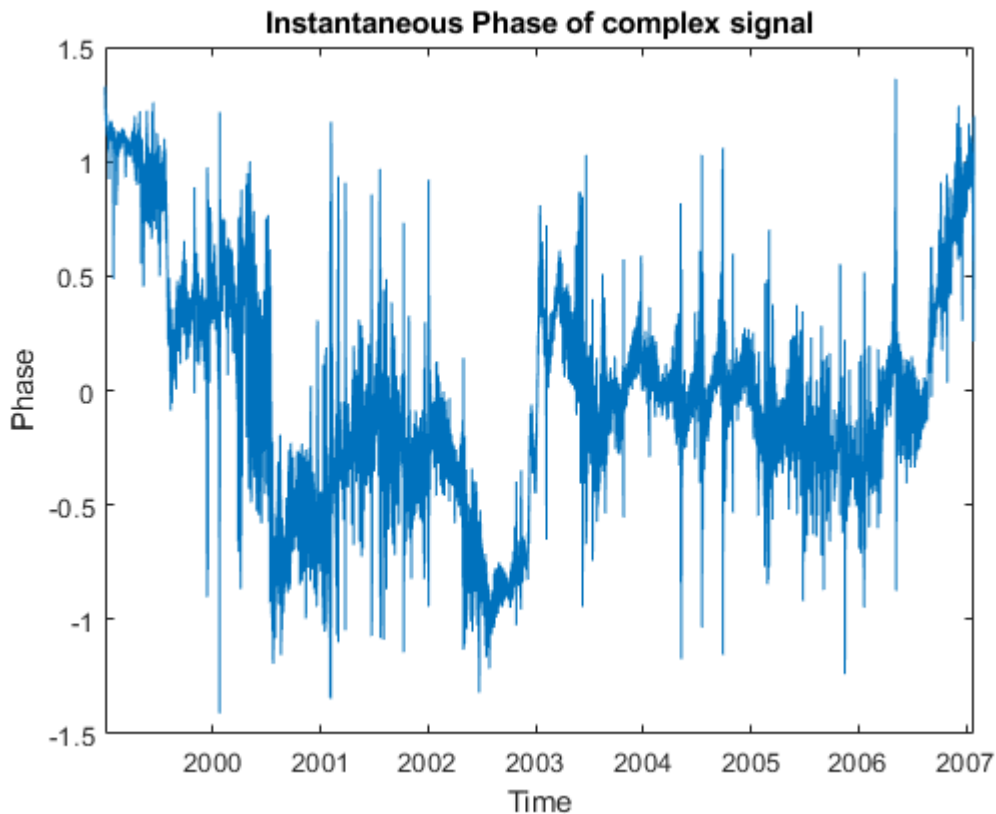


Figure 22: Instantaneous Phase of the complex signal. Published with MATLAB® R2017b

As you can see from figure 22, the phase angle jumps in absolute value whenever the angle increases by  $\pi$ . Using the `unwrap` command in Matlab, which was mentioned in section 3.2, we may ‘untangle’ the phase angle and plot it in a manner which can be analysed. The Matlab code below demonstrates how to use the `unwrap` command:

```

%Let us begin by creating the Nord Pool electricity prices as a financial
%time series by creating a time variable.
%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows. We shall save the transposed variable as 'Time'.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(NordPoolElecPrices);

%plotting the instantaneous phase using Matlab's unwrap command
figure

```

```

plot(Time, unwrap(angle(z)))
title('Instantaneous Phase using unwrap command')
xlabel('Time')
ylabel('Phase')

```

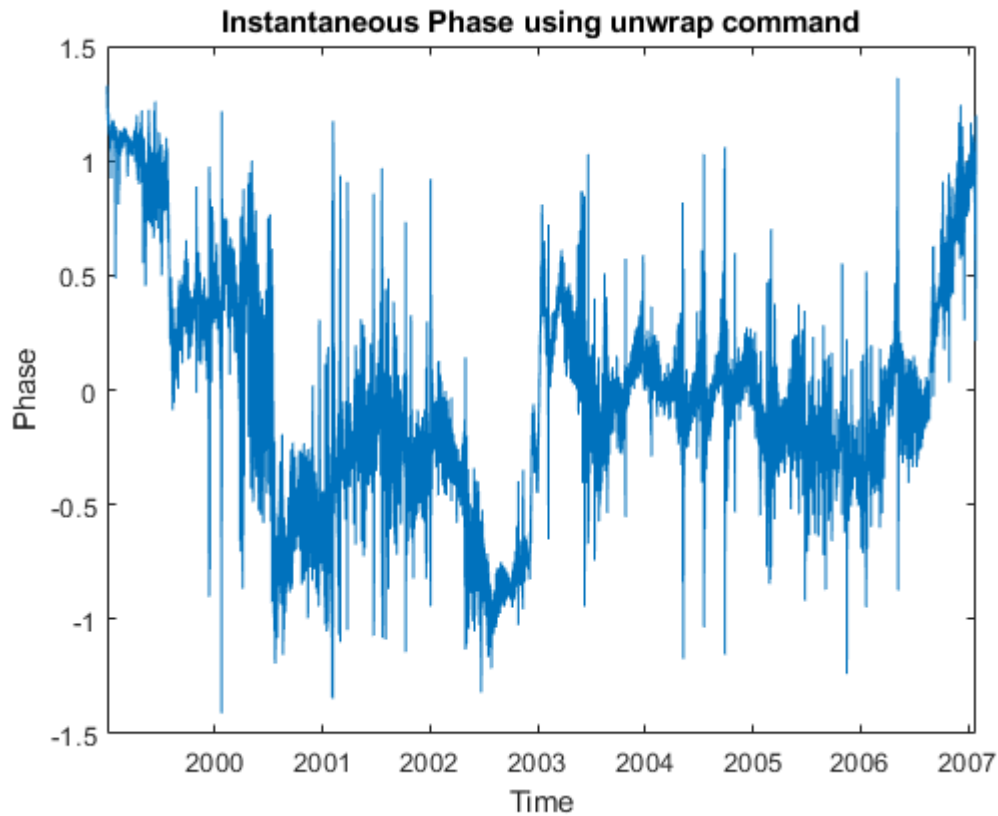


Figure 23: Instantaneous Phase of the complex signal using unwrap command. Published with MATLAB® R2017b

Now looking at the phase in figure 23 you must be thinking how is this different to the phase plot shown in figure 22. On the first attempt of analysing, the phase may just seem like squiggles on a graph! However, why don't we try to plot the graph of the original financial time series, highlighted by the blue oscillating function, alongside the instantaneous phase, highlighted by the orange line, as shown in figure 24.

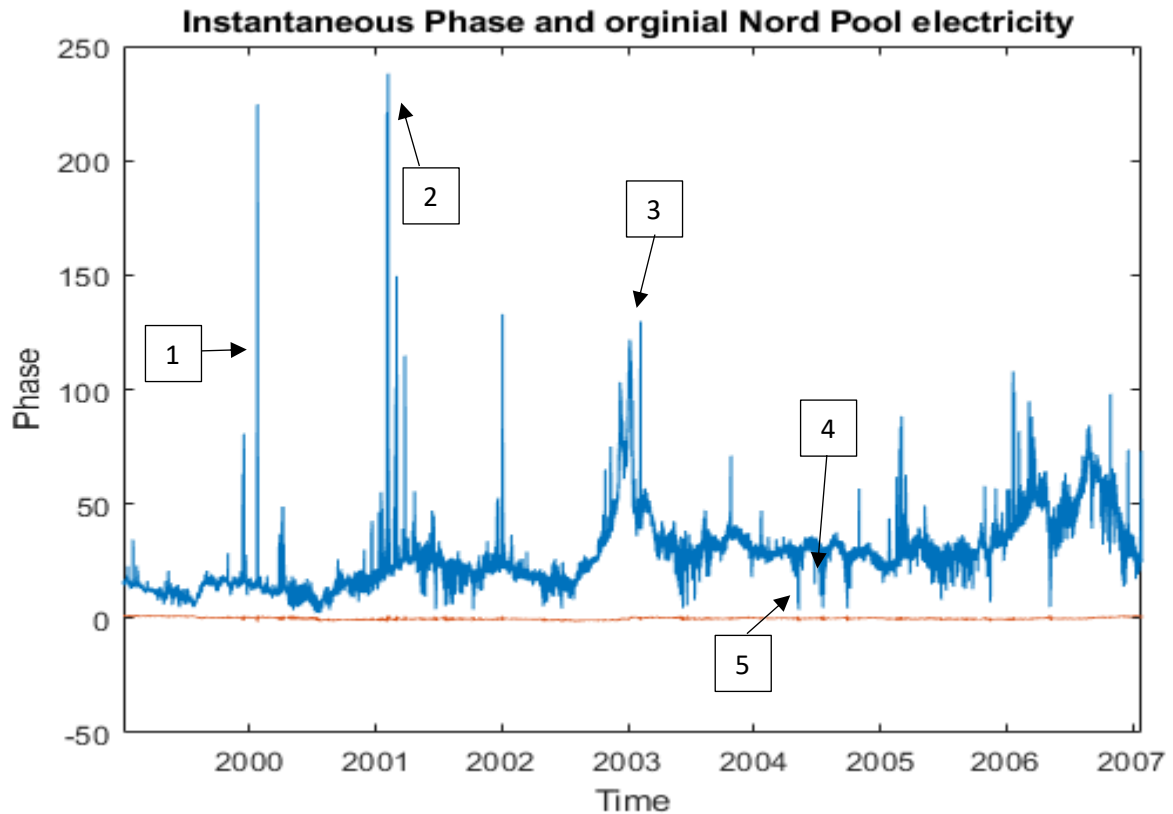


Figure 24.1: Graph of instantaneous phase and Nord Pool electricity signal. Published with MATLAB® R2017b

Observing figure 24.1, we are now able to study the phase of the signal. In order to analyse the phase, we will pick extreme observations, as indicated on the financial time series, and study the corresponding phase in order to deduce whether or not the phase of the signal reveals any information about the occurrence of these particular events. In figure 24.1, each of the extreme observations have been labelled in the order they are analysed.

The first extreme observation studied occurred on the 24<sup>th</sup> January 2000. Figure 24.2 below, shows us the electricity prices and the phase value between the 31<sup>st</sup> December 1999 to the 25<sup>th</sup> February 2000. Between the 31<sup>st</sup> December 1999 and 14<sup>th</sup> January 2000, the electricity prices are fairly periodic, and the corresponding phase value during this period is a fairly straight line. However, after this period we analyse an unusual, but interesting piece of information in the phase of the signal. Up to the 23<sup>rd</sup> January 2000, the phase of the signal seems to be quite constant, however on the 24<sup>th</sup> January 2000 we see a rapid increase in electricity prices. This rapid increase in electricity prices caused a disturbance in the phase of the signal. This disturbance is known as a phase slip. The phase slip occurred on the 24<sup>th</sup> January at 07:00 and stabilised on the 25<sup>th</sup> January at 19:00. We observed a peak price of electricity on the 24<sup>th</sup> January at 09:00. Electricity prices stabilised on the 26<sup>th</sup> January at around 00:00. Gathering this information, we observed a phase slip which lasted for 36 hours, around 2 hours before the electricity prices hit a peak of 224.60 EUR/MWh. As we can see, once the electricity prices stabilised, the phase of the signal returns to being a fairly straight line. We must analyse further extreme observations in the Nord Pool data set, in order to have evidence to support our results.

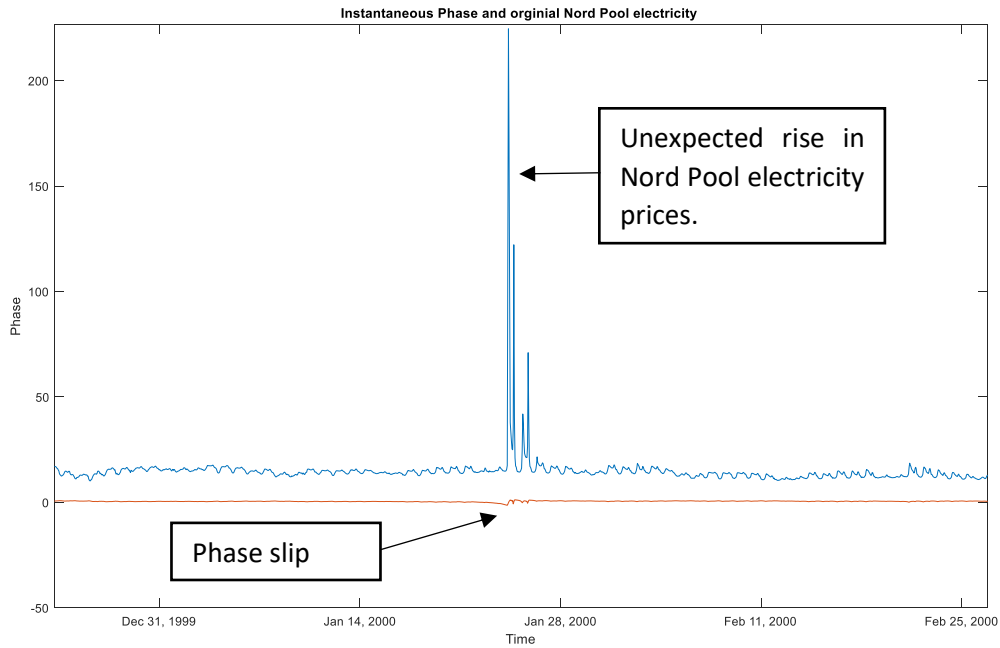


Figure 24.2: Analysis of the first extreme observation. Published with MATLAB® R2017b

The second extreme observation studied occurred on the 5<sup>th</sup> February 2001 at 09:00. This was the highest price electricity has been traded at on the Nordic market between the 1<sup>st</sup> January 1999 and the 26<sup>th</sup> January 2007. Figure 24.3 below, shows us the electricity prices and the phase value between the 19<sup>th</sup> January 2001 and 23<sup>rd</sup> February 2001. Between the 19<sup>th</sup> January 2001 and 2<sup>nd</sup> February 2001, the electricity prices are oscillating with some periodicity, and the corresponding phase value during the period is a fairly straight line. However, on the 2<sup>nd</sup> February 2001, there is a rapid increase in price at which electricity was traded at. This rapid increase caused a phase slip. In figure 24.3, we observe two distinct phase slips. The first occurred on the 2<sup>nd</sup> February at 08:00 and stabilised on the 3<sup>rd</sup> February at 02:00, and the second occurred on the 5<sup>th</sup> February at 07:00 and stabilised on the 6<sup>th</sup> February at 20:00. Comparing this to the peak prices, the first peak in electricity prices appeared on the 2<sup>nd</sup> February at 09:00, which is precisely an hour after the first phase slip. The second peak occurred on the 5<sup>th</sup> February at 09:00, which occurred 2 hours after the second phase slip was observed. Even after 2 observations, it is not enough to conclude our results. Let us take a look at further extreme observations in electricity prices in order to establish a pattern.

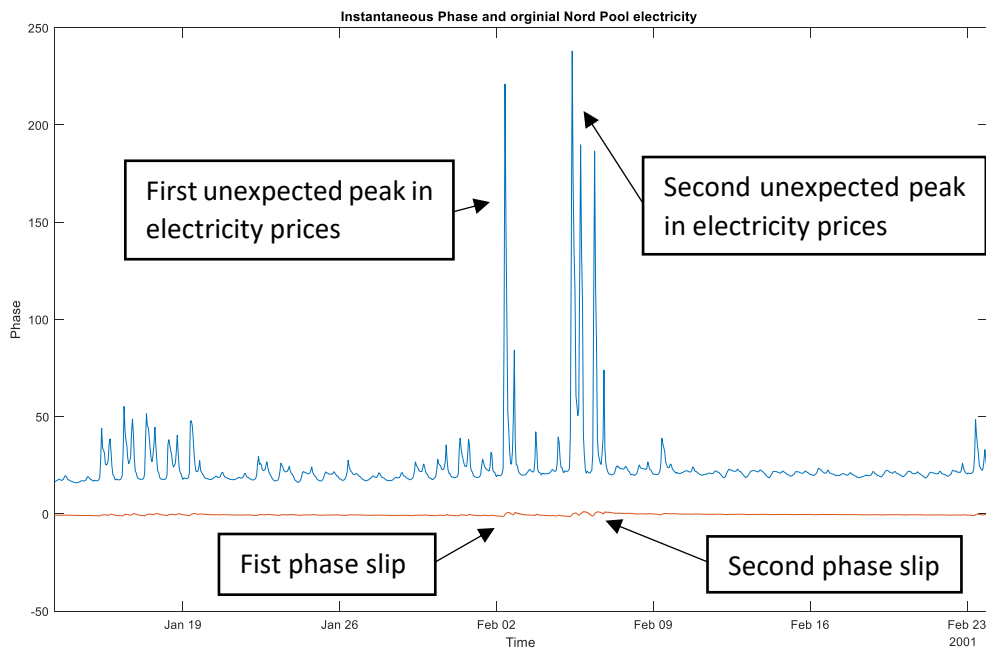


Figure 24.3: Analysis of the second extreme observation. Published with MATLAB® R2017b

Observing the data between July 2002 and May 2003, we examine electricity prices to be slowly increasing and decreasing as the weather changes. During the colder months, electricity is traded at a higher price, than during the warmer months. Between December 2002 and February 2003, we notice the electricity prices fluctuate as they increase and then decrease, with two distinct peaks observed during the period; as shown in figure 24.4. The rate at which prices are changing is not dramatic, as we can see electricity prices gradually increase and decrease, and this has been reflected in the phase of the signal. Between December 2002 and February 2003, there has been no evidence of the occurrence of a phase slip, even though we have seen large fluctuations in electricity prices.

However, electricity prices saw an unexpected rise on the 6<sup>th</sup> February 2003 at 09:00 which then peaked at 129.80 EUR/MWh on the 6<sup>th</sup> February at 18:00; which can be clearly seen in figure 24.5. A phase slip was first recorded on the 6<sup>th</sup> February at 07:00, which was 2 hours before the unexpected rise, and lasted till the 6<sup>th</sup> February at 20:00. Druring this period, the phase saw a further depreciation in its value on the 6<sup>th</sup> February at 17:00, which was an hour before the the electricity prices peaked at 129.80 EUR/MWh. Comapring this to the results found earlier, we are starting to notice that the phase slips mainly occur around a few hours before we see the electrcitity prices to peak or increases to an unexpected price.

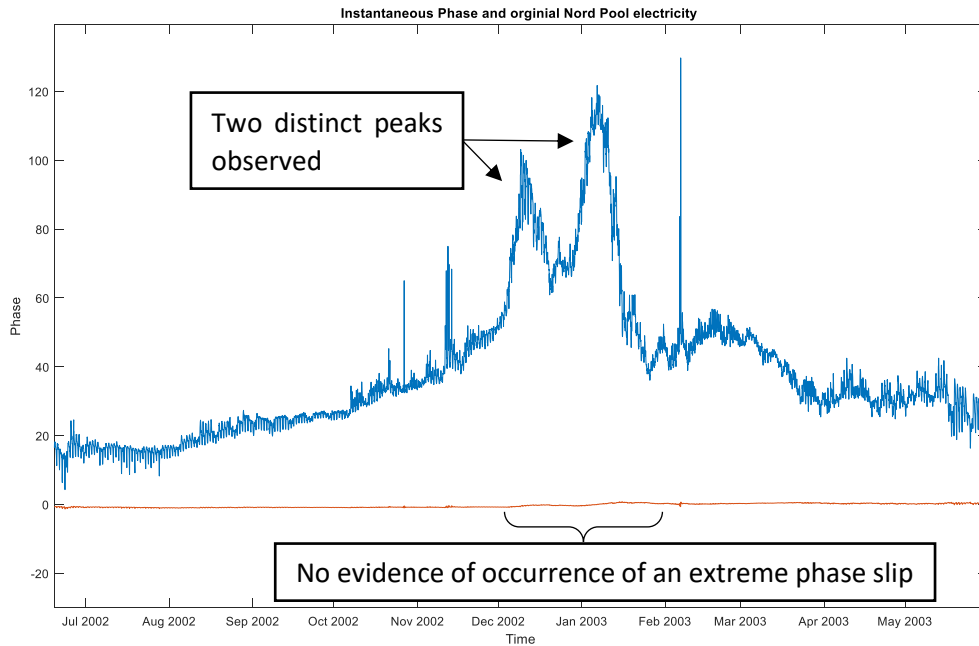


Figure 24.4: Analysis of the fluctuations of electricity between December 2002 and January 2003. Published with MATLAB® R2017b

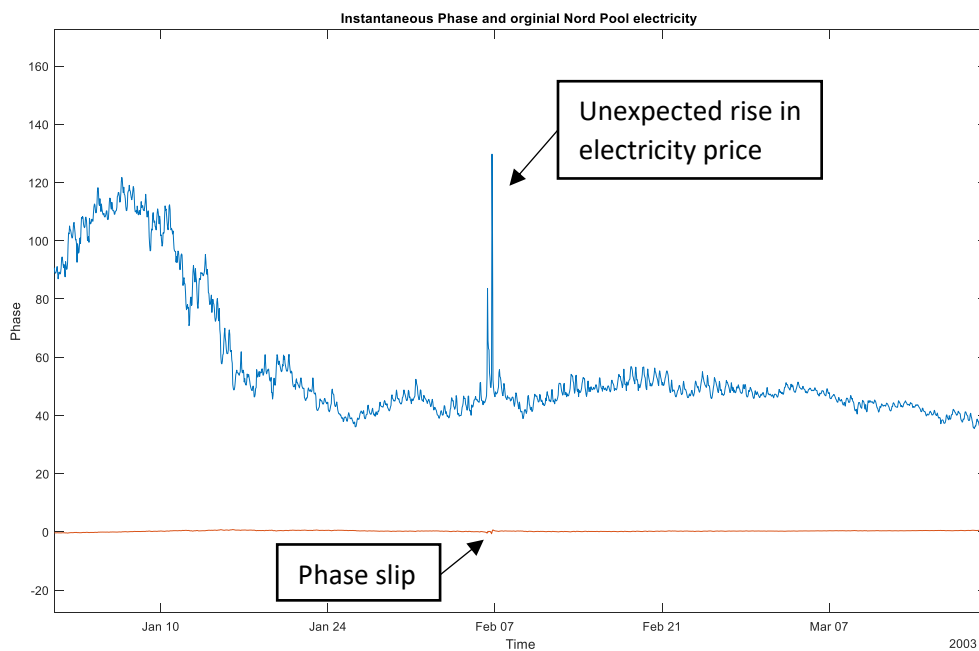


Figure 24.5: Analysis of distinctive unexpected rise in Nord Pool electricity prices on the 6<sup>th</sup> February 2003. Published with MATLAB® R2017b

Before any conclusions are made, I wish to observe the phase of the signal when there is an unexpected depreciation in electricity prices. Studying the time series between January 2004 and January 2005, as shown in figure 24.6, rather than having an extreme increase in electricity price; the opposite has occurred. As you can see from the time series, on three occasions we observe an



unexpected decrease in electricity prices. Studying the corresponding phase of the signal, we may clearly see the phase slip which occurs on all three individual events. Analysing the middle trough closely, as shown in figure 24.7, we see a disturbance in the phase. During this period the electricity prices are at a minimum of 4.64 EUR/MWh on the 18<sup>th</sup> July 2004 at 08:00. A dramatic increase in the phase is detected on the 17<sup>th</sup> July at 23:00, which then peaks on the 18<sup>th</sup> July at 03:00 and then rapidly decreases to a minimum on the 18<sup>th</sup> July at 08:00, the same time the electricity prices hit a minimum during this period. A similar pattern also occurred in May 2004 as shown in figure 24.8.

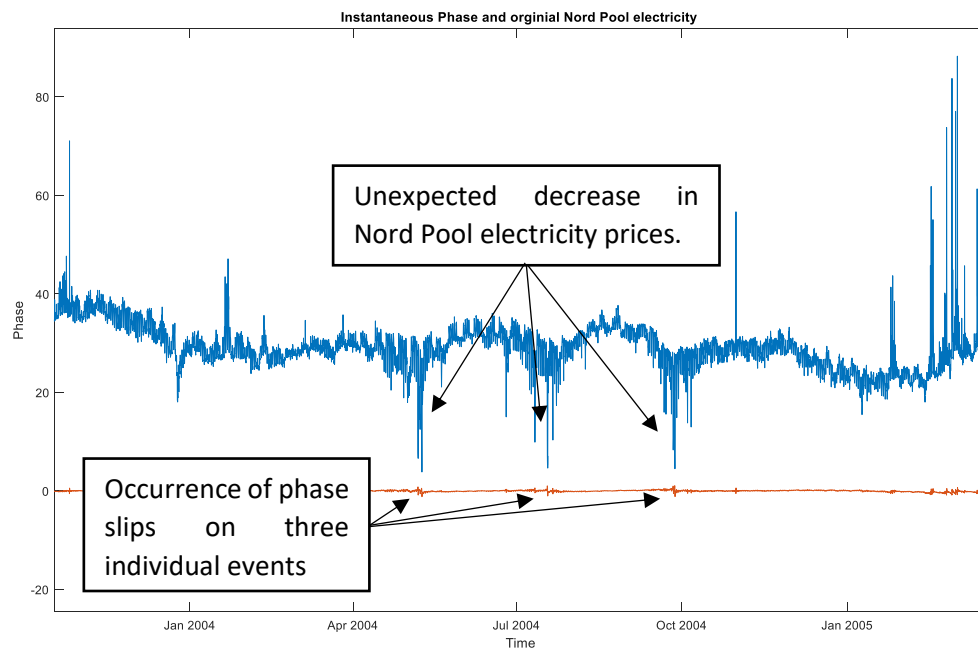


Figure 24.6: Dpreciation in Nord Pool electricity prices occurring between January 2004 and Jan 2005. Published with MATLAB® R2017b

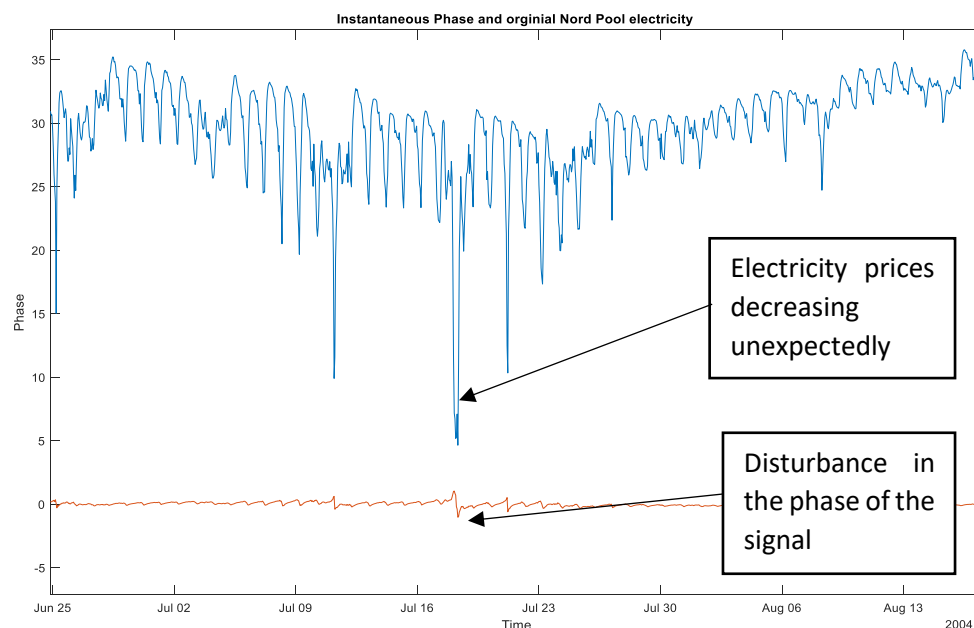
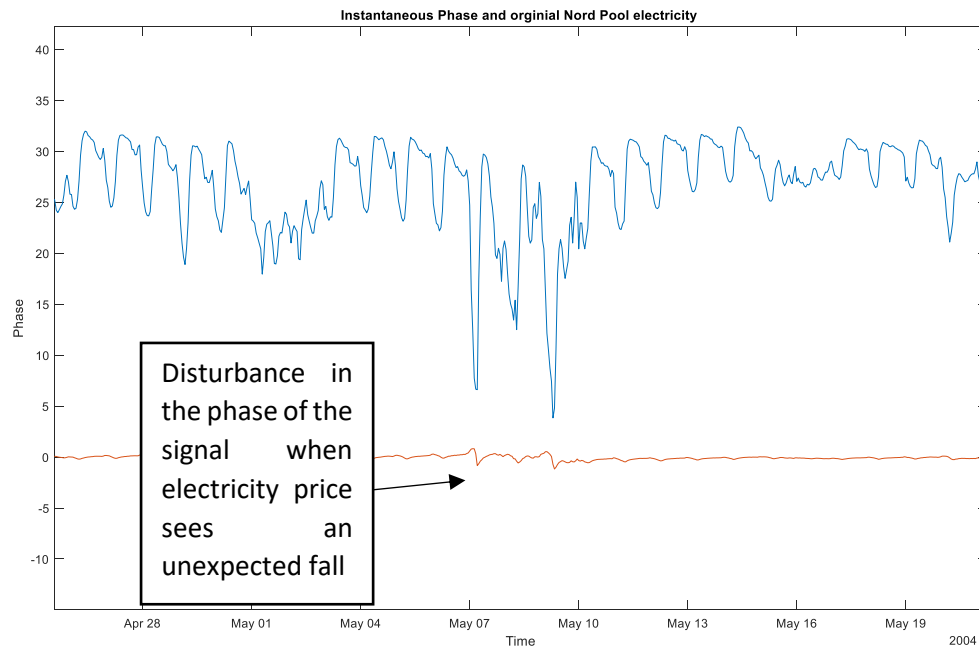


Figure 24.7: Analysis of the middle trough on figure 17. Published with MATLAB® R2017b



*Figure 24.8: Showcasing similar phase slip pattern in May 2004, when Nord Pool electricity prices depreciates. Published with MATLAB® R2017b*

Studying the time series has allowed us to identify a disturbance in the phase slips which has provided us with useful information in regards to the observations of extreme events in the Nord Pool electricity prices time series.

## 5.0 Conclusion

In this thesis, our aim was to discover if there exist some information in the phase of an analytic signal, which would allow one to predict or indicate the occurrence of any extreme events. We used the Nord Pool data set which contained the hourly prices of electricity at which they traded at across 7 years from 1<sup>st</sup> January 1999 to 26<sup>th</sup> January 2007. In this thesis, we assumed, that electricity is a non-storable commodity as well as the time average has been subtracted so the signal has a zero average.

We explored techniques brought to us from Electrical Engineering and applied it to a financial time series. In doing so, we were able to analyse the phase of the financial time series containing the Nord Pool electricity prices and discover some unusual but interesting information, which was embedded in the phase. Over the 5 extreme observations studied, we found the occurrence of phase slips, indicated by a disturbance in the phase, in the neighbourhood of these critical events. Moreover, we found that the phase slips occurred a few hours before the price of electricity peaked, in each of the cases studied. In either case, whether the electricity prices dramatically increased or decreased, the phase gave indication as to massive change in the price.

However, in this thesis, we analysed a time series for a specific type of commodity which has unique characteristics. Before, we make a definite deduction that the phase of a signal for sure provides information regarding critical events, we may need to further our research and consider different factors.

In future work, I would like to consider, firstly if we can apply this methodology to all financial time series of commodities which are both storable and non-storable? Secondly, does having a signal with a non-zero average give us the same results as we found in this thesis? Lastly, the most important factor which would require further research, is there a definite time period in which one would expect to see the occurrence of these phase slip before the existence of critical events, and how will we able to implement such method of extracting the phase of a signal to live data prices extracted from the current markets.

## 6.0 Appendix

An appendix contains the Matlab code used to produce graphs and output which has been explained in this thesis. Each figure below corresponds to a figure which can be found in the main body of the thesis.

Figure 3: Nord Pool Hourly Electricity Prices

```
%Let us begin by plotting the Nord Pool electricity prices as a financial
%time series
%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows and save this as Time.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Let us now plot the financial time series of Nord Pool electricity prices.
figure
plot(Time,NordPoolElecPrices)
title('Nord Pool Hourly Electricity Prices')
xlabel('Time')
ylabel('Electricity Prices (EUR/MWh)')
```

Figure 4 has been obtained by using in the 'zoom in' and 'zoom out' functions, provided by Matlab, on the graph produced by the code above.

Figure 6: Displacement of object in SMH

```
%Let us plot an example of the displacement of an object in Simple Harmonic
%Motion.
t=0:1e-4:50;
%t is the time dependent variable which will vary from 0 to 50
%measured in seconds

%Now we wish to plot the displacement of the object, which we will assume
%follows a sine curve.

figure, plot(t,sin(t)), title('Displacement of object in SMH')
xlabel('Time')
ylabel('Displacement')
```

Figure 7: Graph of  $x(t) = \sin(6t)$  and  $x(t) = \sin(t)$

```
%here we wish to produce an example of a simple harmonic function.
%we will plot the function  $x(t)=\sin(wt)$ 

t=0:1e-4:2*pi;
%The variable t is the time dependent variable.
%In this example we will assume w is 6.

figure, plot(t, sin(6*t))
hold on
plot(t,sin(t)) %let us plot the function  $\sin(t)$  in order to compare the phase
hold off
title('Graph of  $x(t)=\sin(6t)$  and  $x(t)=\sin(t)$ ')
```

```
xlabel('Time, t')
ylabel('x(t)')
```

Figure 8: Graph of signal  $x(t)$

```
%Here we wish to plot the signal
%x(t)=0.5cos(0.9t)+1.5cos(1.1t)

%let the variable t be the time dependent variable
%For this purpose we shall restrict the value of t from 0 to 500.
t=0:1e-4:500;

%let us define the signal x(t)
x= (0.5*cos(0.9*t)) +(1.5*cos(1.1*t));

%plotting the graph
figure, plot(t,x)
title('Graph of signal x(t)')
xlabel('Time, t')
ylabel('Signal, x(t)')
```

Figure 9: Graph of signal  $x(t)$  with Amplitude enveloping

```
%Here we wish to plot the signal
%x(t)=0.5cos(0.9t)+1.5cos(1.1t)

%let the variable t be the time dependent variable
%For this purpose we shall restrict the value of t from 0 to 500.
t=0:1e-4:500;

%let us define the signal x(t)
x= (0.5*cos(0.9*t)) +(1.5*cos(1.1*t));

%Defining our new analytic signal:
z= 0.5*exp(1i*0.9*t) + 1.5*exp(1i*1.1*t);
%Now we can plot our original signal x and over lap it with the amplitude
%A(t) which we have calculated to check whether the amplitude is indeed
%enveloping the signal.

%Defining A(t) in steps:
s= power((1+ (0.5/1.5)*cos((1.1-0.9)*t)),2);
d= power((0.5/1.5)*sin((1.1-0.9)*t),2);
A=1.5* sqrt(s+d);

%Now plotting graph and amplitude on the same axis
figure
plot(t,A)
hold on
plot(t,x)
hold off
xlabel('Time, t')
ylabel('Signal, x(t)')
title('Graph of signal x(t) with Amplitude enveloping')
```

Figure 10: Graph of Instantaneous Phase of signal  $x(t)$  without using unwrap command

```
%Here we wish to plot the signal
%x(t)=0.5cos(0.9t)+1.5cos(1.1t)

%let the variable t be the time dependent variable
%For this purpose we shall restrict the value of t from 0 to 500.
t=0:1e-4:500;

%let us define the signal x(t)
x= (0.5*cos(0.9*t)) +(1.5*cos(1.1*t));

%Defining our new analytic signal:
z= 0.5*exp(1i*0.9*t) + 1.5*exp(1i*1.1*t);

%let us plot the phase, which is the arg(z) using no unwrap command
figure, plot(t, angle(z))
title('Graph of Instantaneous Phase of signal x(t)')
xlabel('Time, t')
ylabel('Phase')
```

Figure 11: Graph of Instantaneous Phase of signal  $x(t)$  using unwrap command

```
%Here we wish to plot the signal
%x(t)=0.5cos(0.9t)+1.5cos(1.1t)

%let the variable t be the time dependent variable
%For this purpose we shall restrict the value of t from 0 to 500.
t=0:1e-4:500;

%let us define the signal x(t)
x= (0.5*cos(0.9*t)) +(1.5*cos(1.1*t));

%Defining our new analytic signal:
z= 0.5*exp(1i*0.9*t) + 1.5*exp(1i*1.1*t);

%let us plot the phase, which is the arg(z) using unwrap command
figure, plot(t, unwrap(angle(z)))
title('Graph of Instantaneous Phase of the signal x(t)')
xlabel('Time, t')
ylabel('Phase')
```

Figure 12 has been obtained by using in the 'zoom in' and 'zoom out' functions, provided by Matlab, on the graph produced by the code above.

Figure 13: Simple harmonic signal

```
%Let us show an example of the application of the Hilbert command
%on a discrete-time signal x(t), where t is the time dependent variable

%First we shall define the time period on which the simple harmonic
```

```

%function will be defined on
t=0:1e-4:500; %1e-4 is Matlab code for 0.0001

x=0.5*sin(0.9*t)+1.5*sin(1.1*t);

%Let us plot the signal x(t)
figure, plot(t,x)
title('Simple harmonic signal')
xlabel('Time')
ylabel('Signal value at each time interval')

```

**Figure 14: Matlab code to produce an analytic signal of  $x(t)$  using Hilbert command**

```

%Let us show an example of the application of the Hilbert command
%on a discrete-time signal x(t), where t is the time dependent variable

%First we shall define the time period on which the simple harmonic
%function will be defined on
t=0:1e-4:500; %1e-4 is Matlab code for 0.0001

x=0.5*sin(0.9*t)+1.5*sin(1.1*t);
%implementing Matlab's Hilbert command in order to extract the analytic
%signal which will be defined by z

z=hilbert(x);

%Let us check the output of signal z are complex numbers
z(1:3) %This command is asking Matlab to display first 3 values of signal z

%We may compare this to the first 3 values of the original real valued
%signal
x(1:3)

```

**Figure 15: Amplitude of the signal extracted using the Hilbert command**

```

%Let us show an example of the application of the Hilbert command
%on a discrete-time signal x(t), where t is the time dependent variable

%First we shall define the time period on which the simple harmonic
%function will be defined on
t=0:1e-4:500; %1e-4 is Matlab code for 0.0001

x=0.5*sin(0.9*t)+1.5*sin(1.1*t);

%implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(x);

%Let us now use the Matlab command abs to plot the amplitude of the signal
figure
plot(t,x) %Plotting the original signal to check whether the amplitude envelopes it.
hold on
plot(t,abs(z))

```

```
hold off
title('Amplitude of the signal x(t)')
xlabel('Time')
ylabel('Signal value at each time interval')
```

Figure 16: Instantaneous Phase:

```
%Let us show an example of the application of the Hilbert command
%on a discrete-time signal x(t), where t is the time dependent variable

%First we shall define the time period on which the simple harmonic
%function will be defined on
t=0:1e-4:500; %1e-4 is Matlab code for 0.0001

x=0.5*sin(0.9*t)+1.5*sin(1.1*t);

%implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(x);

%Let us now plot the instantaneous phase of the signal using Matlabs angle
%command.
figure
plot(t, angle(z))
title('Instantaneous Phase')
xlabel('Time, t')
ylabel('Phase')
```

Figure 17: Instantaneous Phase using unwrap command

```
%Let us show an example of the application of the Hilbert function
%on a discrete-time signal x(t), where t is the time dependent variable

%First we shall define the time period on which the simple harmonic
%function will be defined on
t=0:1e-4:500; %1e-4 is Matlab code for 0.0001

x=0.5*sin(0.9*t)+1.5*sin(1.1*t);

%implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(x);

%plotting the phase using Matlab's unwrap command
figure
plot(t, unwrap(angle(z)))
title('Instantaneous Phase')
xlabel('Time')
ylabel('Phase')
```



Figure 18 has been obtained by using in the 'zoom in' and 'zoom out' functions, provided by Matlab, on the graph produced by the code above.

Figure 19.1: Nord Pool Hourly Electricity Prices

```
%Let us begin by plotting the Nord Pool electricity prices as a financial
%time series
%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows and save this as Time.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Let us now plot the financial time series of Nord Pool electricity prices.
figure
plot(Time,NordPoolElecPrices)
title('Nord Pool Hourly Electricity Prices')
xlabel('Time')
ylabel('Electricity Prices (EUR/MWh)')
```

Figure 19.2-19.5 have all been obtained by using in the 'zoom in' and 'zoom out' functions, provided by Matlab, on the graph produced by the code above.

Figure 20: Matlab code to produce an analytic signal of the financial time series using Hilbert command

```
%Let us begin by creating the Nord Pool electricity prices as a financial
%time series by creating a time variable.
%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows. We shall save the transposed variable as 'Time'.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z
z=hilbert(NordPoolElecPrices);
%Let us check the output of the signal z are complex numbers
z(1:3) %This command is asking Matlab to display first 3 values of signal z

%We may compare this to the first 3 values of the original real valued
%signal(which is the first 3 prices from the data set)
NordPoolElecPrices(1:3)
```

Figure 21: Amplitude of the Nord Pool electricity signal using Hilbert command

```
%Let us begin by creating the Nord Pool electricity prices as a financial
%time series by creating a time variable.
%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows. We shall save the transposed variable as 'Time'.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z
```

```

z=hilbert(NordPoolElecPrices);

%Let us now use the Matlab command abs to plot the amplitude of the complex
%signal
figure
plot(Time,NordPoolElecPrices)%Plotting the original signal to check whether
%the amplitude envelopes it.
hold on
plot(Time,abs(z))
hold off
title('Amplitude of the Nord Pool electricity prices')
xlabel('Time')
ylabel('Electricity Prices (EUR/MWh)')

```

Figure 22: Instantaneous Phase of the complex signal

```

%Let us begin by creating the Nord Pool electricity prices as a financial
%time series by creating the time variable.
%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows. We shall save the transposed variable as 'Time'.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(NordPoolElecPrices);

%Let us now plot the instantaneous phase of the complex signal using
%Matlab's angle command
figure
plot(Time, angle(z))
title('Instantaneous Phase of complex signal')
xlabel('Time')
ylabel('Phase')

```

Figure 23: Instantaneous Phase of the complex signal using unwrap command

```

%Let us begin by creating the Nord Pool electricity prices as a financial
%time series by creating a time variable.
%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows. We shall save the transposed variable as 'Time'.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(NordPoolElecPrices);

%plotting the instantaneous phase using Matlab's unwrap command
figure
plot(Time, unwrap(angle(z)))
title('Instantaneous Phase using unwrap command')
xlabel('Time')
ylabel('Phase')

```

Figure 24.1: Graph of instantaneous phase and Nord Pool electricity signal.

```
%Let us begin by creating the Nord Pool electricity prices as a financial
%time series by creating the time variable.
%The time variable will be stored as 'time'. We must insure to transpose the
%variable as Matlab will store this as rows. We shall save the transposed variable as 'Time'.
time=datetime(1999,1,1,1,0,0):hours(1):datetime(2007,1,27,0,0,0);

%Implementing Matlab's Hilbert command in order to extract the time
%dependent complex signal which will be defined by z

z=hilbert(NordPoolElecPrices);

%plotting the phase using Matlab's unwrap command and the original time
%series on the same graph
figure
plot(Time, NordPoolElecPrices)
hold on
plot(Time, unwrap(angle(z)))
hold off
title('Instantaneous Phase and original Nord Pool electricity ')
xlabel('Time')
ylabel('Phase')
```

Figure 24.2-24.8 has been obtained by using in the 'zoom in' and 'zoom out' functions, provided by Matlab, on the graph produced by the code above.

## 7.0 Glossary

Throughout the thesis, we have used some financial terms which are explained in more depth below.

**Ask Price:** This is the price at which the seller of a financial instrument is willing to accept for their goods or services. The ask price is referred to as the offer price [Boyle, 2018].

**Backwardation:** A market is said to be in backwardation, when the futures spot price is less than the expected future spot price. Speculators who are in a long position want the markets to be in backwardation as they want the futures prices to increase [Ventimiglia, 2018].

**Bid price:** This is the highest price a buyer (also known as the bidder) is offering to pay for the financial instrument [Boyle, 2018].

**Bid-ask spread:** A bid-ask spread (or the bid-offer spread) is the difference in price at which a market maker is willing to buy an asset from the opposing party or sell it to an opposing party [Boyle, 2018].

**Broker:** A broker is an individual or a brokerage firm that arranges transactions between buyers and sellers and in return receives a commission when the deal is executed [Boyle, 2018].

**Clearing house:** A clearing house is a separate entity from the market participants buying or selling the assets. They are set up to enhance market participants assurance against failure to pay on behalf of the losing counterparties. In other words, a clearing house reduces the risk of any clearing firm failing to honour its trade settlement obligations [Boyle, 2018].

**Contango:** A market is said to be in contango, when the futures prices are greater than the expected future spot price. When the market is in contango, futures prices are falling over time as new information brings them in line with the expected future spot price [Ventimiglia, 2018].

**Convenience Yield:** This is the benefit for a user to hold an underlying asset rather than a contract or a derivative product on the underlying asset [Boyle, 2018].

**Day-ahead market:** A day-ahead market is where one can buy or sell a financial contract which has a maturity date of the following day. [Boyle, 2018]

**Derivative:** Also known as a financial derivative, is an economic contract whose value depends on the value of another instrument or underlying asset [Boyle, 2018].

**End user market:** An end user market is a place where an individual or a small institution are able purchase goods for their personal use.

**Forward Contract:** A forward contract is a non-standardised contract between two parties to buy or sell an asset at a specified future time at a price agreed upon today. Forward contracts are private contracts; thus, they do not trade on an organised exchange and the contract specifications are customised according to the parties involved [Boyle, 2018].

**Futures Contract:** A futures contract is a standardised financial contract obligating the buyer to purchase an asset, and the seller to sell an asset at a predetermined date and price. The contract must specify the quality and quantity of the underlying asset. Futures contracts are traded on a futures exchange and the contract may call for physical delivery of the asset (or even cash-settled) [Boyle, 2018].

**Hedge:** Hedging refers to all trading activity that reduces risk and minimises unwanted financial exposure or neutralises portfolio risk. A “well-hedged book” is a portfolio that should experience very few gains or losses regardless to the movement of the market [Boyle, 2018].

**Intraday market:** An intraday market is a trading platform where the buyer or seller of the financial contract must complete all transactions on the same day before the market closes [Boyle, 2018].

**Long-term futures:** Also known as long futures, is a type of financial contract which is used when the company will have to buy the underlying asset in the future [Boyle, 2018].

**Market maker:** A market maker is a company or an individual that quotes both a buy and a sell price of a financial instrument, hoping to make a profit on the bid-offer spread [Boyle, 2018].

**Mark-to-market:** This is the fair value of a financial instrument which has the ability to change over a period of time [Investopedia, 2018].

**Physical power contract:** This is essentially a contract for the physical delivery of electricity for the following day [Boyle, 2018].

**Portfolio:** A portfolio is composed of a range of investments held by a person or an organisation. [Zachariadis, 2017]

**Security:** A financial instrument, such as goods or assets, which holds a value and the property that the asset or goods can be swapped with other assets or goods which possess an equivalent value [Investopedia, 2018].

**Single market:** A single market is a trading bloc, consisting of countries, in which policies and regulations which have been put in place to remove border tariffs on goods traded between these countries. The Nord Pool market is an example of a single market [Boyle, 2018].

**Spot price:** This is the current prevailing value of any goods or services on the market [Boyle, 2018].

**Trader:** An individual or an institution such as an investment bank, who partake in the financial markets by either buying or selling financial instruments [Boyle, 2018].

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