Queen Mary
University of London
Science and Engineering

Radiation Detectors (SPA 6309)

Lecture 19

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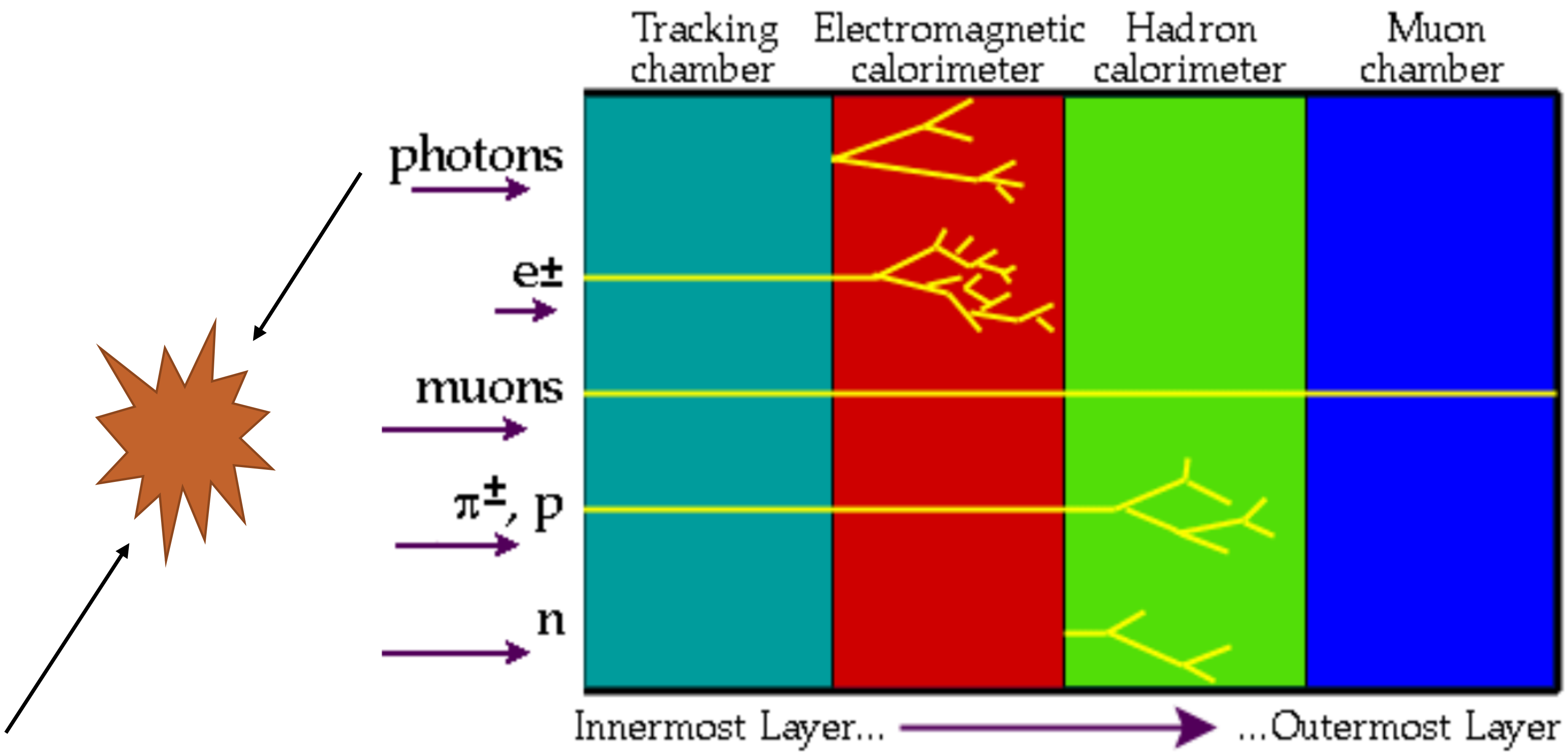
What is this lecture about?

- Tracking
  - Basic principles
  - Momentum resolution
  - Impact parameter resolution
  - Examples

Key points from previous lecture

- Homogeneous and sampling calorimeter key differences
- Crystalline scintillator for homogeneous EM calorimeters
- Many sensor options for sampling calorimeters (EM and hadronic)
- Significant variation of radiation length with material Z, smaller differences for hadronic interaction length λ.
- Challenge in hadron calorimeters of $\pi^0 \rightarrow \gamma\gamma$ generating EM showers which interact differently from the hadronic component (e/π ratio varies with incident hadron energy)

Particle ID (idealised)



Beams collide at centre of a GPD detector such as ATLAS or CMS

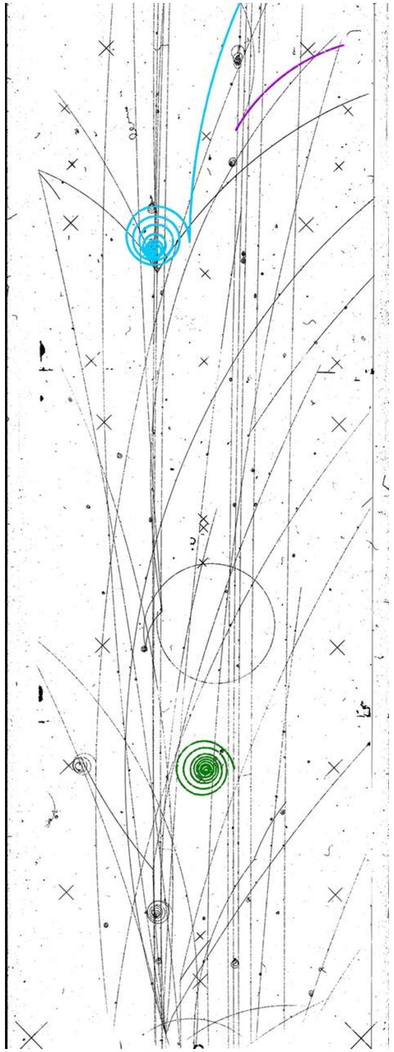
Minimum X_0 here

~ 20 X_0 and ~ 2 λ here

~ 7 λ here

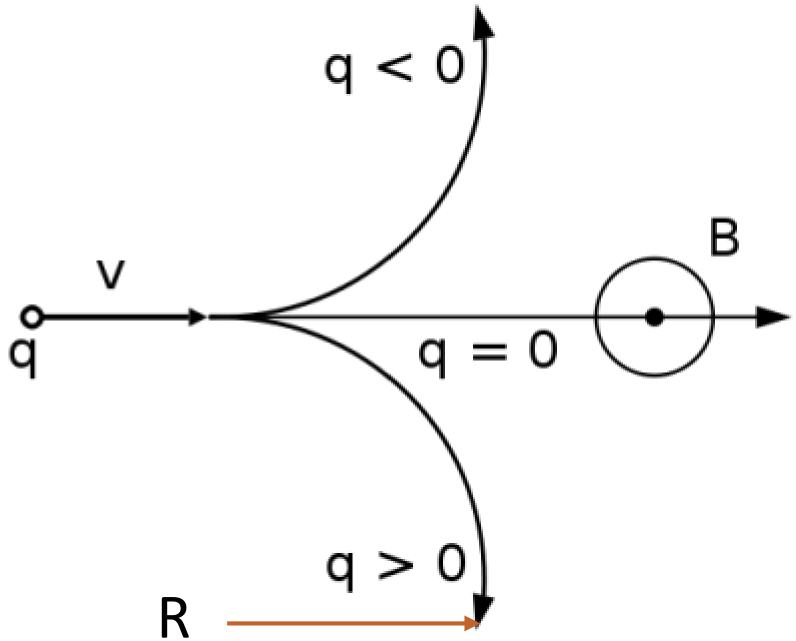
Muons with E > ~3 GeV

Tracking goals



1. Reconstruct charged-particle trajectories (tracks)
   - join points to form a track (pattern recognition)
   - measure direction and position
   - measure momentum and charge (with magnetic field)
   - Two major configurations:
     - inner spectrometers
     - muon systems
2. Reconstruct decay and interaction vertices
   - “primary” vertex: collision point where most particle are produced
   - “secondary” vertices:
     - decay of unstable particles
     - interaction with detector material
   - evaluate compatibility of tracks with primary vertex

Bending in a magnetic field



$$\vec{F} = q\vec{v} \times \vec{B}$$

Lorentz force: is the force on a point charge due to electromagnetic fields

$$\frac{mv^2}{r} = qvB$$

… for a particle in motion perpendicular to a constant B field

$$R = \frac{v\varepsilon}{eBc^2} \qquad \beta\varepsilon = pc \qquad R = \frac{p}{eB}$$

Using units such that the radius, *R*, is in metres, the magnetic field, *B*, is in Tesla and the momentum *p* is in GeV

$$p = 0.299792458RB \Longrightarrow p = 0.3RB$$

Path in a uniform field

The helix can be described in a parametric form

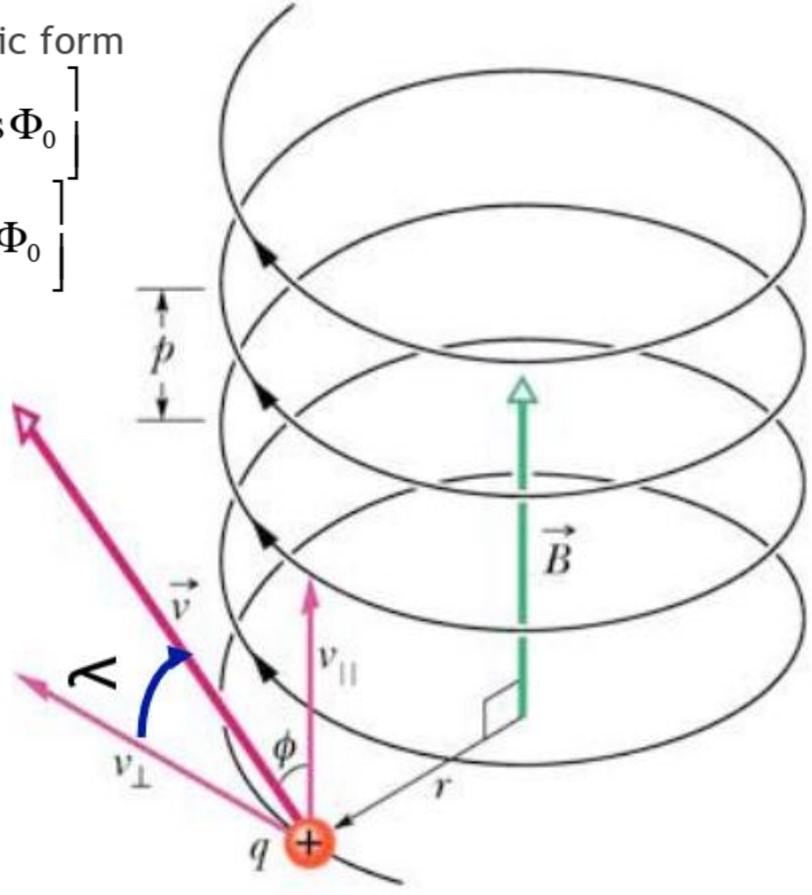
$$x(s) = x_0 + R\left[\cos\left(\Phi_0 + \frac{hs\cos\lambda}{R}\right) - \cos\Phi_0\right]$$

$$y(s) = y_0 + R\left[\sin\left(\Phi_0 + \frac{hs\cos\lambda}{R}\right) - \sin\Phi_0\right]$$

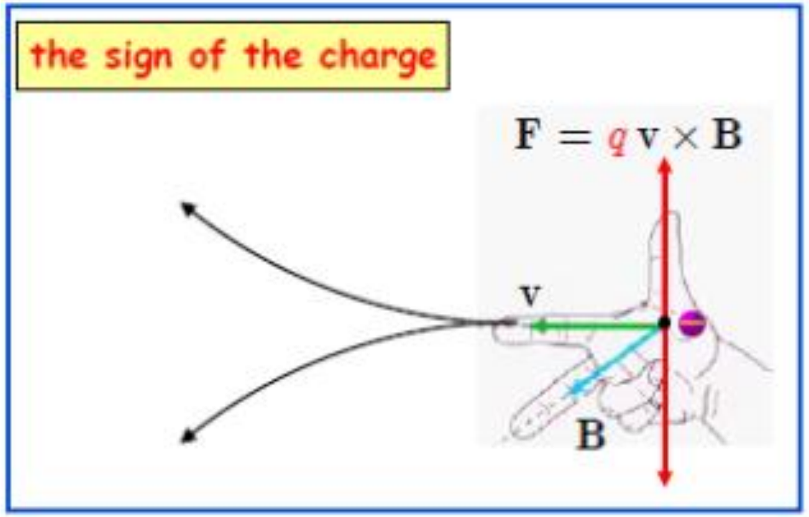
$$z(s) = z_0 + s\sin\lambda$$

λ is the dip-angle

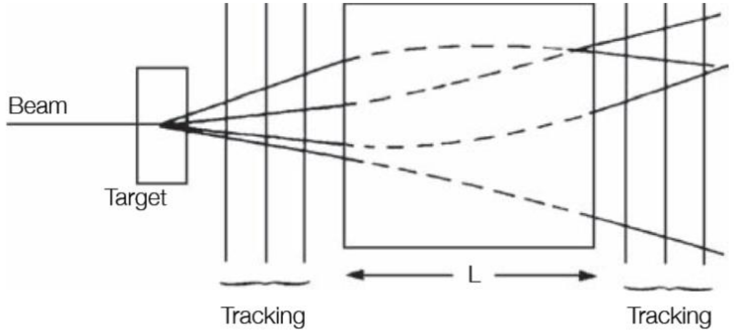
h=±1 is the sense
of rotation of the helix



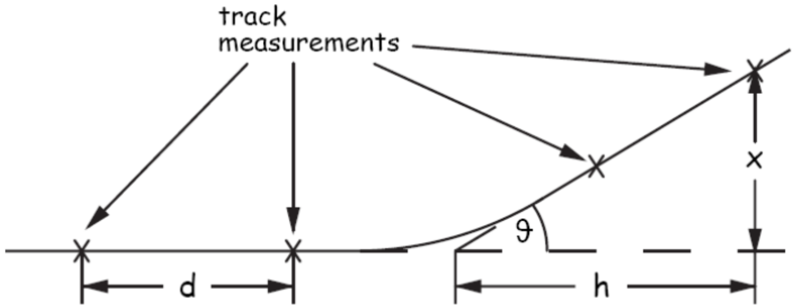
(b)

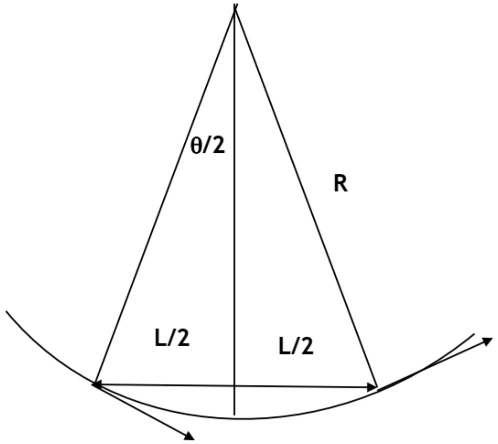


Tracking in a fixed target experiment



A spectrometer with a magnetised region (of length L)





A widely used method consists of the measurements of the bending of the track direction after crossing a magnetic field.

A particle moving across a region with a constant magnetic field will get a pulse of

$$\Delta p_T \approx pL/R = qBL$$

It is therefore possible to determine the momentum of a particle by the angular deviation after crossing a magnetic field:

$$\theta \approx \frac{\Delta p_T}{p_T} = \frac{q\int B dl}{p_T}$$

Determination of σ_p/p:

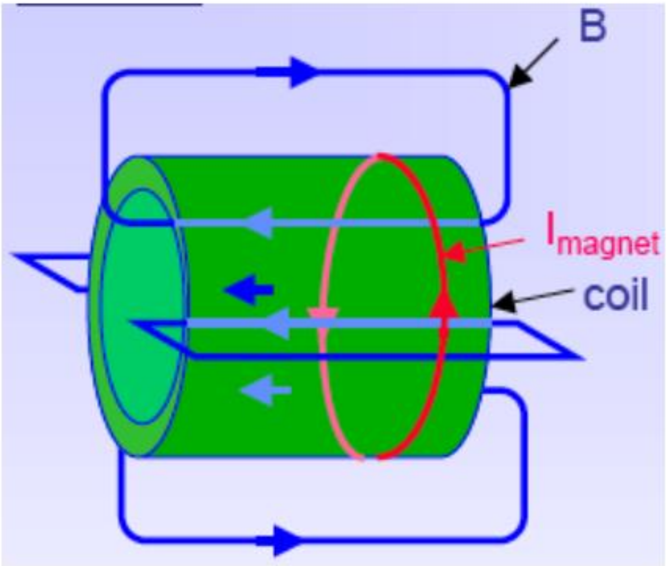
$$\vartheta = \frac{x}{h} \qquad \sigma_\vartheta = \frac{\sigma_x}{h}$$

$$\frac{\sigma_p}{p} = \frac{\sigma_\vartheta}{\vartheta} = \frac{\sigma_x}{h} \cdot \frac{p}{eBL}$$

Magnets at LEP and LHC

Solenoid

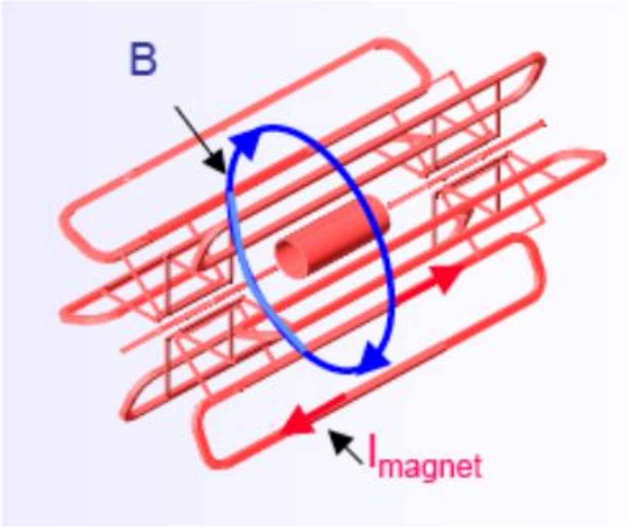
+ Large homogeneous field inside
- Weak opposite field in return yoke
- Size limited by cost
- Relatively large material budget



Examples:
- Delphi: SC, 1.2 T, 5.2 m, L 7.4 m
- L3: NC, 0.5 T, 11.9 m, L 11.9 m
- CMS: SC, 4 T, 5.9 m, L 12.5 m

Toroid

+ Field always perpendicular to p
+ Rel. large fields over large volume
+ Rel. low material budget
- Non-uniform field
- Complex structural design



Example:
- ATLAS: Barrel air toroid, SC, ~1 T, 9.4 m, L 24.3 m

Sensors inside magnetic field

- A widespread method, if it is possible to insert detectors inside the magnetic field, consists of measuring the sagitta of the particle trajectory:

$$s = R\left(1-\cos\frac{\theta}{2}\right) \approx R\frac{\theta^2}{8}$$

$$= \frac{qBL^2}{8p}$$

Taylor series expansion …

Note that resolution *decreases* as p *increases*, in contrast to the energy resolution improvement of a calorimeter

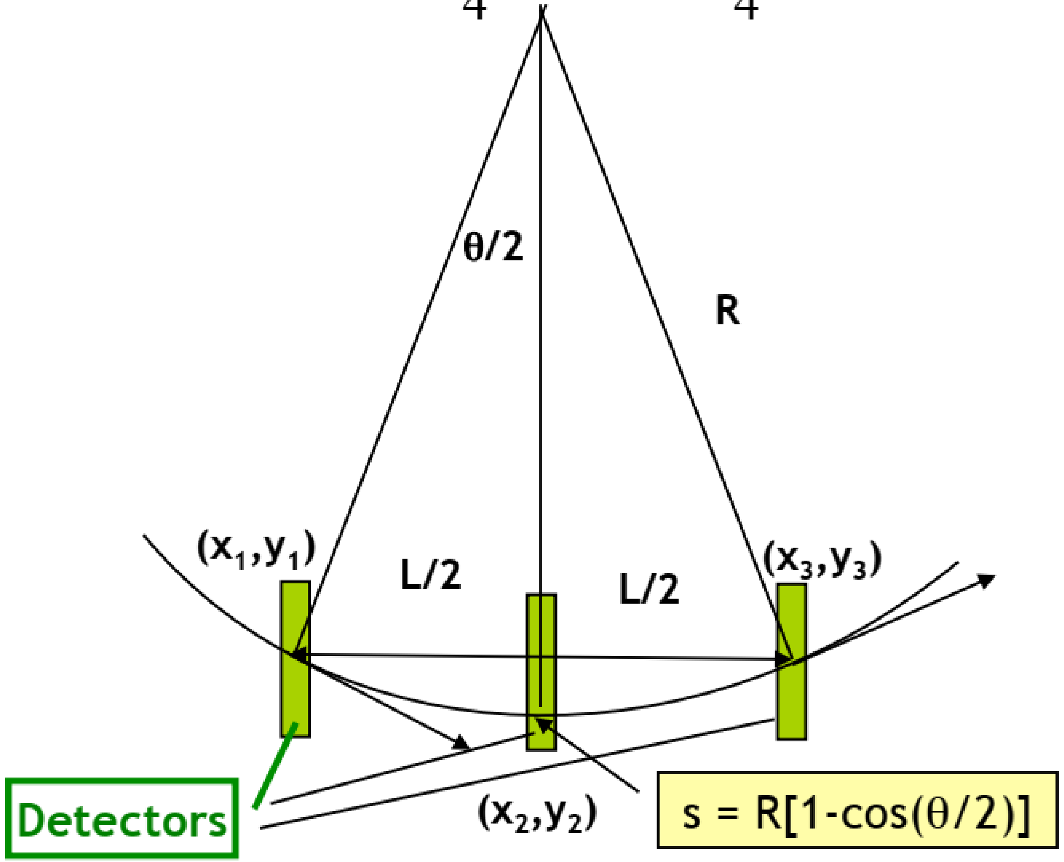
- And the relative momentum resolution is:

$$\frac{\sigma_p}{p} = \frac{\sigma_s}{s} = \frac{8p}{0.3BL^2}\sigma_s$$

- In the case the sagitta is measured by only three detectors:

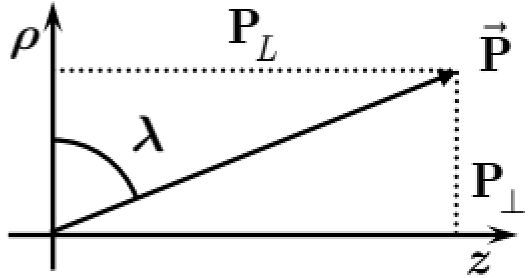
$$s = y_2 - \frac{1}{2}(y_1 + y_3)$$

$$\sigma^2_{s,\text{tracking}} = \frac{1}{4}\sigma^2_{y_1} + \sigma^2_{y_2} + \frac{1}{4}\sigma^2_{y_2}$$

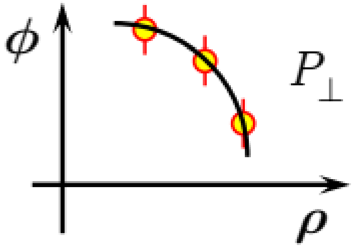


Sensors inside magnetic field

- The momentum of the particle is projected along two directions

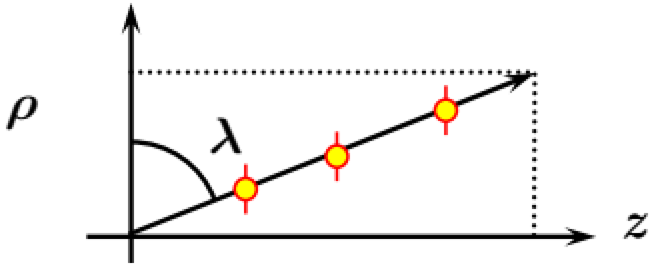


- In ρ - ϕ plane we measure the transverse momentum $P_\perp$



$$P_\perp = P\cos\lambda = 0.3BR$$

- In the ρ - z plane we measure the dip angle λ

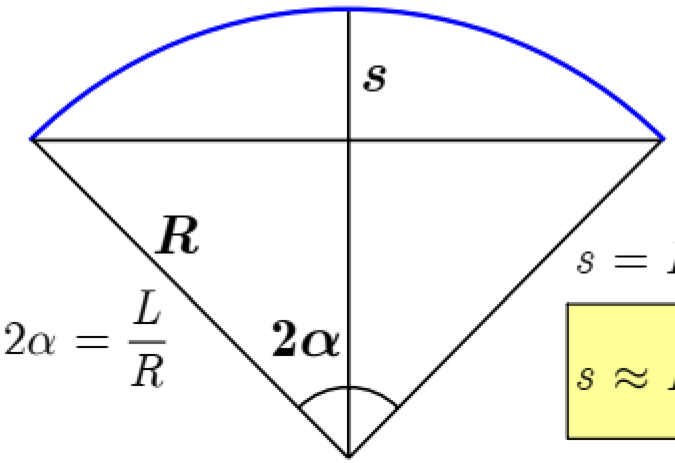


- Orders of magnitude

$$P_\perp = 1 \quad \text{GeV} \quad B = 2\,\text{T} \quad R = 1.67\,\text{m}$$

$$P_\perp = 10\,\text{GeV} \quad B = 2\,\text{T} \quad R = 16.7\,\text{m}$$

- The sagitta s



$$2\alpha = \frac{L}{R}$$

$$s = R(1 - \cos\alpha)$$

$$s \approx R\frac{\alpha^2}{2} = \frac{L^2}{8R}$$

- Assume a track length of 1 m

$$P_\perp = 1 \quad \text{GeV} \qquad s = 7.4\ \text{cm}$$

$$P_\perp = 10\,\text{GeV} \qquad s = 0.74\ \text{cm}$$

Sensors inside magnetic field

- Once we have measured the transverse momentum and the dip angle the total momentum is

$$P = \frac{P_\perp}{\cos\lambda} = \frac{0.3BR}{\cos\lambda}$$

- The error on the momentum is easily calculated

$$\frac{\partial P}{\partial R} = \frac{P_\perp}{R}$$

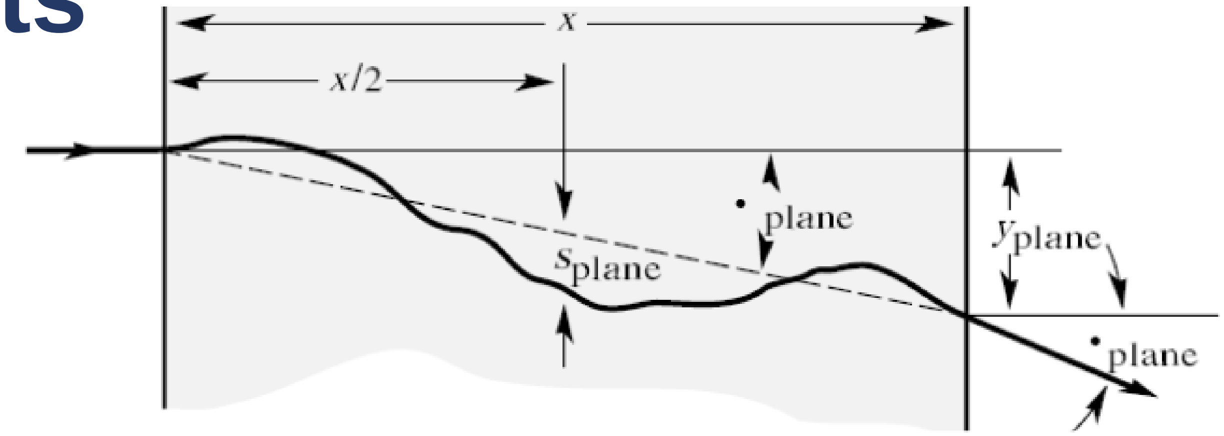
$$\frac{\partial P}{\partial \lambda} = -P_\perp \tan\lambda$$

$$\left(\frac{\Delta P}{P}\right)^2 = \left(\frac{\Delta R}{R}\right)^2 + (\tan\lambda\Delta\lambda)^2$$

- We need to study
  - The error on the radius measured in the bending plane ρ - ϕ
  - The error on the dip angle in the ρ - z plane
- We need to study also
  - Contribution of multiple scattering to momentum resolution

- Comment:
  - In an hadronic collider the main emphasis is on transverse momentum
  - Elementary processes among partons that are not at rest in the laboratory frame
  - Use of momentum conservation only in the transverse plane

Multiple scattering effects



$$\sigma_\phi \approx \frac{14\ \text{MeV}/c}{p}\sqrt{\frac{L}{X_0}}$$

$$\frac{\sigma_p}{p} = \frac{\sigma_R}{R} = \frac{\sigma_\phi}{\phi}$$

as $R = \dfrac{L}{\phi}$

At small momenta this limits resolution of momentum measurement ...

$$\frac{\sigma_p}{p} = \frac{\sigma_\phi}{\phi} = \frac{14\ \text{MeV}/c}{p}\sqrt{\frac{L}{X_0}} \cdot \frac{R}{L} = \frac{14\ \text{MeV}/c}{p}\sqrt{\frac{1}{LX_0}} \cdot \frac{p}{eB} \sim \frac{1}{\sqrt{LX_0}B}$$

momentum independent

- Summarising
  - No Multiple Scattering

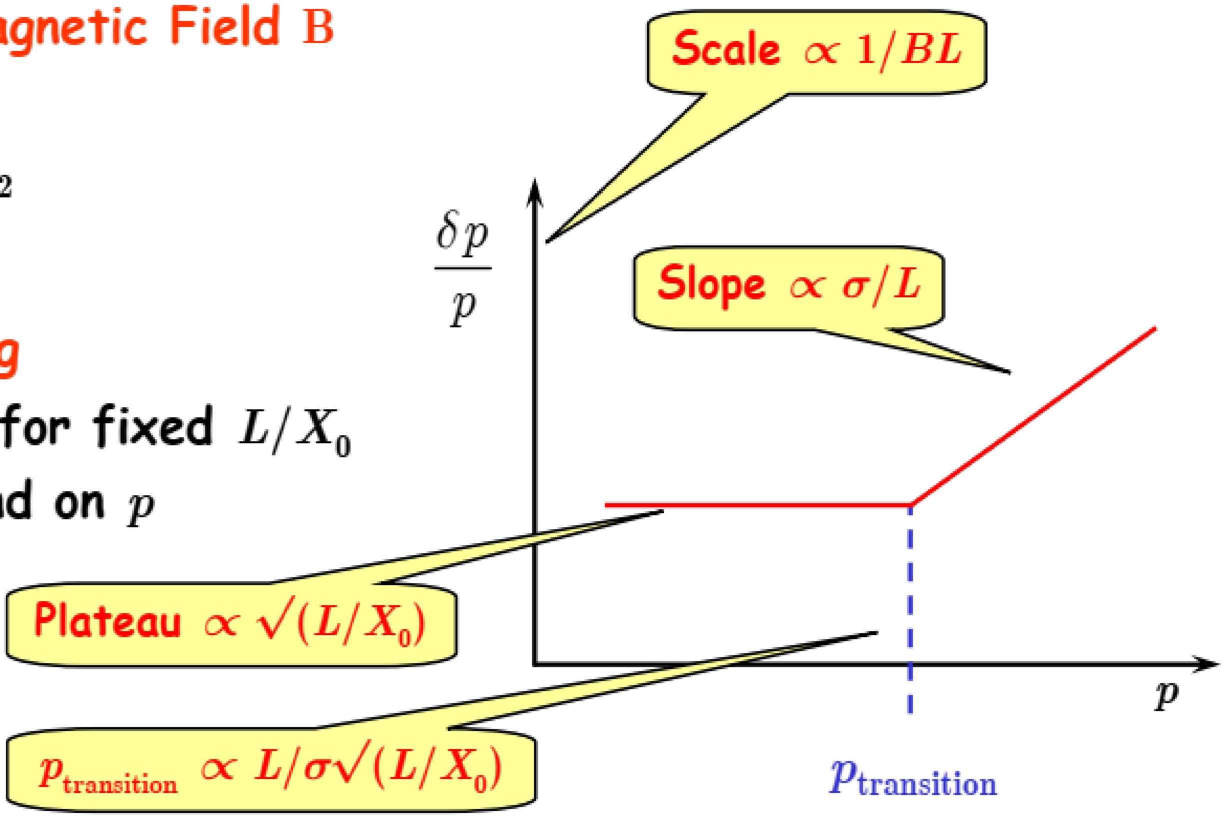
$$\frac{\delta p}{p^2} = \frac{\sigma}{0.3BL^2}\sqrt{4C_N} \qquad C_N = \frac{180N^3}{(N-1)(N+1)(N+2)(N+3)}$$

N is the number of sample points (typically 5 to 15)

  - With Multiple Scattering

$$\frac{\delta p}{p} \sim \frac{1}{0.3B}\frac{0.0136}{\beta}\sqrt{\frac{1.3}{X_0 L}}$$

- Please notice
  - Same dependence on Magnetic Field B
  - No Multiple Scattering
    - $\delta p/p$ improves as L^2
    - $\delta p/p$ worse as p
  - With Multiple Scattering
    - $\delta p/p$ improves as L for fixed L/X_0
    - $\delta p/p$ does not depend on p



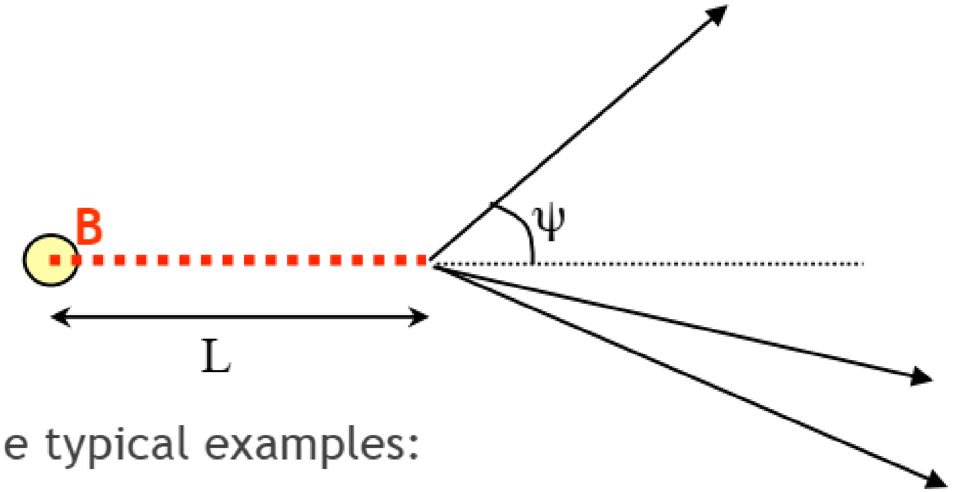
Reconstructing a vertex

In proximity of the interaction region, at first order, it is possible to neglect the curvature:

- focus on position and direction.
- Example: detection of short-lived particles

There is a group of particles with lifetimes of ~1 ps

The flight length L can be measurable: $L = \gamma\beta ct$



Some typical examples:

- Symmetric B-factory:
  Υ(4S) at rest
  γ=1.002, β=0.06, L~30 μm, ψ~1
- Asymmetric B-factory:
  e^- 9 GeV, e^+ 3.1 GeV
  γ=1.15, β=0.5, L~290 μm, ψ~1
- High energy collisions (LEP, Tevatron, LHC)
  γ=5-10, β=1, L=2-3 mm, ψ~0.1

"b tagging" very important at LHC, $c\tau$ is 455 μm for B^0 →

Impact parameter

It is useful to introduce the **impact parameter *d***, defined at the distance between the daugther particle trajectory and the mother particle production point:

$$d = L\sin\psi = O(\gamma\beta c\tau)\times O\left(\frac{1}{\gamma}\right) = O(c\tau)$$

for relativistic particles is approximately independent of the boost.

An experimental apparatus with decay vertex capabilities must be able to separate the production and devay vertices:

$$\sigma_L/L \ll 1$$

As a practical examples, let's consider a relativistic situation, where we can approximate:

$$\tan\psi \approx \psi \approx \sin\psi$$

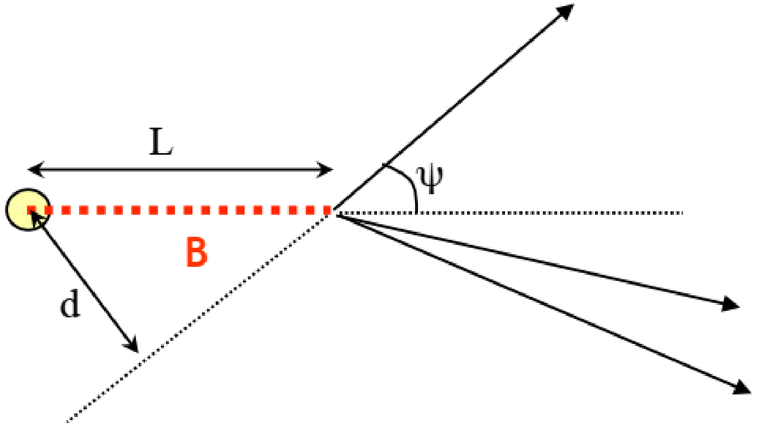
and set the x-axis direction along the mother particle flight direction.

This apparatus reconstructs trajectories

$$y=\tan\psi_i x+d_i$$

with measurement uncertainty σ_d

(σ_ψ is negligible in most practical cases)



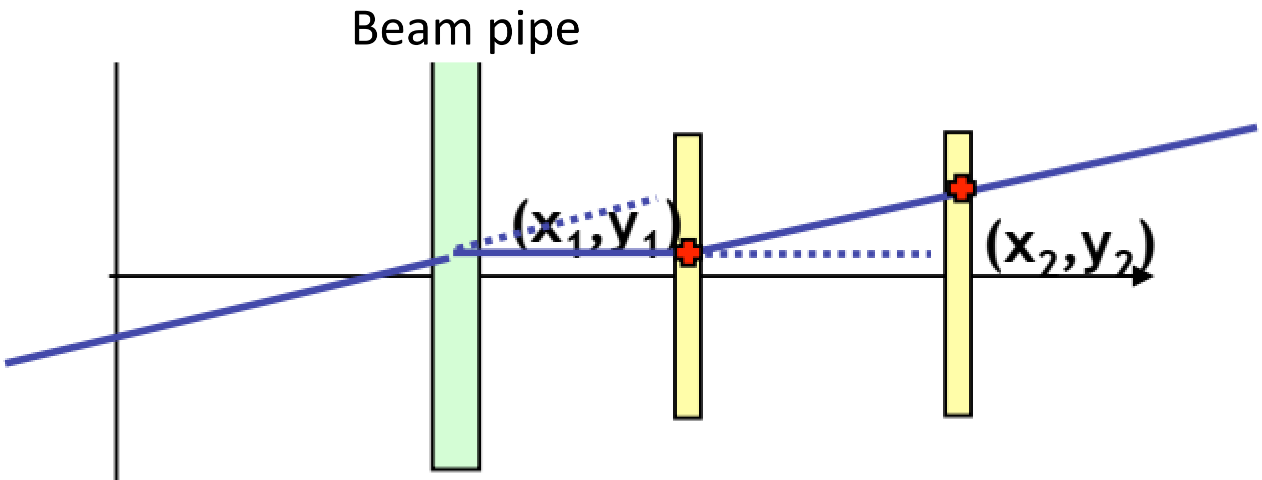
- The decay vertex position is given by the intersection of two trajectories:

$$\begin{cases} y = \tan\psi_1 x + d_1 \\ y = \tan\psi_2 x + d_2 \end{cases}$$

$$L = \frac{d_2 - d_1}{\tan\psi_1 - \tan\psi_2} \approx \frac{d_2 - d_1}{\psi_1 - \psi_2}$$

$$\frac{\sigma_L}{L} = \frac{\sqrt{2}\sigma_d}{d_2 - d_1} = O\left(\frac{\sigma_d}{d}\right) = O\left(\frac{\sigma_d}{c\tau}\right)$$

Impact Parameter resolution



Multiple scattering has a critical role in determining the impact parameter resolution.

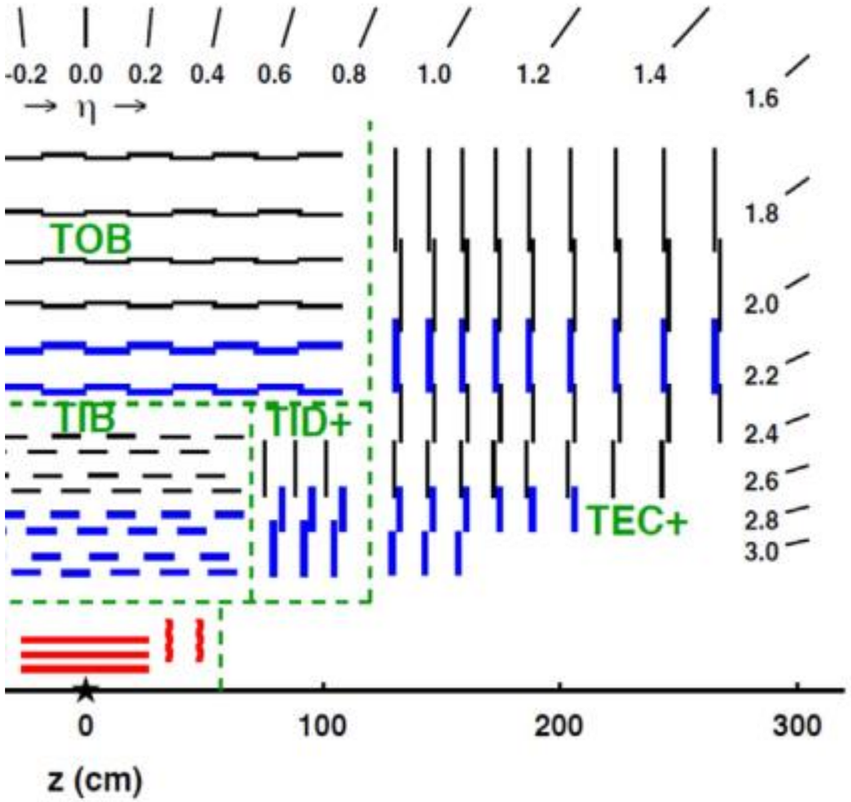
Every material layer crossed by the particle before reaching the detector introduce a random deviations with r.m.s.

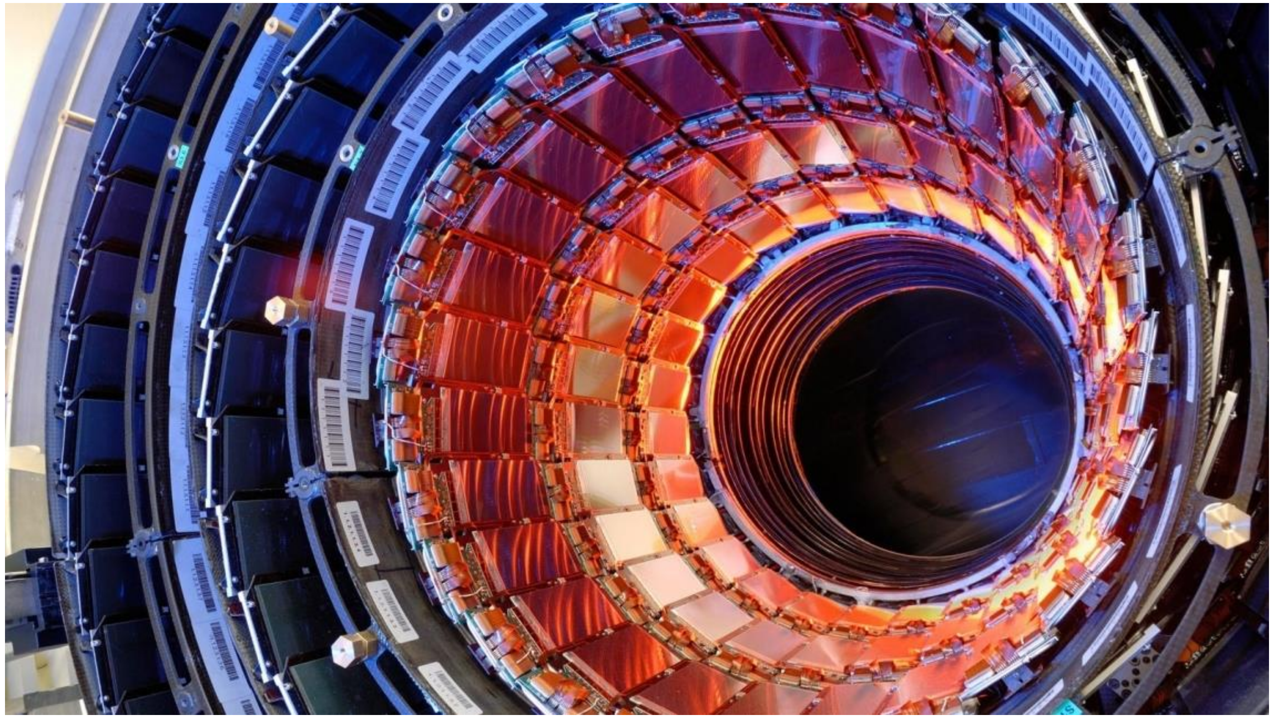
$$\theta_p = \frac{13.6\,\text{MeV}}{\beta c p} z \sqrt{\frac{X}{X_0}} \left(1 + 0.038 \ln \frac{X}{X_0}\right)$$

relevant for thin sensors

radiation length

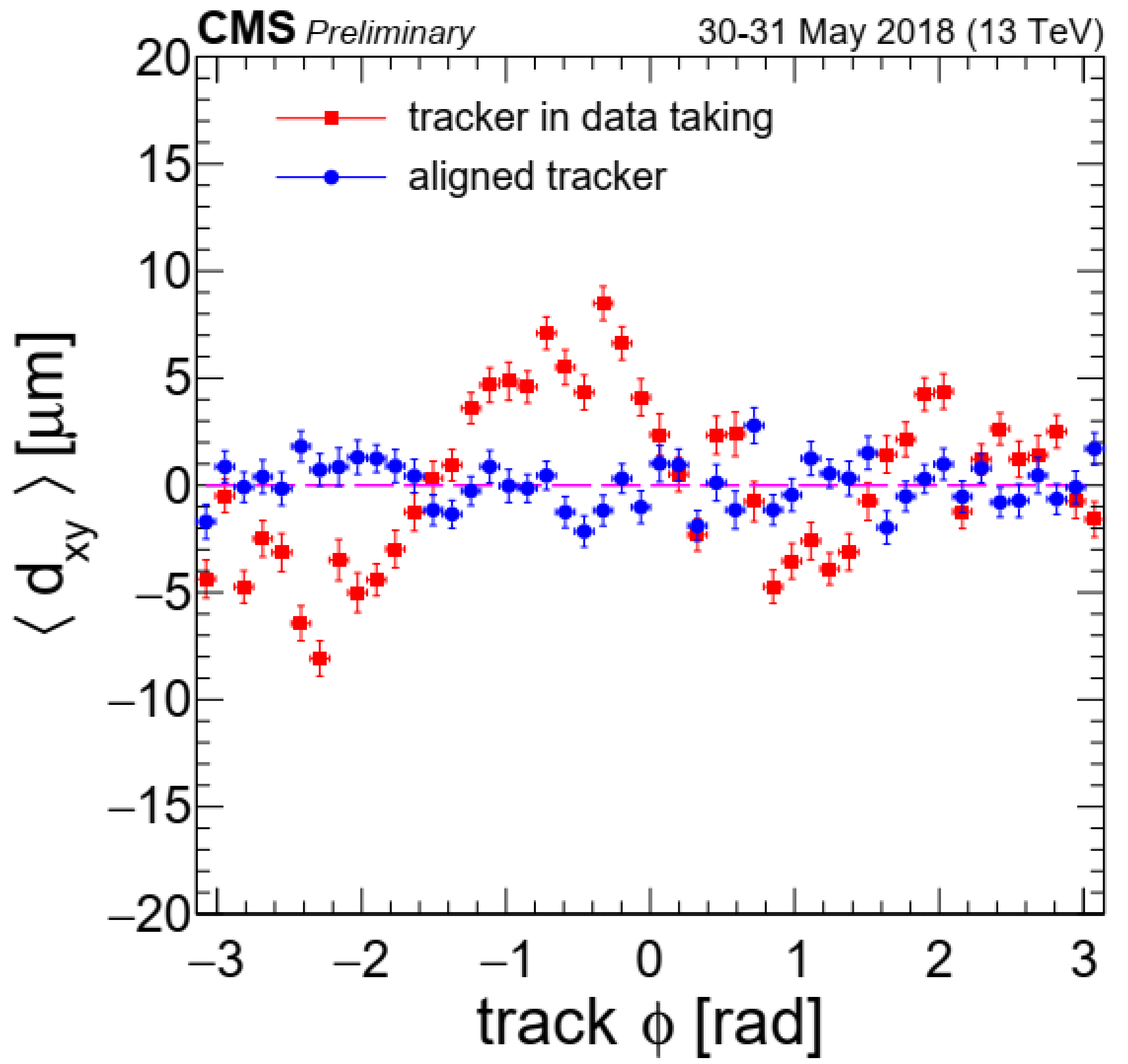
CMS



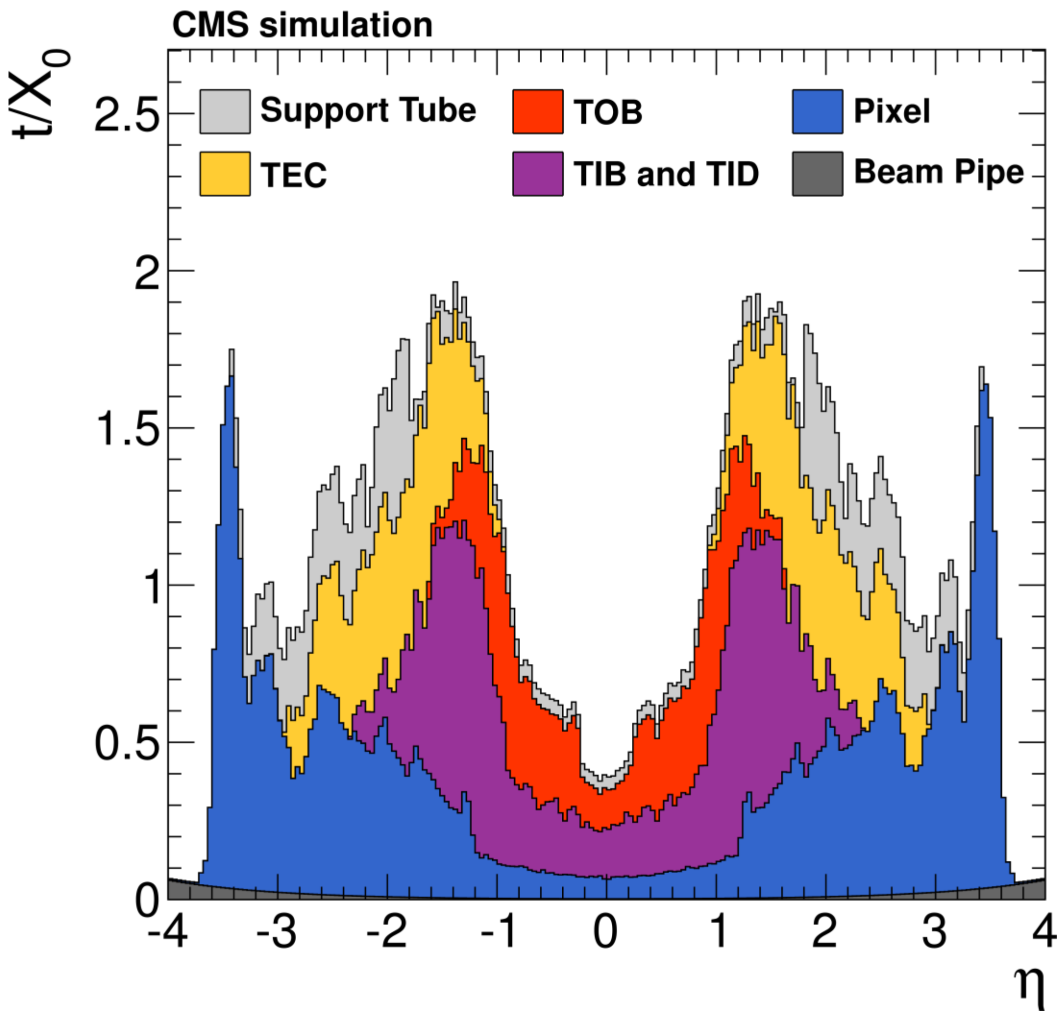


Silicon strip detectors in the “barrel” module (TIB and TOB).

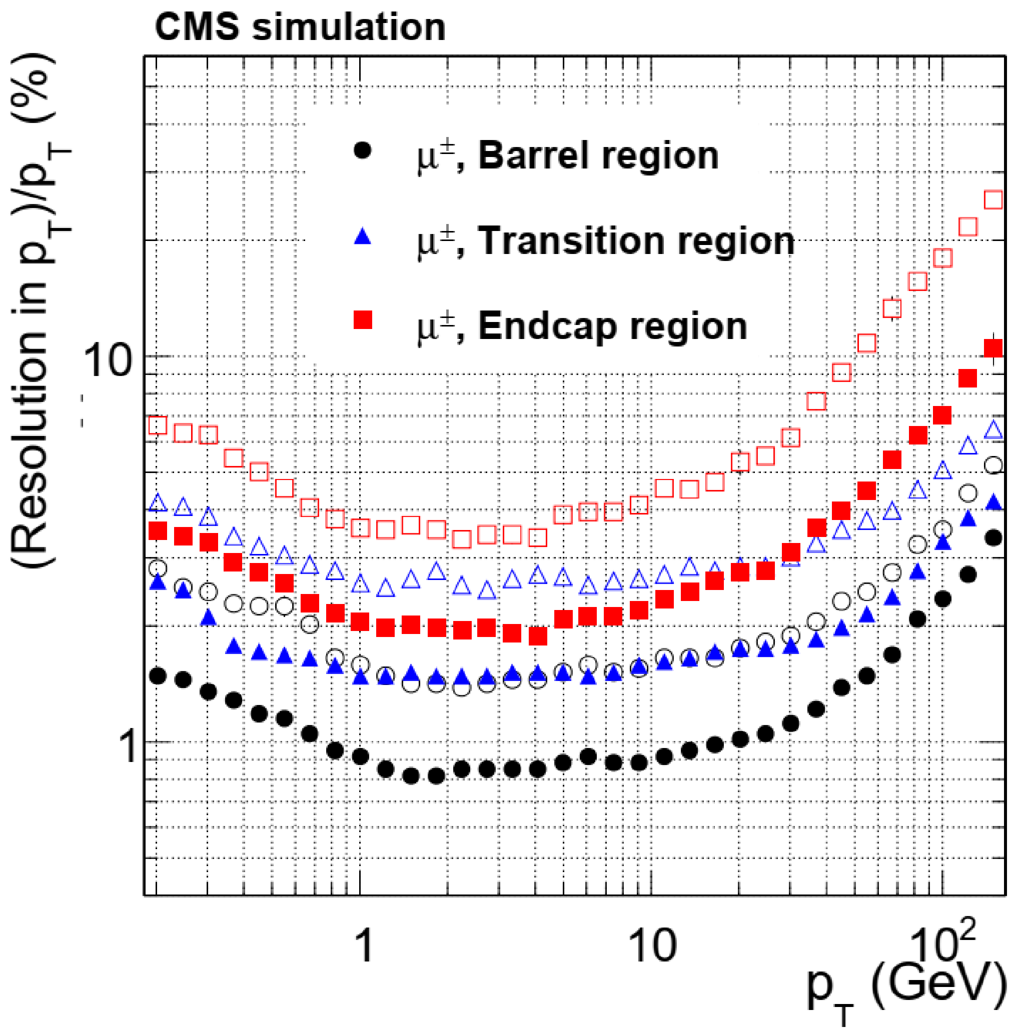
IP performance (2018)



Material budget



Simulated transverse momentum (p_T) resolution



More information

An excellent set of lecture slides by Francesco Ragusa, with a great deal more technical detail on track fitting and errors, is available from here:
http://www0.mi.infn.it/~ragusa/tracking_sns_28.05.2014.pdf

Also this open access paper: F Ragusa and L Rolandi, "Tracking at the LHC" *New J. Phys*. **9** (2007) 336
and this paper on pattern recognition: R Mankel, "Pattern Recognition and Event Reconstruction in Particle Physics Experiments", arXiv:physics/0402039v1 [physics.data-an] (2004)

The CMS plots are publicly available data: http://cms-results.web.cern.ch/cms-results/public-results/publications/