The Linear Time Series Analysis of the Nordic Electricity Market

Abdullahi Dahir Mohamed 100089200

September 10, 2015

Declaration

This MSc project was written for the use of submitting to Queen Mary University of London only. This thesis was completed under the supervision of Dr Wolfram Just and is my original research.

Acknowledgement

I would like to take the chance to express thankfulness to Dr Wolfram Just for taking me under his supervision and providing me the necessary help and guidance to produce this work.

Abstract

The prices of the spot markets have the potential to show interesting dynamical phenomena. Within this project, time series data sets of the Nordic spot electricity market are analysed in order to identify statistical qualities. Standard linear tools are employed to check whether hidden long range correlations can be revealed when comparing the return of the price data.

Contents

1 Introduction **4**

1.1 Research background . . . 4

1.2 Structure of Thesis . . . 5

2 Theory **5**

2.1 The Elspot case study . . . 5

2.2 Time Series . . . 7

2.3 Fourier Transform . . . 13

2.3.1 Background . . . 13

3 Data **16**

3.1 Brief example of Logarithmic return graph . . . 16

3.2 Data Analysis . . . 17

3.2.1 Averages of the price data of 1999 and 2007 . . . 17

3.2.2 Discrete Fourier Transform of the data averages for both years . . . 18

3.2.3 Descriptive Statistics . . . 22

3.3 Time series analysis . . . 24

3.3.1 Stationarity tests . . . 25

3.3.2 White noise test . . . 25

3.3.3 Correlation functions . . . 26

3.3.4 Multiple Regression Trend and Seasonality . . . 28

4 Conclusion **29**

5 Bibliography **30**

6 Excel Formulae **30**

1 Introduction

1.1 Research background

As studies have shown, during the eighties, electricity was manufactured and transported by national enterprises. Operating as a cartel, they frequently had the duty of supplying to businesses and households. A socio-political reform in 1982 led the way for detached generation and distribution companies as widespread privatisation began in 1986, which culminated with the establishment of Nord Pool, the Nordic Market, in Norway in 1992. [1]

The Nordic Pool which is a leading market for trading such power resources is the largest of its kind today, providing a platform for buying and selling power in the Nordic region (Denmark, Finland, Norway and Sweden). In the context of competing electrical energy markets and hotspots, all stakeholders involved, ranging from governments to buyers and sellers to the public, need precise and truthful price projecting tools.

In the Pool, the enterprises that generate electricity place down the bids and matching prices and consumer enterprises do the same with consumption bids. Using market-operators, auctions can be done on an hourly basis to regulate the clearing price as well as the production and consumption bids. Since market clearing prices are readily made-available public information, aggregate supply and demand curves are updated on hourly, weekly and monthly time cycles. Energy firms can then purchase energy from mutual contracts to trade it to their customers. These companies require both short-term and long-term price forecasts to make the most of their respective profits. Just for clarification, we focus on long-term decisions associated to the pool. Therefore, despite using hourly cycles, we will sample weekly, monthly, quarterly and annually computed cycles.

1.2 Structure of Thesis

We will look at evidence from a case study, and some of the features are compared and contrasted in order to understand what they mean and what we can gather. Since electricity prices may display sporadic arrangements on numerous time intervals, it is important to note the effect of these hence we will introduce the maths behind this and the Fourier transform will be briefly discussed. In the third chapter, the data set will be provided and discoursed where we test it before the results are briefed and concluded in Chapter 4.

2 Theory

2.1 The Elspot case study

To comprehend undercurrents of the price fluctuations in electricity markets, it is noteworthy to understand the price setting mechanisms of the electricity market before we discuss our data. As part of an independent case study with hourly spot prices, we look at the Elspot which is the Nord Pool Spots daily auction market (in our case the 2014 week commencing from 19th to 12th July), where electrical power is bought and sold. The competitors who wish to trade energy on the Elspot send their buying orders to the Nord Pool by 12pm a day prior to the power being sent to the grid. And this is similar for conmpetitors who wish to sell power to Elspot.

Elspot price tables and graph

	Sa 19-07	F 18-07	Th 17-07	W 16-07	Tu 15-07	M 14-07	Su 13-07	Sa 12-07
SYS	27,66	28,94	29,30	29,30	28,88	27,44	24,84	26,79
NO1	27,00	26,68	26,78	26,78	26,39	25,24	24,46	25,51
NO2	27,00	26,68	26,78	26,78	26,39	25,24	24,46	25,51
NO3	28,87	30,96	31,12	31,09	30,97	28,78	25,51	29,02
NO4	28,87	30,96	30,23	29,49	29,43	27,73	25,30	27,89
NO5	27,00	26,68	26,78	26,78	26,39	25,24	24,46	25,51
SE1	28,87	30,96	31,12	31,11	30,97	28,79	25,51	29,08
SE2	28,87	30,96	31,12	31,11	30,97	28,79	25,51	29,08
SE3	28,87	30,96	31,12	31,11	30,97	28,79	25,51	29,08
SE4	28,87	30,96	31,12	31,11	30,97	28,79	25,51	29,08
FI	34,85	40,61	39,68	38,21	38,41	36,74	29,08	33,46
DK1	28,87	35,70	36,27	35,59	32,78	28,79	25,62	29,84
DK2	28,90	35,70	36, 27	35,59	32,78	28,79	25,62	29,84
EE	47,63	52,56	52,43	38,21	38,41	37,15	31,59	37,22
LV	58,37	56,99	55,96	53,07	65,75	56,31	56,38	43,99
LT	58,37	56,99	55,96	53,07	65,75	56,31	56,38	43,99

The price setting at Elspot based on information from the European Energy Exchange [2] and the Nordic Pool Spot [3] is a bilateral uniform auctioning price mechanism, where the systematic rate is the crossing of the accumulated supply and demand curves, which we provide. At 12pm, contestants place the bids then offer their bids on an hourly basis of the following day to the administrators of the Market. We know there can be three types of bidding on Elspot. With hourly bidding being the most constructive type where sets of price and volume for every hour are submitted, we look at these the case studies. Nord Pool Spots computers in Norway start computing the next-day prices and publish them. As well as this, the Nord Pool report to the partakers how much electricity theyve purchased or traded for every hour of the next day. This information on trading is then sent to the Transmission system operators in the Nord Pool. The Transmissions system operators utilize this information and compute the balancing power for each participant.

2.2 Time Series

The main goal of a time series analysis is to identify the nature of a variable using a sequence of observations and forecasting future values for it. Regardless of the depth of our understanding and the validity of our interpretation (theory) of this time variable, we can extrapolate the identified pattern to predict future events.

In this section, we present the basics of a time series. We presume that prices are recorded at specific time-intervals. We will attempt to make an analysis based on our data in the next chapter. The models are chosen with an inspection of the key features of the time series. Lets consider the following:

-seasonality;
-trend;
-correlation;
-stationarity
-white noise

Recalling my study of Time Series under Dr Coad [4], time series models with seasonality often took the forms;

Additive

$$X_t = m_t + s_t + Y_t$$

$$t = 0, 1, ..., n$$

Multiplicative

$$X_t = m_t s_t Y_t$$

$$t = 0, 1, ..., n$$

Mixed

$$X_t = m_t s_t + Y_t$$

$$t = 0, 1, ..., n$$

Where m_t is the trend component, s_t is the seasonal effect and Y_t is the random noise component.

Given any natural expected seasonality in our data, various ways have been devised to eliminate it from the trend. One such method is the small trend method. If in the case we find a small trend with a constant period, then given an additive seasonal model, where $E(Y_t) = 0$ and s_t is such that

$$s_t = s_{t-d}$$

d is the length of the period

And

$$\sum_{k=1}^{d} s_k = 0$$

When X_{jk} denotes a time series in season k of year j for k =1,2,...d and j=1,2,...b

The the period average is an unbiased estimator of the trend and given by

$$\hat{m}_j = \frac{1}{d} \sum_{k=1}^{d} X_{jk}$$

Where $E(\hat{m}_j) = m_j$

The seasonal component estimator is therefore

$$\hat{s}_k = \frac{1}{b} \sum_{j=1}^{b} (X_{jk} - \hat{m}_j)$$

Where $E(\hat{S}_k) = S_k$

Thus the residuals are

$$\hat{Y}_{jk} = X_{jk} - \hat{m}_j - \hat{s}_j$$

Another well known method is the classical decomposition. It consists of five steps,

Step 1: Estimating the trend using a moving average filter of period length d;

$$\hat{m}_t = \frac{1}{d}(X_{t-q} + X_{t-q+1} + ... + X_{t+q})$$

where $d = 2q + 1$ and $q + 1 \leq t \leq n - q$

And

$$\hat{m}_t = \frac{1}{d}(\frac{1}{2}X_{t-q} + X_{t-q+1} + ... + X - t + q - 1 + \frac{1}{2}X_{t+q})$$

where $d = 2q$ and $q + 1 \leq t \leq tn - q$

Step 2: Estimate seasonal effects S_k for $k = 1, 2, ..., d$

Compute the averages of the detrended values $(X_l - \hat{m}_l)$, $q < l = k + jd \leq n - q$ for $j = 1, 2, ..., b$

where $k + jd$ is the number of seasons upto year j plus k seasons and b is the number of years

We adjust them so that the seasonal effects meet the model assumptions, that is, the estimate of S_k is

$$\hat{S}_k = (X_l \bar{-} \hat{m}_l)_k - \frac{1}{d}\sum_{i=1}^{d}(X_l \bar{-} \hat{m}_l)_i$$

where $k = 1, 2, ..., d$ and $(X_l \bar{-} \hat{m}_l)_k$ is the average of the detrended data for the k-th season

And $\hat{S}_{d+1} = \hat{S}_1, \hat{S}_{d+2} = \hat{S}_2, ...$

Step 3: Removing the seasonality to obtain

$$D_t = X_t - \hat{S}_t$$

where $t = 1, 2, ..., n$

Step 4 : Re-estimate the trend $\hat{m}_t$ from the deseasonalised variables (D_t)

Step 5 : Calculate the residuals

$$\hat{Y}_t = X_t - \hat{m}_t - \hat{s}_t$$

The last way we will demonstrate removing seasonality is the use of differencing.

By defining a lag d difference operator

$$\nabla_d X_t = X_t - X_{t-d} = (1 - B^d)X_t$$

We apply this to our additive time series to obtain

$$\nabla X_t = (m_t + st + Y_t) - (m_{t-d} + S_{t-d} + Y_{t-d})$$

$$= \nabla m_t + \nabla_d s_t + \nabla_d Y_t$$

$$= \nabla_d m_t + \nabla_d Y_t$$

$$\text{since } \nabla_d s_t = s_t - s_{t-d} = 0$$

Seasonal effect has been removed.

Similarly, we perform differencing to remove trend in the absence of seasonality by using a special kind of linear filter with weights [-1,1] (could be shown via convolution which we will discuss shortly in a different light) and is repeated until a stationary series is obtained. We denote the first differencing operator by Δ (which we just covered but had not hitherto defined)

$$\nabla X_t = X_t - X_{t-1} = X_t - BX_t$$

Where B denotes the backward shift operator. Hence

$$\nabla X_t = X_t - BX_t = (1 - B)X_t$$

And in general,

$$B^j X_t = X_{t-jt}$$

$$\text{where } j \geq 0$$

Where is the trend m_t is a polynomial of degree k, we have the formula

$$\nabla^k X_t = k! B_k + \nabla^k Y_t$$

This means that if the noise fluctuates about zero, then k-th differencing of the time series with a polynomial of degree k should give a stable process with mean $k!B_k$. Next we discuss the autocorrelation function, which is a very helpful tool in assessing the degree of dependence and in recognising what kind of model the time series follows.

We must first define the autocovariance of time series as

$$\delta_{(X_{t+\tau}, X_t)} = cov(X_{t+\tau}, X_t)$$

for all indices t and lags τ

Hence the auto-correlation function becomes

$$p(\tau) = \frac{\delta(\tau)}{\delta(0)} = corr(X_{t+\tau}, X_t)$$

for all t and τ

Note that since $\delta(0) = var(X_t)$ then our autocorrelation function can be written as

$$\frac{cov(X, Y)}{\sqrt{var(X)var(Y)}}$$

In the more specific case i.e. the sample autocorrelation function, we define our sample autocovariance function as

$$\hat{\delta(\tau)} = \frac{1}{n} \sum_{t=1}^{n-|\tau|} (x_t - \bar{x})(x_{t+|\tau|} - \bar{x})$$

$$-n < \tau < n$$

where $\bar{x} = \frac{1}{n} \sum_{t=1}^{n} x_t$

Hence the sample autocorrelation function is defined by

$$\hat{p}(\tau) = \frac{\hat{\delta(t)}}{\hat{\delta}(0)}$$

$$-n < \tau < n$$

A time series X_t is also stationary if the random vectors $(X_t, ..., X_tn)$ is equal in distribution to $(X_t + \tau, ..., X_tn + \tau)^T$. Part of its properties includes the random variables X_t are identically distributed for all t and pairs of random vectors $(X_t, X_t + \tau)^T$ are identically distributed for all t and τ

White noise, which represents those aspects of the time series of interest which could not have been predicted in advance, has a mean zero and variance σ^2 written as

$$\epsilon \, WN(0, \sigma^2)$$

if and only if ϵ_t has zero mean and covariance function as

$$\gamma_\epsilon(h) = [\sigma^2]$$

if h is 0 and

$$\gamma_\epsilon(h) = 0$$

if h is not zero

2.3 Fourier Transform

2.3.1 Background

Our drive in this section is to have an understanding of the characteristics of the Fourier transform from the point of data and graphs in time series. In order to devise good representation techniques we cultivate different tools that allow us to find distinct features of a function - functions such as Amplitude (i.e. loudness) and Phase (the exponent). The study of Fourier transform, its strengths and flaws, is the vocal point towards other periodic functions. For example this happens in our everyday lives and in technology: motions are analysed and interpreted by our senses, a representation is grasped from this analysis and it is sent to the brain. To make sense of the structures of the Fourier Transform we must analyse it having used the works of Feldman [5] and J.A. Peacock [6].

Using a periodic function with period $L > 0$, that is, $f(t + L) = f(t)$. We denote by L^2_T the space of periodic functions of period T which are square integrable. Giving us,

$$\int_{t_0}^{t_0+T} |f(t)|^2 dt < \infty$$

By the Fourier serious the function f can be decomposed as

$$f(s) = \sum_{j=-\infty}^{+\infty} a_j e^{i2\pi\omega_j s}$$

,

$$a_j \subset R$$

We call this decomposition the *Fourier series of f* as the above equation is an orthogonal basis representation of the function f. The Fourier series shows that any periodic function can be broken down as a sum of sines and cosines. The Amplitude combines an amplitude of both sine and cosine and the Phase is the relative proportions of sine and cosine. This decomposition allows us to make an analysis of the frequencies present in a time series model.

To get an exact representation of the function, we compute the frequency amplitude a_j using the equation,

$$a_j = \int_0^L f(u) e^{i2\pi\omega_j u} du,$$

Here we measure a function $x(t)$ that is episodic of period $2L$, while computing the coefficients from the measurements. x(t) being the amplitude of a periodic indicator at time t. Since we cant fix $x(t)$ for some values of t, assume that $x(t)$ is measured for values of t ranging from $0 \leq t < 2L$.

Since we can't compute the complex Fourier coefficient exactly, we can get a Riemann approx by dividing the Integral domain into N intervals with length $\frac{2L}{N}$.

For t in the interval $n\frac{2L}{N} \leq t < (n+1)\frac{2l}{N}$ predict the Integrand $x(t)e^{ik\frac{\pi}{l}t}$ by its value at $t = n\frac{2l}{N}$ that is $x(n\frac{2l}{N})e^{-ik\frac{\pi}{l}n\frac{2l}{N}} = x(n\frac{2l}{N})e^{-2\pi i\frac{kn}{N}}$ hence estimate the integral over $n\frac{2l}{N} \leq t < (n+1)\frac{2l}{N}$ by the area of a rectangle with height $x(n\frac{2l}{N})e^{-2\pi i\frac{kn}{N}}$ and width $\frac{2l}{N}$ which gives us

$$c_k \approx c_k^{(N)} = \frac{1}{2l}\sum_{n=0}^{N-1} x(n\frac{2l}{N})e^{-2\pi i\frac{kn}{N}}\frac{2l}{N} = \frac{1}{N}\sum_{n=0}^{N-1} x(n\frac{2l}{N})e^{-2\pi i\frac{kn}{N}}$$

Since $x[n]$, $\hat{x}[k]$ are both periodic in period N, there is the vector $(\hat{x}[k])_{k=0,1,2,\ldots,N-1}$, defined as,

$$\hat{x}[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-2\pi i\frac{kn}{N}}$$

Which we call our **Discrete Fourier Series** that is used for discrete time periods and models.

So in a brief summary, given a sequence of N samples, a Discrete Fourier Transform is defined as X(K) where

$$X(k) = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-2\pi i\frac{kn}{N}}$$

and the Inverse Discrete Fourier Transform (which had hitherto not been discussed) can be used to determine x(k) where the IDFT is

$$x(k) = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{+2\pi i\frac{kn}{N}}$$

For simplicity purposes, when considering the Fast Fourier Transform, we ignore the scaling factors and simply define the FFT and IFFT as

$$FFT_N(k,x) = \sum_{n=0}^{N-1} x[n]e^{-2\pi i\frac{kn}{N}} = \sqrt{N}X(k)$$

and

$$IFFT_N(n,X) = \sum_{n=0}^{N-1} x[n]e^{+2\pi i\frac{kn}{N}} = \sqrt{N}x(n)$$

Due to its relevancy to understanding linearity of time series models, we will derive some of the properties, namely the conjugation and Parseval relation.

Conjugation

Given $x[n]$ is a discrete-time indication for the period N, $y[n] = x[\bar{n}]$. The k^{th} Fourier coefficient of $y[n]$ is

$$\hat{y}[k] = \frac{1}{N}\sum_{n=0}^{N-1} y[n]e^{-2\pi i\frac{nk}{N}} = \frac{1}{N}\sum_{n=0}^{N-1} x[\bar{n}]e^{-2\pi i\frac{nk}{N}} = \frac{1}{N}\sum_{n=0}^{N-1} \bar{x[n]}e^{-2\pi i\frac{n(-k)}{N}} = \hat{x}[\bar{-}k]$$

We can infer the k^{th} Fourier coefficient of the episodic discrete-time indicator $x[\bar{n}]$ is $\hat{x}[\bar{-}k]$. $x[n]$ is a real value on the condition that $x[\bar{n}]$,for all n, which is correct given that the Fourier coefficients of $x[n]$ and $y[n] = x[\bar{n}]$ are equivalent i.e.

$$x[n] \text{ is real for all } n \longleftrightarrow \hat{x}[\bar{-}k] = \hat{x}[k] \text{ for all } k$$

Parseval's relation

For our Fourier time series we prove via the Fourier Transform;

$$\sum_{k=0}^{N-1} |\hat{x}[k]|^2 = \sum_{k=0}^{N-1} x[\hat{k}]\hat{x}[k]$$

$$= \sum_{k=0}^{N-1} (\frac{1}{N}\sum_{n=0}^{N-1} x[\hat{n}]e^{2\pi i\frac{kn}{N}})\hat{x}[k]$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1} x[\bar{k}]e^{2\pi i\frac{kn}{N}}\hat{x}[k]$$

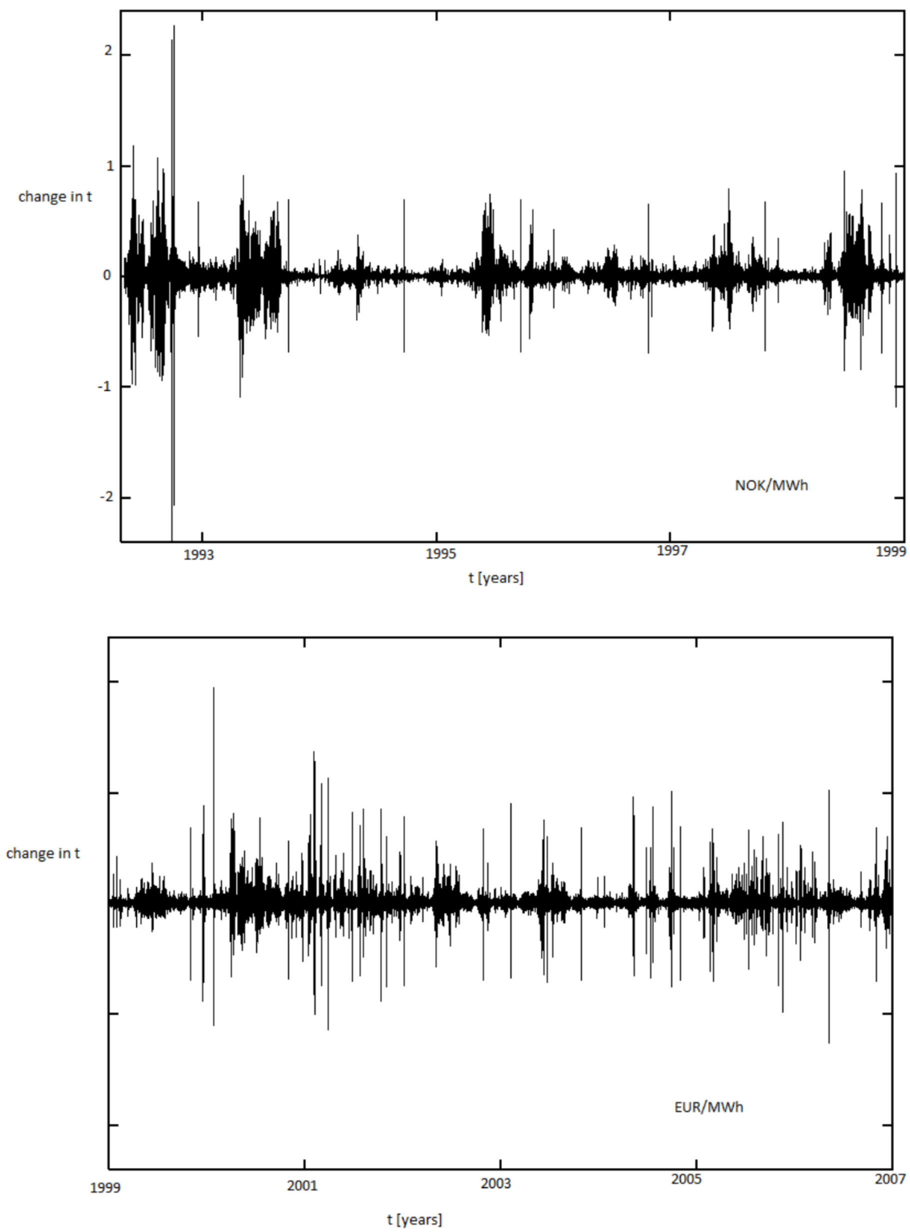
$$= \frac{1}{N}\sum_{n=0}^{N-1} x[\hat{n}](\sum_{k=0}^{N-1} e^{2\pi i\frac{kn}{N}}\hat{x}[k])$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x[\bar{n}]x[n]$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2$$

3 Data

3.1 Brief example of Logarithmic return graph



Top : Hourly logarithmic return for the spot prices in the Nordic electricity market (Nord Pool) from May 1992 until December 1998. Bottom: Hourly logarithmic return for the spot prices from January 1999 until January 2007. This piece of data was used in a similar project supervised by Dr Just and counts as an example of the type of data produced.

3.2 Data Analysis

Having been provided by Dr Just with a data set on an Excel spreadsheet reaching up to 70122 entries of electricity spot market prices ranging from the year 1999 to 2007, i have taken an average of every month of the years 2007 and 1999 in order to simplify the measure of quantitative's and make a comparison. It is worthy to note that throughout all respective data analysis i have used NUMXL features to make derivations as NUMXL is one of the main Microsoft Excel add-ons used to process time series analysis of econometric data.

3.2.1 Averages of the price data of 1999 and 2007

Year (1999)	Price
1	-0.009858772
2	-0.015979035
3	-0.011474983
4	-0.003400207
5	0.00272109
6	0.006770507
7	0.001348618
8	0.004705891
9	0.015968403
10	0.015717416
11	0.008411567
12	0.004500168

Year (2007)	Price
1	-0.01534557
2	0.030707228
3	0.010687669
4	0.003218311
5	0.031625967
6	0.018845855
7	-0.007495776
8	0.045478281
9	0.033482021
10	-0.019810029
11	0.031444308
12	-0.044611141

Based purely on this, there is nothing thus far to suggest a trend setting. The highest electricity price in 1999 was 0.0159 approx. in September and the highest in 2007 was 0.0455 approx. in August.

3.2.2 Discrete Fourier Transform of the data averages for both years

	Fast Fourier Transform (1999)	
step	magnitutde	Phase
0	0.019430663	0
1	0.029289435	2.6965E+308
2	0.04526847	2.6965E+308
3	0.056743453	2.6965E+308
4	0.06014366	2.6965E+308
5	0.05742257	2.6965E+308
6	0.050652063	2.6965E+308

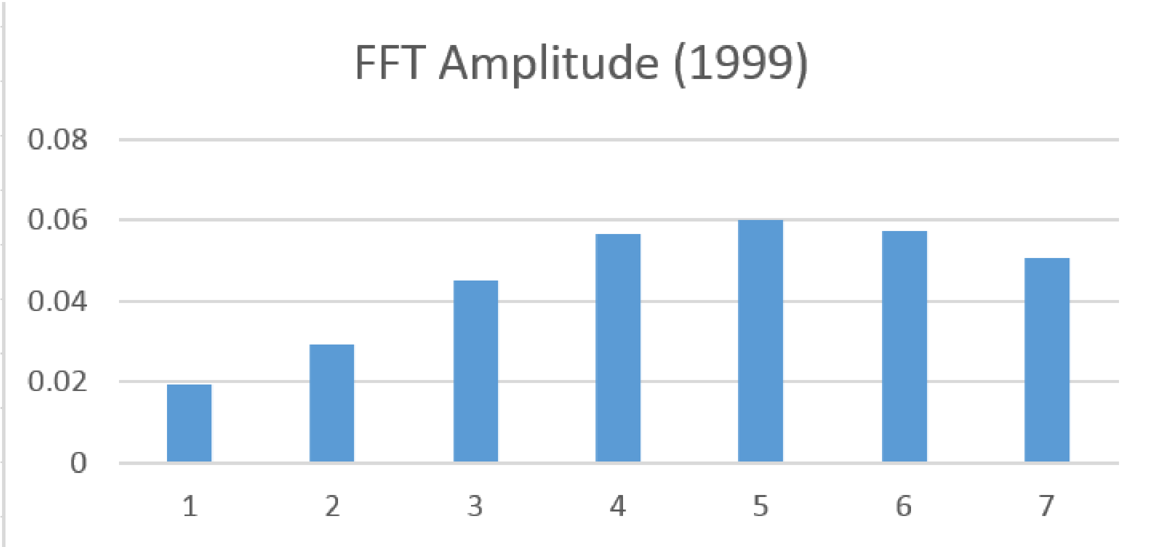
	Fast Fourier Transform (2007)	
step	magnitutde	Phase
0	0.118227125	0
1	0.133572694	2.6965E+308
2	0.102865467	2.6965E+308
3	0.092177798	2.6965E+308
4	0.088959487	2.6965E+308
5	0.05733352	2.6965E+308
6	0.038487665	2.6965E+308

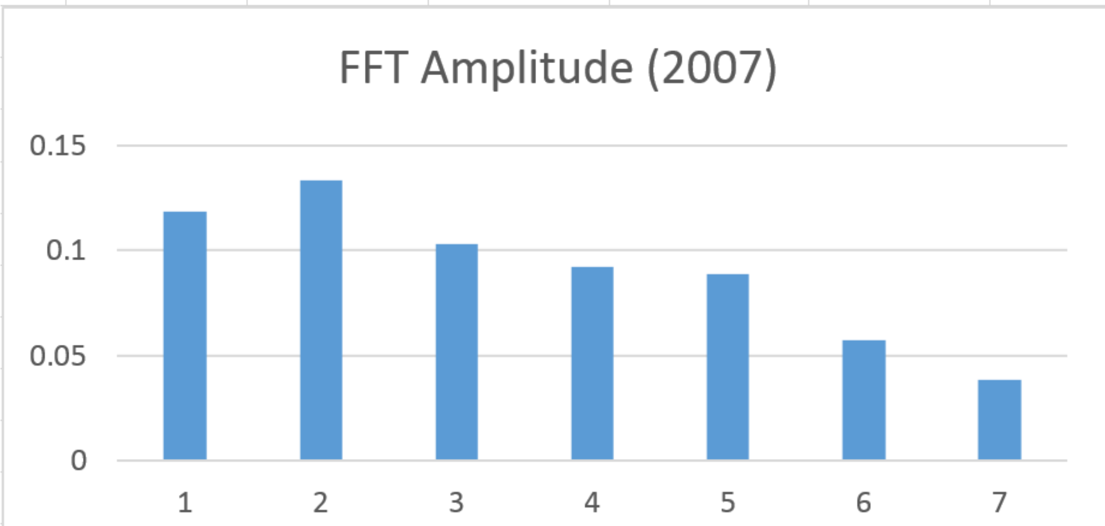
The FTT is the algorithm that computes the discrete Fourier transform (DFT). Here we have generated a table for 6 frequency components of the average data prices (the phase and amplitude are always have the number of cells in data sample which was 12) with amplitude and phase, however as a limitation of our data set, with such smaller decimals the amplitude remains a very small zero decimal as the measure of change is not large enough. But our amplitude values for 2007 are comparatively greater than our 1999 values in 5 steps out of 7.

Both FFT give zero phase for zero frequency components however.

Our phase spectrum measures the relative frequency of signals of each component against the Amplitude as an angle within the range of and and from the results suggests that within the Fourier transform the phase angle reaches the border of on two occasions. But since it is not based on the middle values, there is no equivalent distance to the middle so therefore none of the entries are complex conjugates of each other and therefore they all remain the same - i.e hence have 2.6965E+3-8

Here is a visual description comparing both amplitudes -the 2007 amplitude decreases over time whereas the 1999 amplitude increases over time though they both show normality.





For our Inverse Fourier transform which reconstrusts the Time series we use the subset of the frequency spectrum to give us

IDFT (1999)
-0.005
-0.007
-0.006
0
0
0
0
0
0.01
0.01
0.01
0.003
0

IDFT (2007)
0
0.01
0
0
0.01
0.01
0
0.02
0.02
0
0.01
0
-0.02

It is clear from our values that if we plot these numbers against our original time series for both years we would match something remotely similar.

3.2.3 Descriptive Statistics

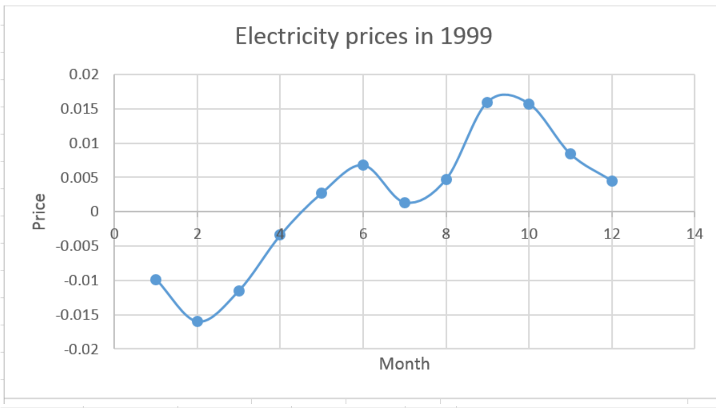
1999	
Mean	0.001619222
Standard Error	0.002931723
Median	0.003610629
Mode	#N/A
Standard Deviation	0.010155788
Sample Variance	0.00010314
Kurtosis	-0.653130888
Skewness	-0.319998625
Range	0.031947438
Minimum	-0.015979035
Maximum	0.015968403
Sum	0.019430663
Count	12
Largest(1)	0.015968403
Smallest(1)	-0.015979035
Confidence Level(95.0%)	0.00645268

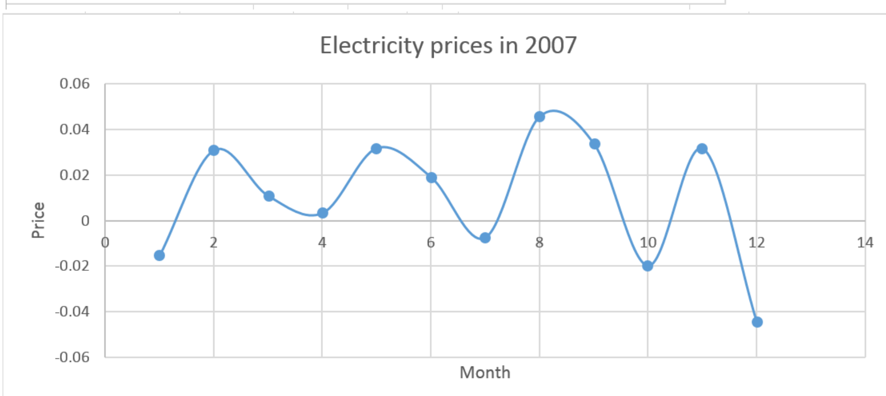
2007	
Mean	-0.015447657
Standard Error	0.006969442
Median	-0.017577799
Mode	#N/A
Standard Deviation	0.024142857
Sample Variance	0.000582878
Kurtosis	-0.238560978
Skewness	0.668816496
Range	0.077104248
Minimum	-0.045478281
Maximum	0.031625967
Sum	-0.18537189
Count	12
Largest(1)	0.031625967
Smallest(1)	-0.045478281
Confidence Level(95.0%)	0.015339639

For quality and assurance purposes, we have covered a table of statistical values to further understand the data set. To begin with, for 1999 we have our mean of 0.001619222 which is higher than our 2007 mean of -0.015447657 though we would not consider it a crucial measurement of the data central propensities since our set of values isn't a set of intervals but rather continuous time data points. Our 1999 of 0.001931723 which is less than the 2007 standard error of 0.006969442 though both are relatively small which suggests that the sample mean is a close representative of the true mean of our greater data set. Since Excel automatically measures uses a margin of error at 95 percent confidence, the standard error + or 2 standard errors from the true mean. We arrived at our median of as the middle value of the sample data. Our Standard deviation, which is an index of variability for both years is below 0.02 which is a calculation of how deviated the data is from the mean which is unsurprisingly small again. One advantage is that the SD maintains the same unit as our sample data but may be victim to the outliers. Similarly, with our negative skewed value of -0.3199 in 2009, the majority of our distribution (in theory) would be focused on the right of the graph which is reflected in the FTT amplitude graph whilst the skewness value of 0.6688 for 2007 is reflective of the 2007 FTT amplitude graph, which shows our data is asymmetric though skewness for discrete distributions. As an application quota,a goodness-of-fit test such as Dagostinos K-squared test would be recommended to better establish if our data sample comes from a more normally distributed pool. The Range, Minimum/Maximum and the Sum/Count is self-intuitive and remains supplementary to our data analysis.

3.3 Time series analysis

Below is a computation of the FFT in a time series of the data values of 1999 and 2007





It is clear from these graphs that we have significant variations about the trend. We can decipher a long term trend as the similar cyclical movements of both graphs and seasonal pattern over a period of 8 years suggests (though we have not discussed white noise fluctuations which we will cover shortly).

3.3.1 Stationarity tests

Stationary Test (1999)					
Test	Stat	P-Value	C.V.	Stationary?	5.0%
ADF					
No Const	-1.3	21.2%	-2.2	FALSE	
Const-Only	-1.4	62.6%	-4.0	FALSE	
Const + Trend	-2.9	0.2%	-1.6	TRUE	
Const+Trend+Trend^2	-20.8	0.0%	-1.6	TRUE	

Stationary Test (2007)					
Test	Stat	P-Value	C.V.	Stationary?	5.0%
ADF					
No Const	-3.4	0.6%	-2.2	TRUE	
Const-Only	-4.8	2.4%	-4.0	TRUE	
Const + Trend	-4.5	0.0%	-1.6	TRUE	
Const+Trend+Trend^2	-5.3	0.0%	-1.6	TRUE	

Using NUMXL to automate a Stationarity test of our data values which encompasses with and without trend, the P value for 1999 without accommodating for trend being high (above 20 percent) indicates there is not enough evidence to reject stationarity hence a FALSE statement, with trend we have our P values below 5 percent which has enough evidence to reject stationarity hence TRUE. For 2007 our P value without trend being less than 1 percent also indicates strong evidence against stationarity hence TRUE. With trend however, another low P value again suggests no stationarity. This suggests that - with exception to our data in 1999 not accommodating for trend - suggests a lack of stationarity because the series displays trends and also seasonality (or changes in variance) though the data can be manipulated using differencing to produce stationarity.

3.3.2 White noise test

White-noise Test (1999)					
Lag	Score	C.V.	P-Value	Pass?	5.0%
1	8.73	3.84	0.3%	FALSE	
2	10.74	5.99	0.5%	FALSE	
3	10.91	7.81	1.2%	FALSE	

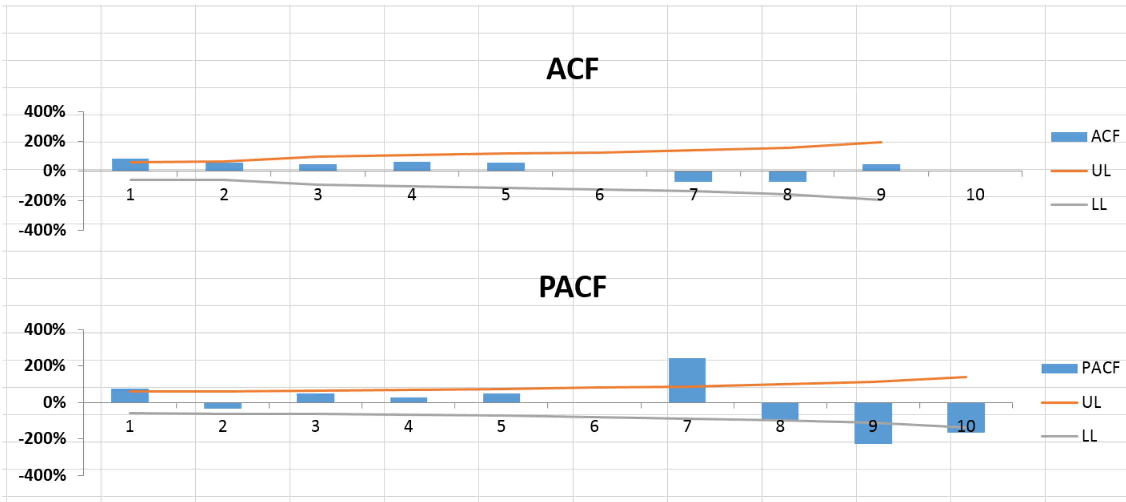
White-noise Test (2007)					
Lag	Score	C.V.	P-Value	Pass?	5.0%
1	1.97	3.84	16.1%	TRUE	
2	2.01	5.99	36.6%	TRUE	
3	2.86	7.81	41.4%	TRUE	

It is clear from the small P values of 1999 that there is white noise evident in the data and agrees with stationarity without trend in the year 1999. In 2007 however, the higher P values show that a lack of white noise must be down to the trend showing a stronger correlation of trend and seasonality.

3.3.3 Correlation functions

For the Year 1999 we can see our ACF and PACF values displayed below

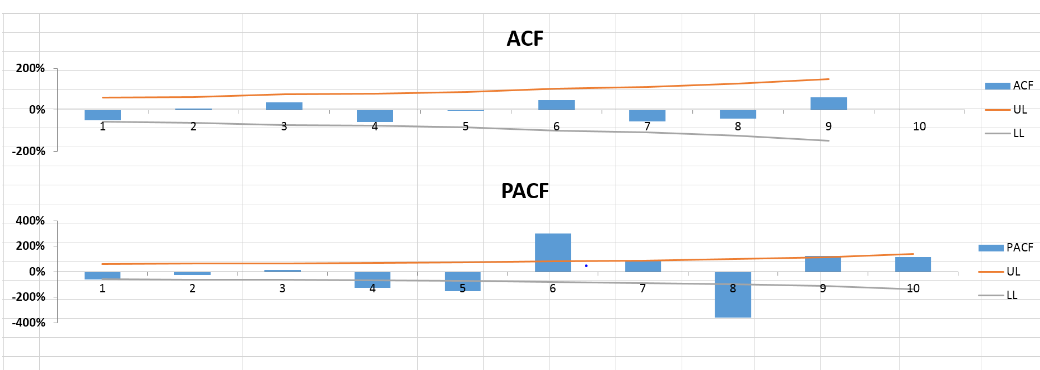
Correlogram Analysis (1999)						
Lag	ACF	UL	LL	PACF	UL	LL
1	81.53%	59.10%	-59.10%	76.50%	59.10%	-59.10%
2	52.00%	61.98%	-61.98%	-35.27%	61.98%	-61.98%
3	42.58%	95.66%	-95.66%	48.83%	65.33%	-65.33%
4	62.50%	106.95%	-106.95%	25.17%	69.30%	-69.30%
5	54.13%	114.77%	-114.77%	47.48%	74.08%	-74.08%
6	-3.05%	123.98%	-123.98%	0.00%	80.02%	-80.02%
7	-74.33%	136.42%	-136.42%	241.59%	87.65%	-87.65%
8	-75.29%	156.51%	-156.51%	-97.87%	98.00%	-98.00%
9	41.74%	194.17%	-194.17%	-226.86%	113.16%	-113.16%
10	#N/A	#N/A	#N/A	-166.77%	138.59%	-138.59%



For the Year 2007 we can see our ACF and PACF values displayed below

Correlogram Analysis (2007)

Lag	ACF	UL	LL	PACF	UL	LL
1	-50.50%	59.10%	-59.10%	-62.30%	59.10%	-59.10%
2	6.82%	61.98%	-61.98%	-24.34%	61.98%	-61.98%
3	35.03%	73.27%	-73.27%	15.63%	65.33%	-65.33%
4	-59.12%	77.87%	-77.87%	-129.25%	69.30%	-69.30%
5	-6.06%	86.19%	-86.19%	-155.16%	74.08%	-74.08%
6	45.37%	100.32%	-100.32%	298.21%	80.02%	-80.02%
7	-56.56%	109.94%	-109.94%	88.02%	87.65%	-87.65%
8	-42.20%	125.15%	-125.15%	-358.78%	98.00%	-98.00%
9	60.24%	148.62%	-148.62%	122.67%	113.16%	-113.16%
10	#N/A	#N/A	#N/A	113.13%	138.59%	-138.59%



Having computed and plotted the correlation functions for 10 lags along with the upper and lower bound values of the significance interval, we can note that for the Year 1999, at lag order 1, both the ACF and PACF are significant (100 percent) suggesting we have unit roots hence non-stationarity (our data for most of the lags show there is significant autocorrelation).

For 2007, the ACF and PACF fluctaute which does not make it likely to have unit roots which shows there is more stationarity in 2007 (which conflicts with our stationarity and white noise tests) which suggests the correlogram analysis is not a reliable tool to understand the interdependency of our data values. However we can also use this to suggest that - without differencing the data - price data correlations can change considerably over the years for external reasons (looking back at the history of electricity prices, World politics can affect the availability and demand for electricity and hence the prices will be affected).

3.3.4 Multiple Regression Trend and Seasonality

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.243677398							
R Square	0.059378674							
Adjusted R Square	-0.079972633							
Standard Error	0.040034222							
Observations	32							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	4	0.002731758	0.000682939	0.426107764	0.788426789			
Residual	27	0.04327395	0.001602739					
Total	31	0.046005708						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.029061396	0.019837582	1.464966667	0.154478449	-0.011641959	0.069764751	-0.011641959	0.069764751
X Variable 1	0.000318069	0.000772177	0.411911607	0.683656147	-0.001266307	0.001902445	-0.001266307	0.001902445
X Variable 2	0.022953122	0.020150708	1.139072712	0.264678462	-0.018392716	0.06429896	-0.018392716	0.06429896
X Variable 3	0.002806244	0.020076597	0.139776892	0.889873749	-0.03838753	0.044000019	-0.03838753	0.044000019
X Variable 4	0.012621706	0.020031999	0.630077208	0.533940635	-0.028480561	0.053723973	-0.028480561	0.053723973

Our intention here was to use the data to develop a multiple regression model to predict prices (both trend and seasonal components) using dummy variables to incorporate the seasonal factor. Instead of comparing the year 1999 and 2007, we predict prices as a whole and evaluate its accuracy. To create the model we made a column of the years and previous quarters (due to a lack of quarters in our original data we just randomly sampled them as four variables). We also create a column called Time which is just a running count of the variables. We then placed a column for all 32 prices of each respective year/quarter. Having run the regression on the analysis add-in on Excel, we see that our value for the Adjusted R squared is -0.079972633 is low, possibly suggesting our model isn't the best model nor a good predictor of prices. Our R square value of roughly 6 percent means our variables explain only 6 percent of the variation in our data, though this is only relevant in a single regression. Our Multiple regression value of roughly 0.24 is a correlation rate of the prices, which is clearly low.

4 Conclusion

So far we have observed the history and structure of the Nordic pool system pricing and introduce some important terminology of production methods relating to what economic and technical factors determine prices in the market. For our mathematical notation we explained the gist of a time series with use of distributed lag models. A further introduction of the Fourier transform listed the essential properties and axioms of periodicity in our data before we finally head to our data analysis where we compared price findings of two different years for various time series measures; suggesting that sample correlations are within the lines though following a random pattern, yet just because we might have white noise more evident in one year to show stationarity, the multiple regression model which we used to predict the prices over all eight years shows it does not fit the data perfectly. We can see that in our results and discussion, the huge uncertainty and the inability to define possible long term forecasts was because a more advanced analysis involving trade volume data would be required, with my suggestion being, using models involving wavelet transforms and Hurst components to utilize the independent behaviour of time series observations since other academic research besides my own have shown this to be fitting to many research complexities in social sciences.

5 Bibliography

References

[1] Walter, E. *Time Series Analysis of Electricity Prices* (Norwegian University of Science and Technology, 2011)

[2] Webpages of the European Energy Exchange (EEX) Available from www.eex.com

[3] Webpages of Nordic Pool Spot [Internet] Visited 19th July, 2014. Available from: www.nordpoolspot.com

[4] Coad, Dr Steve *Time Series* Lecture Notes for School of Mathematical Sciences, Queen Mary University of London

[5] Joel Feldman *Discrete-time Fourier Series and Fourier Transforms* Lecture Notes for Mathematics University of British Columbia, 2007.

[6] Peacock, J.A *Fourier Analysis* Lecture Notes for School of Physics and Astronomy, University of Roehampton session 2013/2014.

6 Excel Formulae

NUMXL automates the formulas which were used in this dissertation including

FAST FOURIER TRANSFORM

```
=DFT(B3:$B$13,1,$I$5,1)
```

INVERSE FAST FOURIER TRANSFORM

```
=IDFT($J$20:$J$26,$K$20:$K$26,12)
```

STATIONARITY

```
=ADFTest(Sheet1!$B$2:$B$13,1,3,1,1,2)
```

WHITE NOISE

```
=WNTest(Sheet1!$B$2:$B$13,1,$A83,2)
```

AUTO CORRELATION FUNCTION

```
=ACF(Sheet1!$E$2:$E$13,1,$A55)
```

PARTIAL AUTO CORRELATION FUNCTION

```
=PACF(Sheet1!$E$2:$E$13,1,$A55)
```