Mathematical Problems of General Relativity

Dr. Juan A. Valiente Kroon
School of Mathematical Sciences
Queen Mary, University of London
j.a.valiente-kroon@qmul.ac.uk

Basic details:

Core audience

First year PhD students in Applied Mathematics.

Format

Extended course.

Course details:

Introduction

General Relativity is sometimes described as the flagship of Mathematical Physics. The study of the mathematical properties of the solutions to the equations of General Relativity —the Einstein field equations— has experienced a great development in recent years. Work in this area has been based on a systematic use of the so-called *initial value problem for the Einstein field equations*. As such, it requires the use of ideas and techniques from various branches of Mathematics —especially Differential Geometry and partial differential equations (elliptic and hyperbolic). Current mathematical challenges in the area include the analysis of global existence of solutions to the Einstein field equations, the uniqueness of stationary black holes, the non-linear stability of the Kerr spacetime, and the construction of initial data sets of geometrical or of physical interest.

Objectives

The main objective of the course is to provide a discussion of General Relativity as an initial value problem. In addition, the course will serve as an introduction to applied methods of Differential Geometry and Partial Differential Equations. It is expected that at the end of the course the student will have a good overview of the main ideas and methods of modern mathematical General Relativity.

Topics to be covered

i. Review of the key ideas of General Relativity and the vacuum field equations.
ii. Selected topics of Lorentzian and Riemannian Differential Geometry including embeddings and submanifolds, intrinsic metrics, extrinsic curvature, Gauss-Codazzi and Codazzi-Mainardi equations.
iii. The constraint equations of General Relativity including selected topics of elliptic PDE theory.
iv. The evolution equations including selected topics of hyperbolic PDE theory.
v. Time independent solutions to the Einstein field equations.
vi. Energy and momentum in General Relativity.

Keywords

Initial value problem in General Relativity, initial data sets, evolution equations, static and stationary solutions, mass in General Relativity.

Prerequisites

A first course in General Relativity covering the Schwarzschild spacetime. Acquaintance with the basic notions of modern Differential Geometry will also be of advantage.

Format:

The format of the course will combine slides presentations with some blackboard calculations. The course will be based on personal notes on the above topics. A typeset version of the notes will be provided. Four problem sheets will be handed out, and solutions provided. The course contains a *light assessment* in the form of a take home set of problems to be handed back in two weeks.

Course homepage:

Resources for the course (slides, typeset notes, problem sheets, solutions, additional material) will be posted at:

www.maths.qmul.ac.uk/~jav/LTCC

Bibliography:

If required, a good general introduction to modern General Relativity is given in

R.M. Wald. *General Relativity*. Chicago University Press, 1984.

A number of introductory texts to various topics on mathematical General Relativity have appeared in recent years. Most notably,

Y. Choquet-Bruhat. *General Relativity and Einstein's equations*. Oxford University Press, 2008.

D. Christodoulou. *Mathematical problems of General Relativity. I.* Zürich lectures in Advanced Mathematics. European Mathematical Society, 2008.

A.D. Rendall. *Partial differential equations in General Relativity*. Oxford University Presss, 2008.

H. Ringström. *The Cauchy Problem in General Relativity*. European Mathematical Society, 2009.